Permutation & Combination :

Permutation :

A permutation is an arrangement of 'n' objects in which can be taken some (or) all at a time.

$$nP_r = \frac{n!}{(n-r)!}$$

(or) $P(n,r)$

Note:

$0! = 1$, $nP_0 = 1$, $nP_n = n!$

Problems

1. How many different bit strings are there of length 7?

No. of bit strings of length $7 = 7!$
$= 5040$

2. In how many ways can 6 persons occupy 3 vacant seats?

No. of persons $n = 6$
vacant seats $r = 3$

Total no. of ways $= nP_r = 6P_3 = 6 \times 5 \times 4 = 120$ ways

(or)

$$nP_r = \frac{n!}{(n-r)!} = \frac{6!}{(6-3)!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$$
$= 120$ ways

3. How many permutations are there in the world for the following.

(i) MISSISSIPPI (ii) Radar (iii) Mathematical (iv) unusual

(i) No. of letters, $n = 11$

Repeated letters: $I \to 4$, $S \to 4$, $P \to 2$ (r)

Required No. of permutations $= \frac{n!}{r!\,r!\,r!} = \frac{11!}{4!\,4!\,2!}$

$= 34,650$

(ii) No. of letters : $n = 5$

Repeated letters : $a \to 2$, $r \to 2$ (r)

Required No. of permutations $= \frac{5!}{2!\,2!} = 30$ ways

(iii) mathematical

No. of letters : $n = 12$

Repeated letters : $m \to 2$, $t \to 2$, $a \to 3$

Required No. of permutations $= \frac{12!}{3!\,2!\,2!}$

$= 1995840$

(iv) unusual

No. of letters : $n = 7$

Repeated letters : $u \to 3$, $n \to 1$, $s \to 1$, $a \to 1$, $l \to 1$

Required No. of permutations $= \frac{7!}{3!} = 840$ ways

4). Suppose there are 6 boys and 4 girls

(i) In how many ways can they sit in a row?

(ii) In how many ways can they sit in a row of the boys and girls each sit together?

(iii) In how many ways they can sit in a row of the girls can sit together?

(iv) How many seating arrangements are there with no two girls sitting together?

6 boys can sit in a row in 6! ways.

4 girls can sit in a row in 4! ways

(i) No. of ways can they sit in a row is $6! + 4!$

$= 10!$

$= 3,628,800$

(ii) No. of ways can they sit on a row if the boys and girls each sit together is

$4!\,6!\,2! = 34,560$

(iii) No. of ways they can sit on a row if the girls can sit together is $7!\,4! = 120960$

(iv) No. of seating arrangements are there with no two girls sitting together is $7P_4 \times 6!$

$= \frac{7!}{3!} \times 6!$

$= 604800$

Permutation :

The password for a computer system consists of 8 distinct alphabetic characters. Find the number of passwords possible that

a) End in the string MATH

b). Begin with the string CREAM

c). Contain the word COMPUTER as a string

Soln.

No. of alphabetic characters → 26

a). End with the string MATH

$= {}^{22}P_4$

$= \frac{22!}{18!}$

$= 175560$

$= 22 \times 21 \times 20 \times 19$

$= 175560$

b). Begin with the string CREAM

$= {}^{21}P_3$

$= \frac{21!}{18!} = 7980$

$= 21 \times 20 \times 19$

$= 7980$

c). Contain the word COMPUTER as a string

$= 1!$

$= 1$