



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)



Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

THE PRINCIPLE OF INCLUSION - EXCLUSION:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$(or) |A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

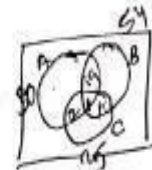
$$|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |B \cap C \cap D| + |A \cap C \cap D| - |A \cap B \cap C \cap D|$$

1) In a survey of 100 students, it was found that 30 studied mathematics, 54 studied statistics, 25 studied Operations Research, 1 studied all the three subjects, 20 studied mathematics and statistics, 3 studied mathematics and operations research and 15 studied statistics and operations research.

- How many students studied none of these subjects?
- How many students studied only mathematics?

Soln: Let A denote the students who studied mathematics
Let B denote the students who studied statistics
Let C denote the students who studied OR.

Then $|A| = 30$; $|B| = 54$; $|C| = 25$
 $|A \cap B| = 20$; $|A \cap C| = 3$; $|B \cap C| = 15$
 $|A \cap B \cap C| = 1$





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(i) No. of students studied none of these subjects = $100 - |A \cup B \cup C|$

\therefore By principle of Inclusion - Exclusion,

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$= 30 + 54 + 25 - 20 - 3 - 15 + 1$$

$$= 72$$

$$\therefore \text{No. of students studied none of these subjects} = 100 - 72 = 28$$

$$\text{(ii) No. of students studied only Maths \& statistics} = n(A \cap B) - n(A \cap B \cap C) = 20 - 1 = 19$$

$$\text{No. of students studied only Maths \& OR} = n(A \cap C) - n(A \cap B \cap C) = 3 - 1 = 2$$

$$\text{Then, No. of students studied only Mathematics} = 30 - 19 - 2 = 9$$

(2) How many positive integers not exceeding 1000 are divisible by 7 or 11?

Let A denote the set of +ve integers not exceeding 1000 that are divisible by 7.

Let B denote the set of +ve integers not exceeding 1000 that are divisible by 11.

$$\text{Then } |A| = \left[\frac{1000}{7} \right] = [142.8] = 142$$

$$|B| = \left[\frac{1000}{11} \right] = [90.9] = 90$$



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$$|A \cap B| = \left[\frac{1000}{7 \times 11} \right] = [12.9] = 12$$

∴ No. of +ve integer not exceeding 1000 that are divisible by either 7 or 11 is $|A \cup B|$

By principle of Inclusion-Exclusion,

$$\begin{aligned} |A \cup B| &= |A| + |B| - |A \cap B| \\ &= 142 + 90 - 12 = 220 \end{aligned}$$

③ Find the no. of integers between 1 to 250 that are not divisible by any of the integers 2, 3, 5 & 7.

Soln Let A denote the integer from 1 to 250 that are divisible by 2
Let B " " " " " 3
Let C " " " " " 5
Let D " " " " " 7

$$|A| = \left[\frac{250}{2} \right] = 125$$

$$|B| = \left[\frac{250}{3} \right] = 83$$

$$|C| = \left[\frac{250}{5} \right] = 50$$

$$|D| = \left[\frac{250}{7} \right] = 35$$

$$\begin{aligned} \text{Now, the no. of integer btwn 1 to 250} \\ \text{that are divisible by 2 \& 3} \end{aligned} \left. \vphantom{\begin{aligned} \text{Now, the no. of integer btwn 1 to 250} \\ \text{that are divisible by 2 \& 3} \end{aligned}} \right\} = |A \cap B| = \left[\frac{250}{2 \times 3} \right] = 41$$

$$\begin{aligned} \text{The no. of integer divisible by 2 \& 5} &= |A \cap C| = \left[\frac{250}{2 \times 5} \right] \\ &= 25 \end{aligned}$$



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$$\therefore \text{Similarly, } |A \cap D| = \left[\frac{250}{2 \times 7} \right] = 17$$

$$|B \cap C| = \left[\frac{250}{3 \times 5} \right] = 16$$

$$|B \cap D| = \left[\frac{250}{3 \times 7} \right] = 11$$

$$|C \cap D| = \left[\frac{250}{5 \times 7} \right] = 7$$

$$\therefore \text{No. of integers divisible by 2, 3 & 5} = |A \cap B \cap C| = \left[\frac{250}{2 \times 3 \times 5} \right] = 8$$

$$\text{Similarly, } |A \cap B \cap D| = \left[\frac{250}{2 \times 3 \times 7} \right] = 5$$

$$|A \cap C \cap D| = \left[\frac{250}{2 \times 5 \times 7} \right] = 3$$

$$|B \cap C \cap D| = \left[\frac{250}{3 \times 5 \times 7} \right] = 2$$

$$\therefore |A \cap B \cap C \cap D| = \left[\frac{250}{2 \times 3 \times 5 \times 7} \right] = 1$$

\therefore No. of integers btwn. 1-250 that are divisible by integers 2, 3, 5 & 7 is $|A \cup B \cup C \cup D|$

\therefore By principle Inclusion-Exclusion,

$$\begin{aligned} |A \cup B \cup C \cup D| &= |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - \\ &\quad |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| - \\ &\quad |A \cap B \cap C \cap D| \\ &= 125 + 83 + 50 + 35 - 41 - 25 - 17 - 16 - 11 - 7 + 8 + 5 + 3 + 2 - 1 \\ &= 193 \end{aligned}$$

$$\begin{aligned} \therefore \text{No. of integers btwn. 1 to 250 that are not divisible by} \\ \text{2, 3, 5 \& 7 is } 250 - |A \cup B \cup C \cup D| \\ = 250 - 193 = 57 \end{aligned}$$

