RECURRENCE RELATIONS:

An equation that expresses a_n, the general term of the sequence $\{a_n\}$ in terms of one or more of the previous terms of the sequence, namely, $a_0, a_1, \ldots a_{n-1}$ for all integers n with $n \geq n_0$ where n_0 is a non -ve integer is called a recurrence relation for $\{a_n\}$.

Eg: Fibonacci series : 0, 1, 1, 2, 3, 5, 8, 13, ...

which can be represented by the recurrence relation

$$f_n = f_{n-1} + f_{n-2},\ n \geq 2 \text{ with } f_0 = 0,\ f_1 = 1.$$

HOMOGENEOUS RECURRENCE RELATION:

A linear homogeneous recurrence relation of degree k with constant co-efficients is a recurrence relation of the form, $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \ldots + c_k a_{n-k}$, where $c_1, c_2, \ldots c_k$ are real numbers, and $c_k \neq 0$.

Working Rule:

Step 1: Find the characteristic equation & find the roots. (CE)

Step 2: Find the roots for the characteristic equation.

Case (i) If the roots are real and distinct then

char. eqn. is $a_n = A(a_1)^n + B(a_2)^n + C(a_3)^n + \dots + k a_k^n$

case (ii) If the roots are real and equal (same)

then char. eqn is, $a_n = (A+Bn)(a)^n$ (2 roots)

$a_n = (A + Bn + Cn^2)(a)^n$ (3 roots)

Step 3: Find the particular solution.

Case (i) If RHS = 0 then PS = 0

case (ii) If RHS = constant then

put $a_n = a_{n-1} = a_{n-2} = a_{n-3} = d$

Case (iii): If RHS = a linear function then

put $a_n = d_0 + d_1(n)$

$a_{n-1} = d_0 + d_1(n-1)$

$a_{n-2} = d_0 + d_1(n-2)$

$a_{n-3} = d_0 + d_1(n-3)$

Case (iv): If RHS $= k^n$ then,

put $a_n = dk^n$, if k is not the root of char. eqn.

$a_n = dnk^n$, if " one of the " " .

$a_n = dn^2k^n$, if " repeated " " .

Step 4: General soln = char eqn. + particular soln.

1) If the sequence $a_n = 3 \cdot 2^n, n \geq 1$, then find the corresponding recurrence relation.

Given: For $n \geq 1$, $a_n = 3 \cdot 2^n$

Now $a_{n-1} = 3 \cdot 2^{n-1}$

$= 3 \cdot \frac{2^n}{2}$

$2a_{n-1} = 3 \cdot 2^n$

$= a_n$

$\therefore \quad a_n = 2a_{n-1}$, for $n \geq 1$

2) Find the recurrence relation for $S(n) = 6(-5)^n, n \geq 0$

Given: $S(n) = 6(-5)^n, n \geq 0$

Now $S(n-1) = 6(-5)^{n-1}$

$= \frac{6(-5)^n}{-5}$

$-5\, S(n-1) = S(n)$

$\therefore S_n = -5 S_{n-1}, n \geq 0$