Tautology:

A statement formula which is true always irrespective of the truth values of the individual variables is called a tautology.

Eg:
$P \vee \neg P$ is a tautology.

Contradiction:

A statement formula which is always false is called a contradiction (or) absurdity.

Eg:
$P \wedge \neg P$ is a contradiction.

Contingency:

A statement formula which is neither tautology nor contradiction is called contingency.

Eg:
$P \leftrightarrow Q$ is contingency.

1. Show that $[\neg P \wedge (P \wedge q)] \rightarrow q$ is a tautology.

p	q	$\neg P$	$P \wedge q$	$\neg P \wedge (P \wedge q)$	$[\neg P \wedge (P \wedge q)] \rightarrow q$
T	T	F	T	F	T
T	F	F	F	F	T
F	T	T	F	F	T
F	F	T	F	F	T

Since all the entries in the resulting column is true, the given expression is a tautology.

2. Show that $(P \wedge Q) \wedge \neg (P \vee Q)$ is a contradiction.

P	Q	$P \wedge Q$	$P \vee Q$	$\neg (P \vee Q)$	$(P \wedge Q) \wedge \neg (P \vee Q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Since all the entries in the resulting column are F, the given expression is a contradiction.

3. Identify the given expression

$\neg(P \vee Q) \vee (\neg P \vee \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$\neg P \vee \neg Q$	$\neg(P \vee Q) \vee (\neg P \vee \neg Q)$
T	T	F	F	T	F	F	F
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	F	T	T	T

Since the entries in the resulting column are T as well as F, the given expression is a contingency.

Examine whether $[(P \vee Q) \to r] \leftrightarrow [\wedge r \to \sim(P \vee q)]$ is a tautology?

Equivalence :

Two statement formulas P and Q are equivalent iff $P \leftrightarrow Q$ is a tautology. It is denoted by $P \Leftrightarrow Q$.

Show that the propositions are logically equivalence for the following. (i) $P \to Q \Leftrightarrow \neg P \vee Q$

(ii) $P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$

(i) $P \to Q \Leftrightarrow \neg P \vee Q$

P	Q	$P \to Q$	$\neg P$	$\neg P \vee Q$	$P \to Q \Leftrightarrow \neg P \vee Q$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Since $(P \to Q) \leftrightarrow (\neg P \vee Q)$ is a tautology.

Hence $(P \to Q) \Leftrightarrow (\neg P \vee Q)$

(ii) $P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$

P	Q	R	$Q \vee R$	$P \to (Q \vee R)$	$P \to Q$	$P \to R$	$(P \to Q) \vee (P \to R)$	$P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
F	T	T	T	T	T	T	T	T
T	F	F	F	F	F	F	F	T
F	F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Here $[P \to (Q \vee R)] \leftrightarrow (P \to Q) \vee (P \to R)$ is a tautology.

Hence $P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$.

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