1. Show that the Propositions are logically equivalence for the following

   $P \rightarrow Q \Leftrightarrow \neg P \vee Q$

2. All humans are mortal. Sachin is a human. Therefore he is mortal.

3. Show that among 1000 people, atleast 12 of them were born in the same month.

4. How many positive integers not exceeding 1000 are divisible by 7 or 11?

5. Explain for $n > 2$ is the graph C_n bipartite?.

6. If 10 people each shake hands with each other, how many handshakes took place? What does this question have to do with graph theory?

7. What is algebraic structure? List properties of the algebraic system.

8. Define Monoid with an example.

9. $(Z^+, /)$ is a lattice?

10. Draw Hasse diagram where $X = \{1, 2, 3, 4, 5, 6\}$ and " / " is a partial ordering relation on X.

Prove that every homomorphism image of a graph G is isomorphic to some quotient group of G.

For any Boolean algebra, prove the following

i) $x \vee y = x \vee z$ and $\bar{x} \vee y = \bar{x} \vee z \Rightarrow y = z$

ii) $x \vee y = 0$ iff $x = 0$ and $y = 0$

iii) $x \leq \bar{y}$ iff $x \wedge y = 0$

iv) $x \wedge y = 1$ iff $x = 1$ and $y = 1$.

) Prove that D_{110}, the set of all positive divisors of positive integer 110, is a Boolean algebra and find all sub algebra.

) Verify the validity of the following argument.

"Every living thing is a plant or an animal"

"John's gold fish is alive and it is not a plant"

"All animals have hearts". Therefore, "John's gold fish has a heart.

Obtain PDNF and PCNF for the following using truth table.

$P \rightarrow [(P \rightarrow Q) \wedge (\neg Q \vee \neg P)]$

Solve the recurrence relation $a_n - a_{n-1} - 6\,a_{n-2} = -30$, $a_0 = 0$, $a_1 = -5$, $n \geq 2$.

Using generating function, solve the recurrence relation $a_{n+2} - 2\,a_{n+1} - a_n = 2^n$, $a_0 = 2$, $a_1 = 1$.

Prove that a simple graph with n vertices and k component cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges.

Show that a connected graph is Eulerian if and only if every vertex is of even degree.

Show that the union of two subgroups of a group G is a subgroup iff one is contained in the other.