10/8/16

UNIT-2

DESIGN OF SHAFTS AND COUPLING.

Shaft

Shaft is used to transmit the power from prime movers to machine.

Classification of Shaft

Line Shaft:-
It is a shaft which transmits power to several machines.

Spindle Shaft:-
Spindle is a short revolving shaft.

Stub Spindle:-
A shaft that is integral with prime movers or machine is known as stub shaft

Counter Shaft:-
A shaft that connect prime movers to the line shaft of the machine is called counter shaft

i) Based on Torsional Rigidity...

$$\frac{T}{J} = \frac{\tau}{r} = \frac{C\theta}{l}.$$

ii) Based on Strength.

$$\sigma_{eq} = M/z \quad , \quad \tau_{eq} = \frac{16T}{\pi d^3}$$

iii) Based on critical speed of shaft

$$\frac{1}{(\omega_c)^2} = \frac{1}{(\omega_1)^2} + \frac{1}{(\omega_2)^2} + \frac{1}{(\omega_3)^2} + \frac{1}{(\omega_5)^2}$$

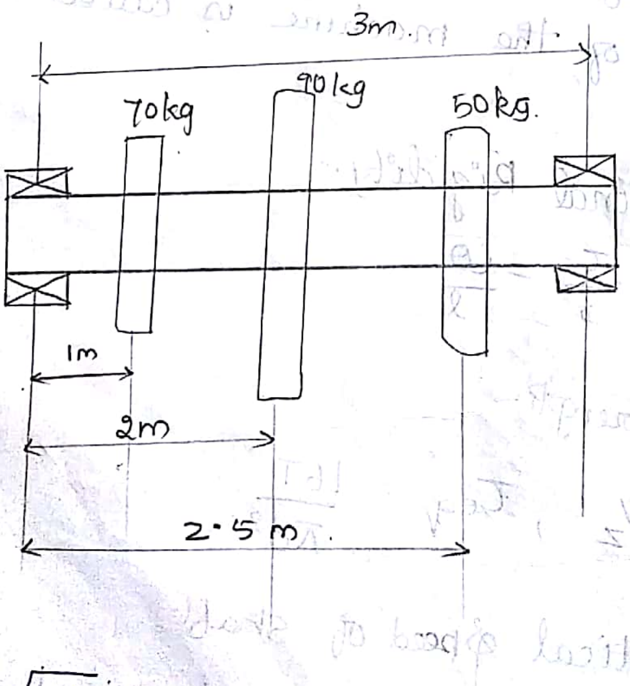
$\omega_1 = \sqrt{g/\delta_1}$ $\omega_2 = \sqrt{g/\delta_2}$ $\omega_3 = \sqrt{g/\delta_3}$

$$\omega_s = \left(\sqrt{\frac{g}{\delta_s}}\right) 1.12$$

$$\delta_1 = \frac{W l_1^2 l_2^2}{3EIl}$$

$$\delta_s = \frac{5Wl^4}{384\,EI}$$

1\. A turbine shaft of 125 mm diameter has mass of 100 kg per metre length. It is simply supported between the bearings at its ends and carries a 3 rotors of masses 70 kg, 90 kg, 50 kg, at 1m, 2m, and 2.5 m respectively from left support. Find the critical speed of shaft. Assume Young's modulus $E = 2\times10^{11}$ N/m². Total length = 3m.



$$\omega_1 = \sqrt{g/\delta_1}$$

$$\delta_1 = \frac{m_1 g \times l_1^2 \times l_2^2}{3EIl}$$

$$= \frac{70 \times 9.81 \times 1^2 \times 2^2}{3 \times 2 \times 10^{11} \times \frac{\pi}{64}(0.125)^4 \times 3}$$

$$= 1.27 \times 10^{-4} m$$

$$\omega_1 = \sqrt{g/\delta_1} = \sqrt{\frac{9.81}{1.27 \times 10^{-4}}}$$

$$= 277.92 \text{ rad/s}$$

$$\delta_2 = \frac{m_2 g \times l_1^2 \times l_2^2}{3EIl}$$

$$= \frac{90 \times 9.81 \times 2^2 \times 1^2}{3 \times 2 \times 10^{11} \times \frac{\pi}{64}(0.125)^4 \times 3}$$

$$= 1.636 \times 10^{-4} m.$$

$$\omega_2 = \sqrt{g/\delta_2}$$

$$= \sqrt{\frac{9.81}{1.636 \times 10^{-4}}}$$

$$\omega_2 = 244.87 \text{ rad/s}.$$

$$\delta_3 = \frac{m_3 g \times l_1^2 \times l_2^2}{3EIl}$$

$$= \frac{50 \times 9.81 \times (2.5)^2 \times (0.5)^2}{3 \times 2 \times 10^{11} \times \frac{\pi}{64}(0.125)^4 \times 3}$$

$$= 3.5 \times 10^{-5} m.$$

$$\omega_3 = \sqrt{g/\delta_3}.$$

$$\omega_3 = 525.46 \text{ rad/s}$$

$$\delta_s = \frac{5 W l^4}{384 EI}$$

$$= \frac{5 \times 100 \times 3^4 \times 9.81}{384 \times 2 \times 10^{11} \times \frac{\pi}{64}(0.125)^4}$$

$$= 4.316 \times 10^{-4}\ m$$

$$\omega_s = \sqrt{g/\delta_s}$$

$$= 150.75\ rad/s$$

$$\frac{1}{\omega_c^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} + \frac{1}{\omega_s^2}$$

$$= \frac{1}{77239.5} + \frac{1}{59961.3} + \frac{1}{276108.2} + \frac{1}{22725.5}$$

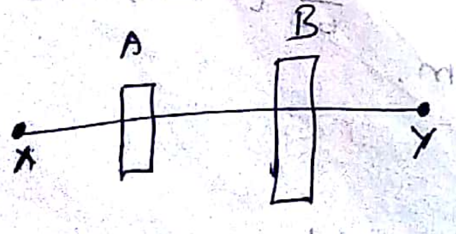
$$\frac{1}{\omega_c^2} = 7.7249 \times 10^{-5}$$

$$\omega_c^2 = 12945.0$$

$$\omega_c = 113.77$$

Whirling & critical Speed of Shaft :-

The speed at which amount of deflection become infinite is called whirling speed of shaft

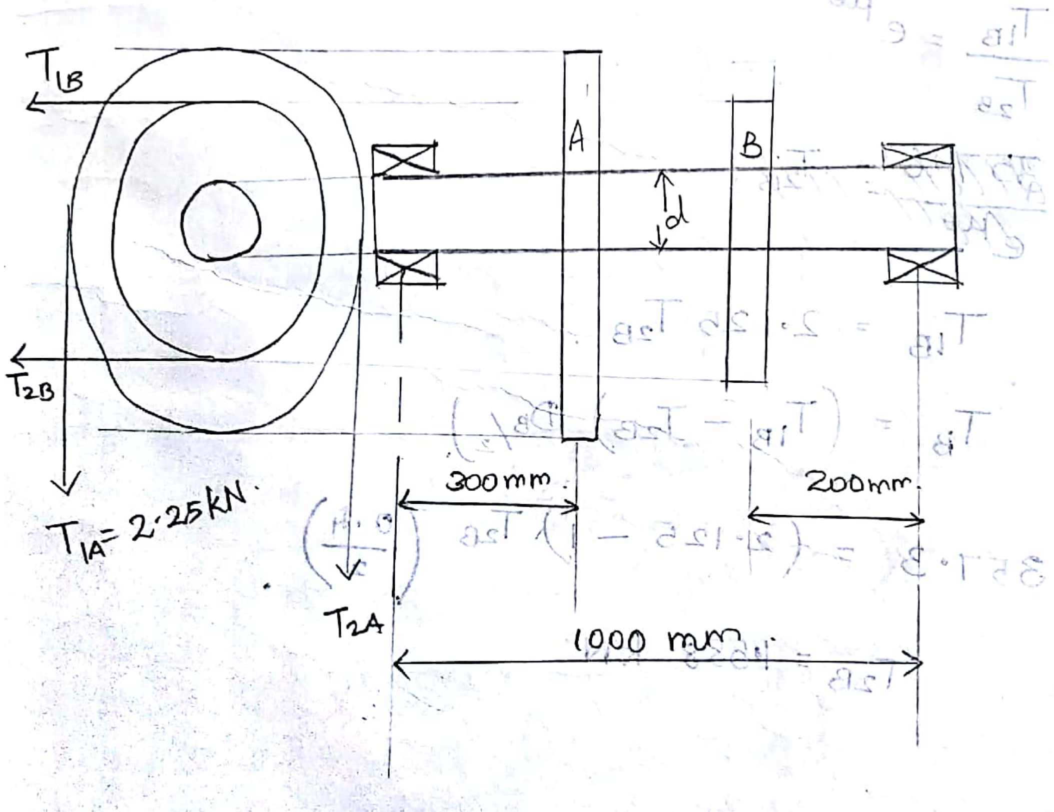


$$(M_v)_A = \sqrt{(M_A)_V^2 + (M_A)_H^2}$$

$$(M_v)_B = \sqrt{(M_B)_V^2 + (M_B)_H^2}$$

$$(M_A)_V = R_{AV}\, X$$

1) - A shaft is supported by 2 bearings placed 1m apart, a 600mm diameter pulley is mounted at a distance of 300 mm to the right of left side bearing and drives pulley below it with the help of belt having the maximum tension of 2.25 kN. Another pulley of 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electrical motor and the belt which is placed horizontally to the right. The angle of contact for both pulley is 180° & coefficient of friction 0.24. Allowable tensile stress is 60 MPas. Allowable shear stress is 40 MPa. Assume that torque on the one pulley is equal to the other pulley. Determine the diameter of shaft.



$\mu = 0.24$

$\sigma = 60\ MPa$

$\tau = 40\ MPa$

$$\frac{T_{1A}}{T_{2A}} = e^{\mu\theta}$$

$$\frac{T_{1A}}{T_{2A}} = e^{(3.14)(0.24)}$$

$$T_{2A} = 1.059\ kN.$$

$$T_A = (T_{1A} - T_{2A})\ D_A/2$$

$$= (2250 - 1059)\ 0.6/2$$

$$= 357.3\ N\text{-}m$$

$$T_A = T_B.$$

$$\frac{T_{1B}}{T_{2B}} = e^{\mu\theta}$$

~~[illegible]~~ = T_{2B}

$$T_{1B} = 2.125\ T_{2B}.$$

$$T_B = (T_{1B} - T_{2B})\ D_B/2)$$

$$357.3 = (2.125 - 1)\ T_{2B}\left(\frac{0.4}{2}\right).$$

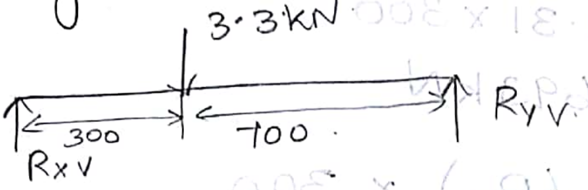
$$T_{2B} = 1.588\ kN$$

$T_{1B} = 2.125 \times 1.588$

$= 3.3745 \text{ kN}$.

$T_A + T_B = 33$

Considering Vertical load.



$R_{xv} + R_{yv} = 3.308 \times 10^3 \text{ N}$

$(R_{xv} \times 0) - ((3.3)(300) + (R_{yv}) \times 1000) = 0.$

$1000 R_{yv} = 990.4$

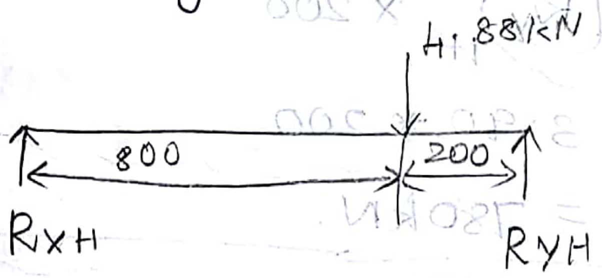
$R_{yv} = 0.99 \text{ kN}.$

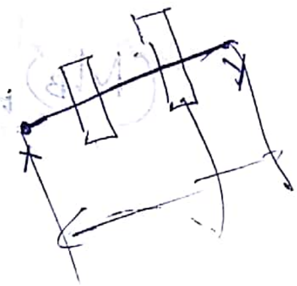
$R_{xv} = 2.318 \text{ kN}.$

On pulley 'B' the force acting horizontal let assume in vertical

$T_{1B} + T_{2B} = 4.88 \text{ kN}.$

Considering Horizontal load.





$R_{xH} + R_{yH} = 4.88 \times 10^3 \text{ N}.$

$(-4.88 \times 800) + (R_{yH} \times 1000) = 0.$

$1000 R_y = 3904$

$R_{yH} = 3.904 \text{ kN}.$

$R_{xH} = 0.976 \text{ kN}.$

$$M_A = \sqrt{(M_A)_V^2 + (M_A)_H^2}$$

$$(M_A)_V = (R_x)_V \times 300.$$

$$= 2.31 \times 300$$

$$= 693\ kN.$$

$$(M_A)_H = (R_x)_H \times 300$$

$$= 0.976 + 300.$$

$$= 292.8\ kN.$$

$$M_A = \sqrt{(693)^2 + (292.8)^2}$$

$$= 752.31\ kN.mm.$$

$$M_B = \sqrt{(M_B)_V^2 + (M_B)_H^2}.$$

$$(M_B)_V = (R_Y)_V \times 200$$

$$= 0.99 \times 200$$

$$= 198\ kN.$$

$$(M_B)_H = (R_Y)_H \times 200$$

$$= 3.90 * 200$$

$$= 780\ kN.$$

$$T_{eq} = \sqrt{(K_b M)^2 + (K_t T)^2}$$

M_B is maximum.

$$T_{eq} = \sqrt{(1 \times 804 \times 10^3)^2 + (1 \times 357 \times 10^3)^2}$$

$$= 879695.95$$

$$= 879.695\ kN.mm.$$

$$M_B = \sqrt{(198)^2 + (-780)^2}$$

$$M_B = 804 \text{ kN·mm}$$

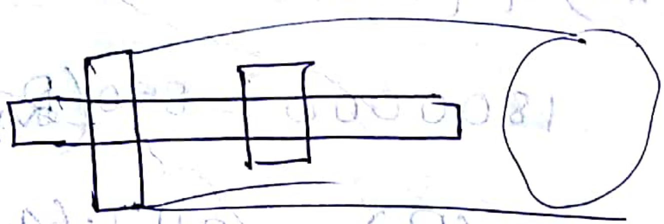
$$T_{eq} = \frac{\pi}{16} \tau d^3$$

$$d^3 = \frac{16 T_{eq}}{\pi \tau}$$

$$d^3 = \frac{16 \times 879.695 \times 10^3}{\pi \times 40}$$

$$d = 48.20 \text{ mm}$$

3. A pulley weights 1.2 kN and 500 mm diameter is driven by horizontal belt drive the power transmitted to solid shaft by pinion key to shaft which in turn meshes with gear. The belt tension and component of gear reaction on pinion is shown in figure. Design the shaft if shock & fatigue factor are $K_b = 2$ & $K_t = 1.5$ $\tau = 55$ N/mm².



Torque (T_p) of pulley

$= (T_1 - T_2) \times D/2$

$= (5000 - 1500) \times 250$

$= 875000$

$T_p = 875 \times 10^3$ N-mm

$F = T_1 + T_2$

$= 6500$ N.

Moment

$(R_x)_v + (R_y)_v = 8000$ N

$(R_x \times 0) + (8000 \times 225) - ((R_y)_v \times 850) = 0$

$1800000 = 850 (R_y)_v$

$(R_y)_v = 2117.64$ N

$(R_x)_v = 8000 - 2117.64$

$= 5882.35$ N.

$$(R_x)_H + (R_y)_H = 9500$$

$$[(R_x)_H \times 0] - [3000 \times 225] - [6500 \times 600] + ((R_y)_H \times 850) = 0$$

$$(R_y)_H \times 850 = 4575000$$

$$(R_y)_H = 5382.35 \text{ N}$$

$$(R_x)_H = 9500 - 5382.35$$

$$= 4117.64 \text{ N}$$

Moment due to "gear"

$$M_G = \sqrt{(M_G)_V^2 + (M_G)_H^2}$$

$$(M_G)_V = (R_x)_V \times 225$$

$$= 1.32 \times 10^6 \text{ N-mm}$$

$$(M_G)_H = (R_x)_H \times 225$$

$$= 926469 \text{ N-m}.$$

$$M_{Gt} = \sqrt{(1.32\times10^6)^2 + (926469)^2}$$

$$= 1612682.4$$

$$= 1.612 \times 10^6 \text{ N.mm}$$

Moment due to pulley.

$$M_p = \sqrt{(M_p)_V^2 + (M_p)_H^2}$$

$$(M_p)_V = (R_y)_V \times 250$$

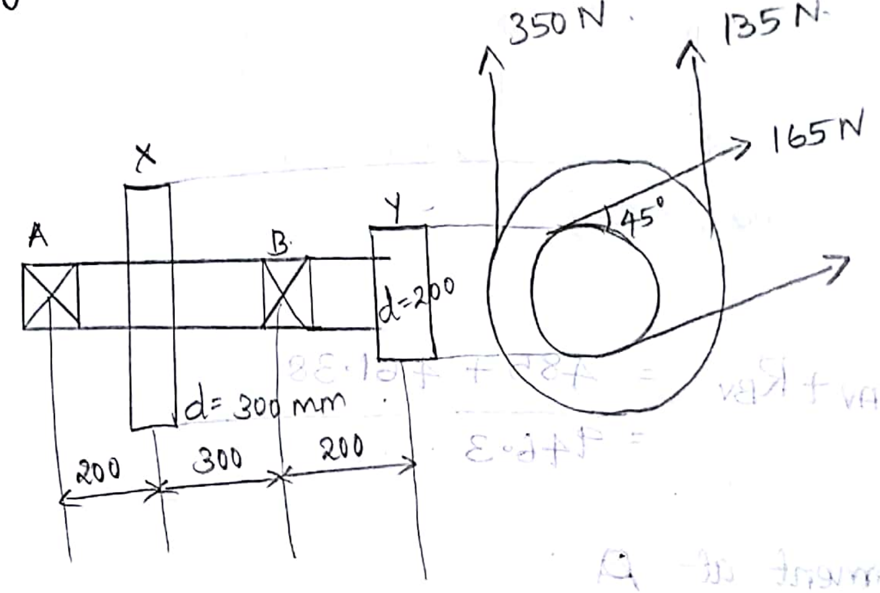
$$= 529410 \text{ N.mm}.$$

$$(M_p)_H = (R_y)_H \times 250$$

$$= 1345587.5 \text{ N.mm}.$$

$$M_p = \sqrt{(529410)^2 + (1345587.5)^2}$$

1. A steel shaft made of 40 C8 is used to drive a machine pulley xy and bearing ab are located is shown in figure. The allowable shear stress of shaft is 94·5 N/mm². Assume Torque on one pulley is equal to the torque on other pulley. Determine the diameter of shaft.



Given

$T_{1x} = 350 N$. Tension $x = 485 N$.

$T_{2x} = 135 N$.

$Torque_X = (T_{1x} - T_{2x}) D/2$

$= (350 - 135) 150$.

$= 32250$ N.mm.

$T_{2Y} = 165 N$.

$T_x = T_Y$.

$T_Y = (T_{1Y} - T_{2Y}) D/2$

$= (T_{1Y} - 165) 100$.

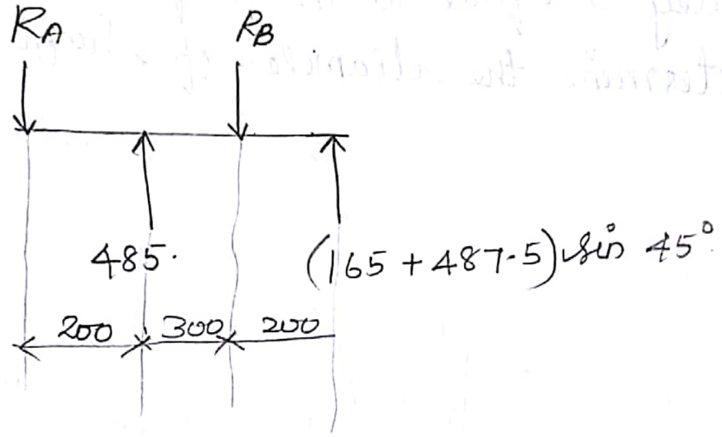
$(T_{1Y} - 165) 100 = 32250$.

$T_{1Y} - 165 = 322·5$

$T_{1Y} = 487·5 N$.

$$T_{max} = \frac{16\,T_{eq}}{\pi d^3}$$

$$T_{eq} = \sqrt{(K_b M)^2 + (K_t T)^2}$$



$$R_{AV} + R_{BV} = 485 + 461.38$$

$$= 946.3$$

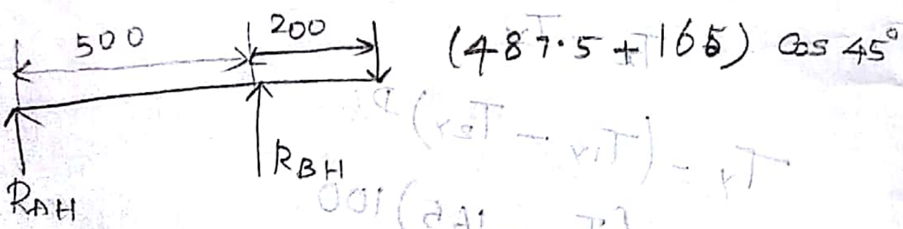
Moment at A

$$(R_{AV} \times 0) + (485 \times 200) - (R_{BV} \times 500) + (461.2 \times 700) = 0$$

$$97000 - 500\,R_{BV} + 322840 = 0$$

$$R_{BV} = 839.68\ N$$

$$R_{AV} = 106.52\ N$$



$$R_{AH} + R_{BH} = 461.38$$

$R_{BH} \times 500 - (~~R_{BH}\cdot 66~~ \times 700) = 0$

$R_{BH} = ~~581.153~~ \; 645.94 \text{ N}$

$R_{AH} = -184.56 \text{ N}$

$M_x = \sqrt{(M_{xH})^2 + (M_{xV})^2}$

$M_{xV} = R_{AV} \times 200$

$= 21324 \text{ N.mm}$

$M_{xH} = R_{AH} \times 200$

$= -36912 \text{ N.mm}$

$M_x = 42628.73 \text{ N.mm}$

$M_y = \sqrt{(M_{yH})^2 + (M_{yV})^2}$

$M_{yV} = R_{BV} \times 200$

$= 839.68 \times 200$

$= 167936 \text{ N.mm}$

$M_{yH} = R_{BH} \times 200$

$= 645.94 \times 200$

$= 129188 \text{ N.mm}$

$M_y = 211877.4 \text{ N.mm}$

$T_{eq} = \sqrt{(K_b M_y)^2 + (K_t T)^2}$

$= \sqrt{(211877.4)^2 + (32250)^2}$

$= 214317.74 \text{ N.mm}$

$$T_{max} = \frac{16 \times 214317.74}{\pi d^3}$$

$$94.5 = \frac{16 \times 214317.74}{\pi d^3}$$

$$d^3 = \frac{16 \times 214317.74}{\pi \times 94.5}$$

$$d = 22.6 \text{ mm}$$

DESIGN OF KEYS AND KEYWAYS.

Purpose of keys:

Key is a machine element used on the shaft to secure rotating element like gear, pulley and sockets and prevent the relative motion between the two.

Keyway is a slot or recess in a shaft and hub of the rotating element to accomodat key.

Classification of keys:-

- Saddle key and Sunk keys
- Kennedy keys (Square with 90°)
- Round keys and Taper pins
- Tangent keys (Rectangular with 90°)
- Splines

Sunk keys:

Sunk keys are provided half in the keyway of shaft and half in the hub

Splines (High torque Requirement)
Multiple keys integral with shaft.

Saddle key:
Keyways either in shafts or hubs

Gib head key :-
Head at one side Square or Rectangular.

1. A square keys is used to be used to fix a gear of 35mm diameter the hub length of gear is 60 mm. Both the shafts and keys are made of same material having an allowable shear stress of 55 N/mm². If a torque is to be transmitted is 395 N·m. Determine the dimension of key cross section?

$\tau = 55\ N/mm^2$

$d = 35\ mm$

$l = 60\ mm$

$h = w$

$T = 395 \times 10^3\ N \cdot mm.$

$$\tau_d = \frac{2T}{dwl}$$

$$55 = \frac{2 \times 395 \times 10^3}{35 \times w \times 60}$$

$$w = \frac{2 \times 395 \times 10^3}{35 \times 55 \times 60}$$

$= 6.3\ mm$