SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade (Cycle III)
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT302 – TRANSMISSION LINES AND ANTENNAS

III YEAR/ V SEMESTER

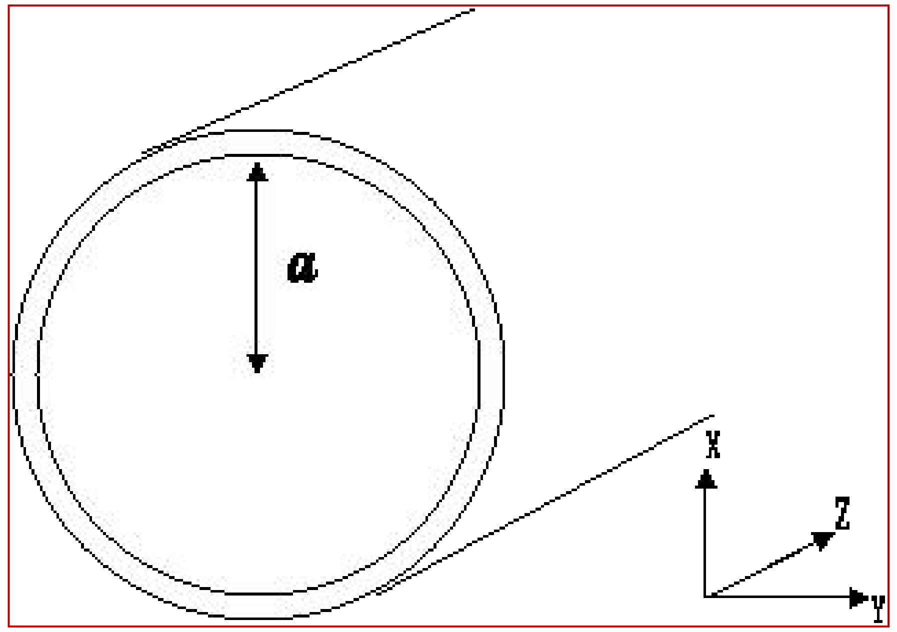
UNIT 5 – CIRCULAR WAVEGUIDES AND RESONATORS

TOPIC – SOLUTION OF FIELD EQUATIONS IN CYLINDRICAL CO-ORDINATES

How to find the fields of a circular waveguide ?

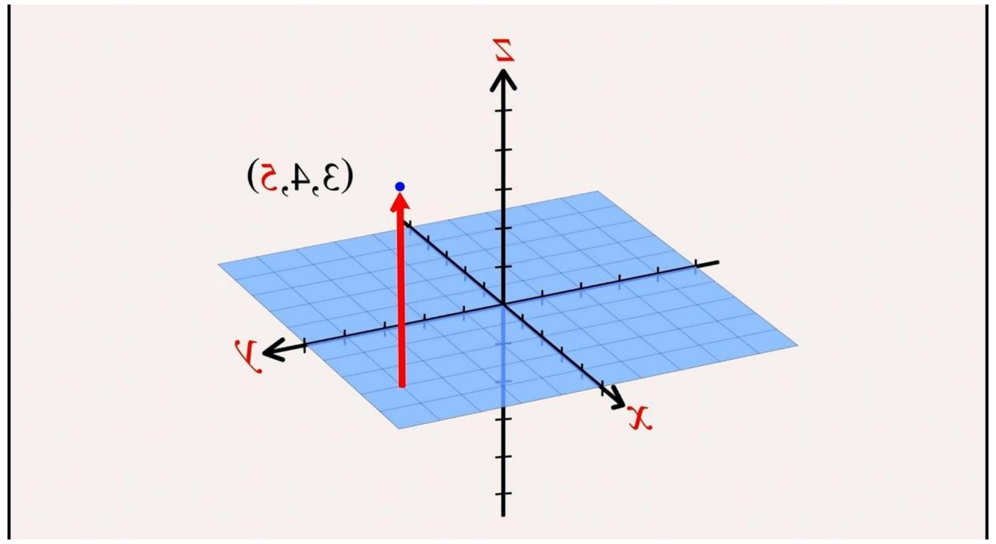
SOLUTION OF FIELD EQUATIONS

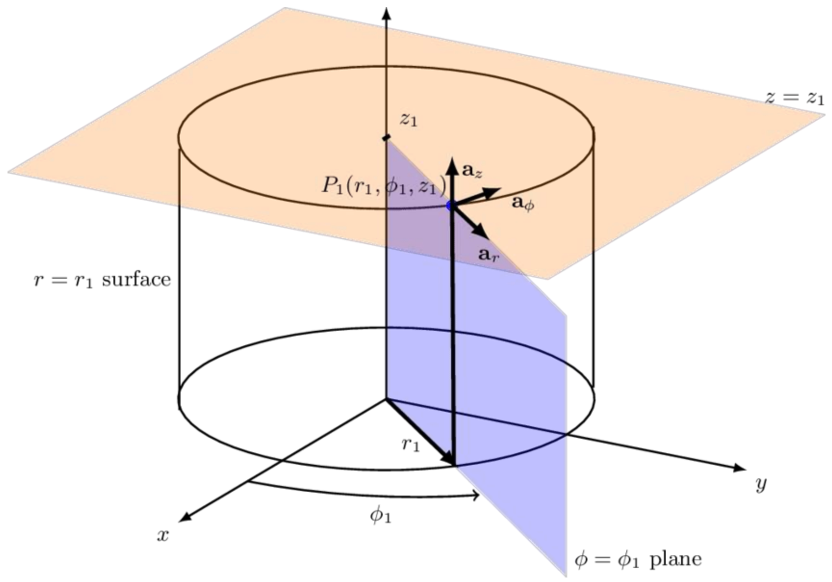
- Figure shows a cylindrical waveguide which is simply a hollow tube with circular cross section of radius ***a*** and extending along the Z direction.
- There are two sets of modes, TE and TM modes, which can propagate in a cylindrical waveguide.



COMPARISION – CARTESIAN AND CYLINDRICAL CO-ORDINATE SYSTEMS

- While Cartesian coordinates are attractive because oftheir simplicity, there are many problems whose symmetry makes it easier to use a different system of coordinates.
- For example, there are times when a problem has cylindrical symmetry (the fields produced by an infinitely long, straight wire, for example)
- In this case it is easier to use cylindrical coordinates.





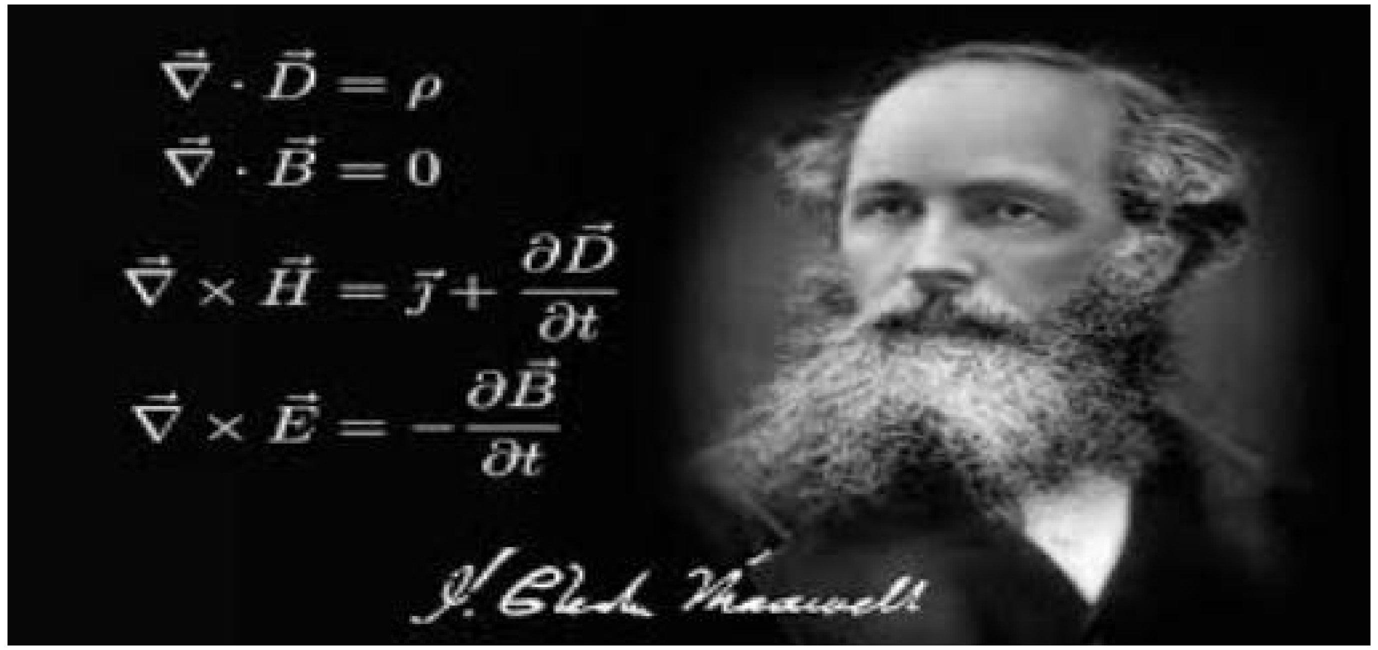
SOLUTION OF FIELD EQUATIONS

From Maxwell's Equations, for non – conducting medium inside the waveguide

$\hat{z}$

$$\nabla \times E = -j\omega\mu H$$

$$\nabla \times H = j\omega\varepsilon E$$



SOLUTION OF FIELD EQUATIONS

In Matrix Form

$\hat{z}$

$$\nabla \times H = \begin{vmatrix} \frac{1}{\rho}\vec{a}_\rho & \vec{a}_\phi & \frac{1}{\rho}\vec{a}_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ H_\rho & \rho H_\phi & H_z \end{vmatrix}$$

SOLUTION OF FIELD EQUATIONS

Solving,

solving we get

$$\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \bar{\gamma} H_\phi = j\omega\varepsilon E_\rho \rightarrow (1)$$

$$-\bar{\gamma} H_\rho - \frac{\partial H_z}{\partial \rho} = j\omega\varepsilon E_\phi \rightarrow (2)$$

$$\frac{1}{\rho}\left[\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi}\right] = j\omega\varepsilon E_z \rightarrow (3)$$

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In Matrix form

$\hat{z}$

$$\nabla \times E = \begin{vmatrix} \frac{1}{\rho}\vec{a}_\rho & \vec{a}_\phi & \frac{1}{\rho}\vec{a}_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ E_\rho & \rho E_\phi & E_z \end{vmatrix}$$

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SOLUTION OF FIELD EQUATIONS

Solving,

$\hat{z}$

Solving we get

$$\frac{\partial E_z}{\rho\,\partial \phi} + \gamma E_\phi = -j\omega\mu H_\rho \rightarrow (4)$$

$$-\gamma E_\rho - \frac{\partial E_z}{\partial \rho} = -j\omega\mu H_\phi \rightarrow (5)$$

$$\frac{1}{\rho}\left[\frac{\partial(\rho E_\phi)}{\partial \rho} - \frac{\partial E_\rho}{\partial \phi}\right] = -j\omega\mu H_z \rightarrow (6)$$

ACTIVITY

IF

3 + 5 + 6 = 151872
5 + 5 + 6 = 253094
5 + 6 + 7 = 303585
5 + 5 + 3 = 251573

Then

9 + 4 + 7 = ??????

SOLUTION OF FIELD EQUATIONS

These equations are simultaneously solved to get solution.

Ex: From Eq (1)

$$E_\rho = \frac{1}{j\omega\varepsilon}\left[\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \gamma H_\phi\right]$$

subs. in eqn (5)

$$-\gamma \times \frac{1}{j\omega\varepsilon}\left[\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \gamma H_\phi\right] - \frac{\partial E_z}{\partial \rho} = -j\omega\mu H_\phi$$

Rearranging, we get

$$h^2 H_\phi = -j\omega\varepsilon\left[\frac{\partial E_z}{\partial \rho} - \frac{\gamma}{\rho}\frac{\partial H_z}{\partial \phi}\right] \rightarrow (7)$$

SOLUTION OF FIELD EQUATIONS

IIIly

$$h^2 H_\rho = \frac{j\omega\varepsilon}{\rho}\frac{\partial E_z}{\partial \phi} - \bar{\gamma}\frac{\partial H_z}{\partial \rho}$$

$$h^2 H_\phi = -j\omega\varepsilon \frac{\partial E_z}{\partial \rho} - \frac{\bar{\gamma}}{\rho}\frac{\partial H_z}{\partial \phi}$$

$$h^2 E_\rho = -\bar{\gamma}\frac{\partial E_z}{\partial \rho} - \frac{j\omega\mu}{\rho}\frac{\partial H_z}{\partial \phi}$$

$$h^2 E_\phi = \frac{-\bar{\gamma}}{\rho}\frac{\partial E_z}{\partial \phi} + j\omega\mu\frac{\partial H_z}{\partial \rho}$$

where $h^2 = \bar{\gamma}^2 + \omega^2\mu\varepsilon$

CONCLUSION

There are two possible waves can propagate through circular waveguides

1. TE Waves – Electic field in the direction of propagation is zero(Ez =0)
2. TM Waves – Magnetic field in the direction of propagation is zero(Hz =0)

ASSESSMENT

1. What is the need for cylindrical co-ordinate system?
2. What are the modes can propagate in circular waveguides?
3. Mention the Maxwell's Equations used in circular waveguide field analysis.
4. What is the value of propagation constant in waveguide analysis?
5. State the basic field equations of circular waveguides.

THANK YOU