PROPOSITION :

A proposition (or statement) is a declarative sentence which is either true or false but not both.

Eg:
New Delhi is the capital of India (True)
Moscow is the capital of spain (False)

NOT PROPOSITION :

Questions, commands, exclamation are not proposition.

eg: close the door. (Command)
what a wonderful city ? (exclamation)
Do you speak Hindi ? (question)

TYPES OF PROPOSITION :

SIMPLE PROPOSITION :

A declarative sentence which cannot be further split up into simple sentences are called primary or atomic or primitive statements.

Eg: Lotus is a flower.

COMPOUND PROPOSITION:

Statements which contain one or more primary statements and some connectives are called compound or molecular or composite statements.

Eg: p: Tiger is a wild animal and it is national animal of India.

CONNECTIVES:

Connective is an operation which is used to connect two or more than two statements. There are five basic connectives.

TRUTH TABLE:

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Logical connectives	Name	Symbols	Type of operator
① NOT	Negation (or) Denial	$\neg$ (or) $\sim$	Unary
Eg: p: It is raining today; $\neg p$: It is not raining today			
② AND	Conjunction	$\wedge$	Binary
Eg: p: It is cold; q: It is raining $p \wedge q$: It is cold and It is raining.			
③ OR	Disjunction	$\vee$	Binary
Eg: p: Roses are red in color; q: Roses are white in color. $p \vee q$: Roses are red or white in colour.			
④ If ... then	Conditional (or) Implication	$\rightarrow$	Binary
Eg: p: There is a flood; q: The crop will be destroyed $p \rightarrow q$: If there is a flood then The crop will be destroyed.			
⑤ If and only if	Biconditional	$\rightleftarrows$ (or) $\leftrightarrow$	Binary
Eg: p: students will come to college q: Friday is a working day $p \leftrightarrow$ Students will come to college if and only if Friday is a working day.			

(1) Using the statements: A: John is rich
B: John is happy.

write the following statements in symbolic form:

(a) John is poor ~~but~~
(b) John is poor but happy.
(c) John is rich or unhappy
(d) John is neither rich nor happy.
(e) John is poor or he is both rich and unhappy.

Soln: (a) John is poor $\Rightarrow \neg A$

(b) John is poor but happy $\Rightarrow \neg A \wedge B$

(c) John is rich or unhappy $\Rightarrow A \vee \neg B$

(d) John is neither rich nor happy $\Rightarrow \neg A \wedge \neg B$

(e) John is poor or he is both rich and unhappy
$\Rightarrow \neg A \vee (A \wedge \neg B)$

(2) write the statements for the following symbolic form.

p: It is hot day
q: The temperature is $45°c$.

(i) $\neg p$ (ii) $\neg(p \vee q)$ (iii) $p \wedge q$ (iv) $\neg(\neg p)$
(v) $\neg p \wedge \neg q$ (vi) $\neg p \vee \neg q$ (vii) $\neg(\neg p \vee \neg q)$

Soln: (i) $\neg p \Rightarrow$ It is not hot day.

(ii) $\neg(p \vee q) \Rightarrow$ It is false that it is hot day or the temperature is 45°c.

(iii) $p \wedge q \Rightarrow$ It is hot day and the temperature is 45°c

(iv) $\neg(\neg p) \Rightarrow$ It is hot day.

(v) $\neg p \wedge \neg q \Rightarrow$ It is not hot day and the temperature is not 45°c (or)
Neither it is hot day nor the temperature is 45°c.

(vi) $\neg p \vee \neg q \Rightarrow$ It is not hot day or the temperature is not 45°c (or)
Either it is not hot day or the temp is not 45°c

(vii) $\neg(\neg p \vee \neg q) \Rightarrow$ It is false that it is not hot day or the temperature is not 45°c (or)
It is hot day or the temp. is 45°c.

② Let p: Triangle ABC is an isoceles
Q: Triangle ABC is an equilateral
R: Triangle ABC is an equiangular.

Translate each of the following notations into a statement

(i) $Q \rightarrow p$ (ii) $\neg p \rightarrow \neg Q$ (iii) $Q \leftrightarrow R$ (iv) $p \rightarrow \neg Q$

(v) $R \rightarrow p$ (vi) $(p \vee Q) \rightarrow R$ (vii) $(\neg p \wedge Q) \rightarrow \neg R$.

Soln:

(i) $Q \to P \Rightarrow$ If triangle ABC is an equilateral then triangle ABC is an isoceles.

(ii) $\neg P \to \neg Q \Rightarrow$ If triangle ABC is not an isoceles then triangle ABC is not an equilatera

(iii) $Q \leftrightarrow R \Rightarrow$ Triangle ABC is an equilateral iff Triangle ABC is an equiangular.

(iv) $P \to \neg Q \Rightarrow$ If triangle ABC is an isoceles then triangle ABC is not an equilateral.

(v) $R \to P \Rightarrow$ If triangle ABC is an equiangular then triangle ABC is an isoceles.

(vi) $(P \vee Q) \to R \Rightarrow$ If either triangle ABC is an isoceles or triangle ABC is an equilateral then triangle ABC is an equiangular.

(vii) $(\neg P \wedge Q) \to \neg R \Rightarrow$ If triangle ABC is not an isoceles but and equilateral then triangle ABC is not an equiangular.

(5) Construct the truth table for the following :

(i) $(p \to q) \wedge (q \to p)$

(ii) $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$

(i) $(p \to q) \wedge (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(ii) $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$	$\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

TAUTOLOGY:

A statement formula which is true always irrespective of the truth values of the individual variables is called a tautology. Eg: $P \vee \neg P$ is a tautology.

CONTRADICTION:

A statement formula which is always false is called a contradiction (or) absurdity.
Eg: $P \wedge \neg P$ is a contradiction.

CONTIGENCY:

A statement formula which is neither tautology nor contradiction is called contigency.
Eg: $P \leftrightarrow Q$ is a contigency.

1) Show that $(Q \vee (P \wedge \neg Q)) \vee (\neg P \wedge \neg Q)$ is a tautology.

P	Q	$\neg P$	$\neg Q$	$P \wedge \neg Q$	$Q \vee (P \wedge \neg Q)$	$\neg P \wedge \neg Q$	$(Q \vee (P \wedge \neg Q)) \vee (\neg P \wedge \neg Q)$
T	T	F	F	F	T	F	T
T	F	F	T	T	T	F	T
F	T	T	F	F	T	F	T
F	F	T	T	F	F	T	T

Since all the entries in the resulting column are T, the given expression is a tautology.

2) Show that $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction.

p	q	$p \wedge q$	$p \vee q$	$\neg(p \vee q)$	$(p \wedge q) \wedge \neg(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Since all the entries in the resulting column are F, the given expression is a contradiction.

3) Identify the given expression. $\neg(p \vee q) \vee (\neg p \vee \neg q)$

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \vee \neg q$	$\neg(p \vee q) \vee (\neg p \vee \neg q)$
T	T	F	F	T	F	F	F
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	F	T	T	T

Since all the entries in the resulting column are T as well as F, the given expression indicates it is a contingency.