Energy and power signals :-

Energy signal :- (Joules)

A signal is said to be an energy signal if its total energy is finite and non-zero $0 < E < \infty$

CT :- $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$ (or) $E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt$

DT :- $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$ (or) $E = \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2$

POWER SIGNAL :- (Watts)

A signal is said to be an power signal if its normalized power is finite and non-zero $0 < P < \infty$

RMS = $\sqrt{\text{power}}$

CT :- $P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$ (or) $\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$

DT :- $P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

Comparison between Energy and power signal :-

Energy Signal	Power Signal
* Total energy is finite & non-zero $0 < E < \infty$	* Normalized power is finite & non-zero $0 < P < \infty$
* Non-periodic signals are energy signals	* periodic signals are power signals
* $E = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 \, dt$	* $P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t) \, dt$
* Power of the energy signal is zero over infinite time	* Energy of the power signal is infinite over infinite time

1) Prove that the power of an energy signal is zero over infinite time

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[\lim_{T \to \infty} \int_{-T}^{T} x^2(t)\, dt \right]$$

$$= \lim_{T \to \infty} \frac{1}{2T} \underbrace{\int_{-\infty}^{\infty} x^2(t)\, dt}$$

$$= \lim_{T \to \infty} \frac{1}{2T} \cdot E$$

$$P = \frac{1}{\infty} \cdot E = 0$$

power of an energy signal is zero over infinite time

2) prove that the energy of power signal is infinite over infinite time

$$E = \lim_{T \to \infty} \int_{-T/2}^{T/2} x^2(t)\, dt$$

Multiply and divide by T

$$= \lim_{T \to \infty} T \cdot \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$= \lim_{T \to \infty} T \underbrace{\left[\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt \right]}_{P}$$

$$= \lim_{T \to \infty} T \cdot P$$

$$= \infty \cdot P$$

$$\boxed{E = \infty}$$

Energy of the power signal is infinite over infinite time.

Problems :-

1) $x(t) = \cos t$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} \cos^2 t \, dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} \frac{1+\cos 2t}{2} \, dt$$

$$= \frac{1}{2}\left[\lim_{T\to\infty} \int_{-T}^{T} 1 \, dt + \lim_{T\to\infty} \int_{-T}^{T} \cancel{\cos 2t}^{\,0} \, dt\right]$$

$$= \frac{1}{2} \lim_{T\to\infty} [T+T] \Rightarrow \frac{1}{2} \lim_{T\to\infty} 2T$$

$$= \frac{1}{2} \cdot \infty \qquad \boxed{E = \infty} \text{ Joules}.$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \cos^2 t \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{1+\cos 2t}{2} \, dt$$

$$= \frac{1}{2}\left[\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt + \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \cancel{\cos 2t}^{\,0} \, dt\right]$$

$$= \frac{1}{2} \lim_{T\to\infty} \frac{1}{2\cancel{T}} [2\cancel{T}]$$

$$P = \frac{1}{2} \text{ Watts}$$

2) Determine power & RMS value of the signal :-

$$x(t) = e^{j\alpha t} \cos \Omega_0 t \qquad e^{j\alpha t} = 1$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t) \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \cos^2 \Omega_0 t \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{1+\cos 2\Omega_0 t}{2} \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \frac{1}{2} \int_{-T}^{T} dt$$

$$= \lim_{T\to\infty} \frac{1}{4T} [2T]$$

$P = 1/2$ Watts

$$Rms = \sqrt{power}$$
$$= \sqrt{1/2}$$

(3) $x(n) = e^{j\left(n\pi/2 + n\pi/8\right)}$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} 1$$

$$\sum_{n=-N}^{N} 1 = N_2 - N_1 + 1$$

$$= \lim_{N\to\infty} \sum_{n=-N}^{N} (1)$$

$$= \lim_{N\to\infty} (N+N+1)$$

$$= \lim_{N\to\infty} (2N+1)$$

$\boxed{E = \infty}$ joules.

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} N-(-N)+1$$

$$= \lim_{N\to\infty} \frac{1}{\cancel{2N+1}} \cancel{(2N+1)}$$

$P = 1$ Watts

(4) what is the total energy of DT Signal $x(n)$ which takes the value of unity at $n = -1, 0, 1$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-1}^{1} |1|^2 \Rightarrow \sum_{n=-1}^{1} 1$$

$$\sum_{n=-N_1}^{N_2} 1 = N_2 - N_1 + 1$$

$$= 1 - (-1) + 1$$

$E = 3$ joules

(5) Determine energy & power of the signal :-

$$x(t) = e^{j\left(2t + \pi/4\right)}$$

$P = 1$ Watts

$E = \infty$ joules