SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

Unit 1 : TRANSFORMS

TRANSFORMS

- Introduction and Sampling theorem
- ECG signal conversion system
- Discrete Fourier Transform (DFT)
- Decimation in time FFT
- Decimation in time FFT Problems
- Decimation in frequency FFT
- Decimation in frequency FFT Problems
- Multi rate Signal Processing
- Wavelet Transform

Discrete-Time Fourier Transform

- **Derive an inverse transform:**

$$\tilde{x}[n] = \sum_{k=<N>}^{<N>} (\frac{1}{N}) X\left(e^{jk\omega_0}\right) e^{jk\ \ {}_0 n} = \frac{1}{2\pi} \sum_{k=<N>}^{<N>} X\left(e^{jk\omega_0}\right) e^{jk\omega_0 n} (\frac{2\pi}{N}) = \frac{1}{2\pi} \sum_{k=<N>}^{<N>} X\left(e^{jk\omega_0}\right) e^{jk\omega_0 n} \omega_0$$

As

$$N \to \infty, \quad \tilde{x}[n] \to x[n], \ \omega_0 = \left(\frac{2\pi}{N}\right) \to 0, \ \sum \omega_0 \to \int d\omega$$

- **This results in our Discrete-Time Fourier Transform:**

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X\left(e^{j\omega}\right) e^{j\omega n} d\omega \quad \text{(synthesis equation)}$$

$$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \quad \text{(analysis equation)}$$

Discrete Fourier transform

DFT of x (n) is given by,

$$X(k) = \sum_{n=0}^{N-1} \mathrm{x(n)}.\, e^{\frac{-j2\pi kn}{N}} \;;\; \text{where k} = 0,1,2,3,4, \dots, N-1$$

IDFT of X(k) is given by,

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} \mathrm{X(k)}.\, e^{\frac{j2\pi kn}{N}} \;;\; \text{where n} = 0, 1, 2,3,4, \dots, \mathrm{N} - 1$$

Compute DFT of the following sequence x (n) ={ 0, 1, 2, 3}

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$$X(k) = \sum_{n=0}^{N-1} x(n).\, e^{\frac{-j2\pi kn}{N}} \;;\; \text{where } k = 0,1,2,3,4,\ldots,N-1$$

$$W_N^{nk} = e^{\frac{-j2\pi nk}{N}}$$

$$[W_4] = \begin{array}{c} \\ k=0 \\ k=1 \\ k=2 \\ k=3 \end{array} \begin{array}{c} \begin{matrix} n=0 & n=1 & n=2 & n=3 \end{matrix} \\ \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \end{array}$$

Compute DFT of the following sequence x (n) ={ 0, 1, 2, 3}

$$W_4^0 = e^{\frac{-j2\pi(0)}{4}} = 1$$

$$W_4^1 = e^{\frac{-j2\pi(1)}{4}} = \cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right) = -j$$

$$W_4^2 = e^{\frac{-j2\pi(2)}{4}} = \cos(\pi) - j\sin(\pi) = 1$$

$$W_4^3 = e^{\frac{-j2\pi(3)}{4}} = \cos\left(\frac{3\pi}{2}\right) - j\sin\left(\frac{3\pi}{2}\right) = j$$

$$W_4^4 = e^{\frac{-j2\pi(4)}{4}} = \cos(2\pi) - j\sin(2\pi) = 1$$

$$[W_4] = \begin{array}{c} \\ k=0 \\ k=1 \\ k=2 \\ k=3 \end{array} \begin{array}{cccc} n=0 & n=1 & n=2 & n=3 \\ \end{array}$$

$$[W_4] = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \quad (\text{rows } k=0,1,2,3;\ \text{columns } n=0,1,2,3)$$

$$[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Compute DFT of the following sequence x (n) ={ 0, 1, 2, 3}

$$X_N = [W_N]\, x_N$$

$$\text{With } N = 4, X_4 = [W_4]\, x_4$$

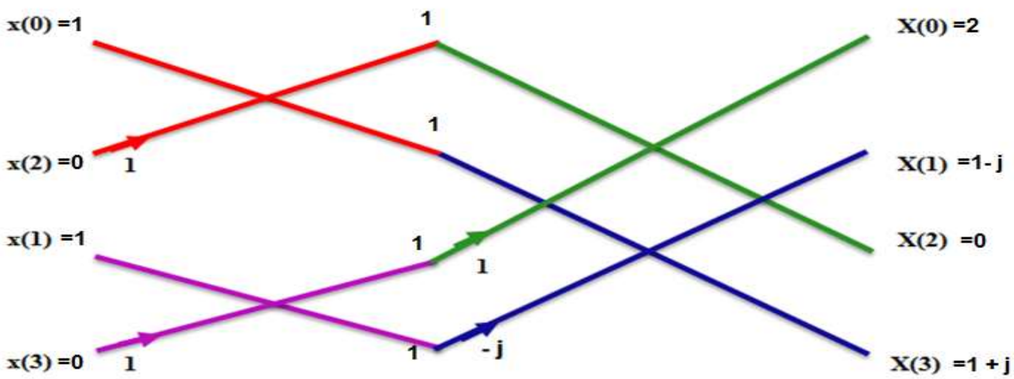
Putting values of W_4 and x_4,

$$X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0+1+2+3 \\ 0-j-2+3j \\ 0-1+2-3 \\ 0+j-2-3j \end{bmatrix} = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

Thus we obtained 4 point DFT as,

$$X_4 = \begin{bmatrix} X(0) = 6 \\ X(1) = -2+2j \\ X(2) = -2 \\ X(3) = -2-2j \end{bmatrix}$$

Find the DFT for the sequence x(n)={1,1,0,0}

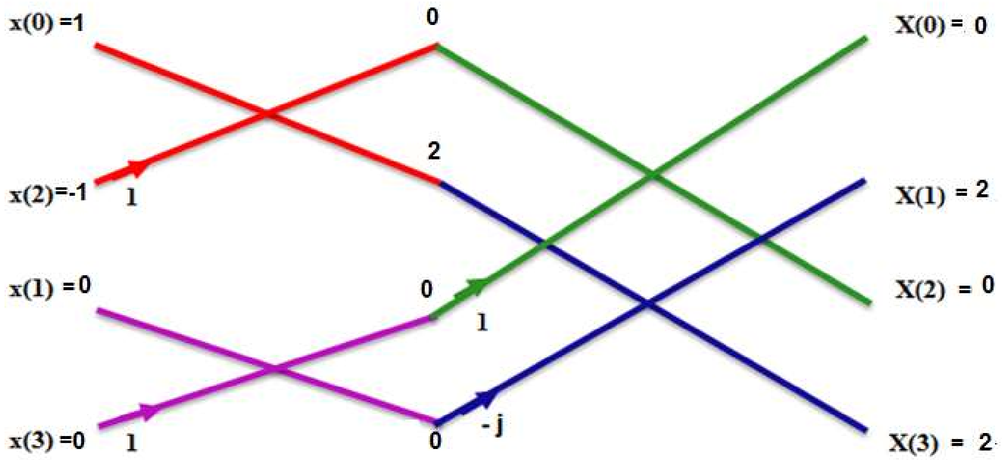


$x(n) = \{1,1,0,0\}.$

$X(k) = \{2, 1-j, 0, 1+j\}$

Input	S1	Output
1	1 + (0*1)=1	1+1(1)=2
0	1 - (0*1)=1	1+1(-j)=1-j
1	1 + (0*1)=1	1-1(1)=0
0	1 - (0*1)=1	1-1(-j)=1+j

Find the DFT for the sequence x(n)={1,0,-1,0}

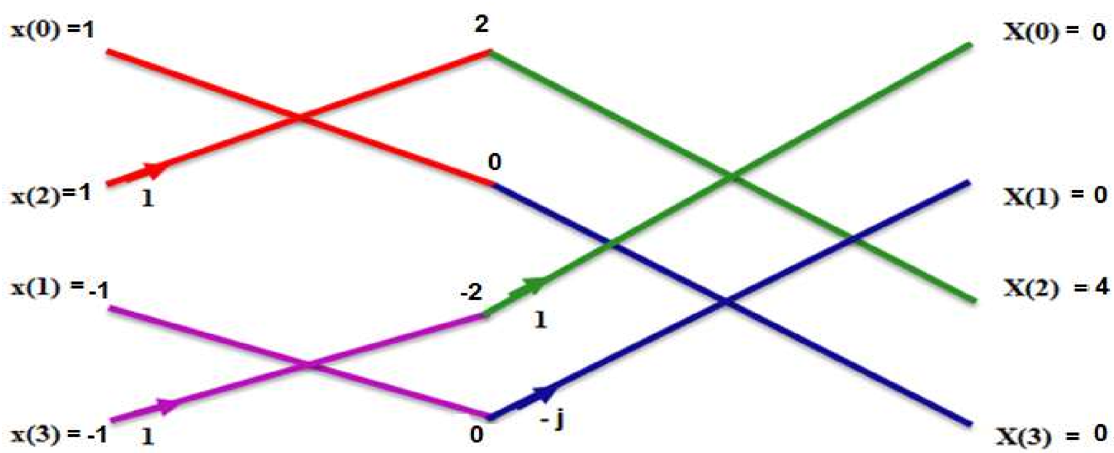


Input	S1	Output
1	1+(-1)(1)=0	0+0(1)=0
-1	1-(-1)(1)=2	2+0(-j)=2
0	0+0(1)=0	0-0(1)=0
0	0-0(1)=0	2-0(-j)=2

$$x(n) = \{1,0,-1,0\}.$$

$$X(k) = \{0,2,0,2\}$$

Find the DFT for the sequence x(n)={1,-1,1,-1}

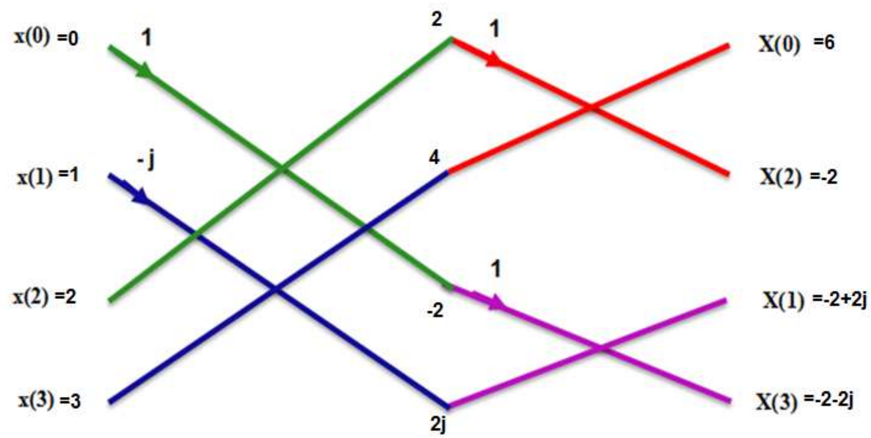


Input	S1	Output
1	1+1(1)=2	2+(-2)(1)=0
1	1-1(1)=0	0+0(-j)=0
-1	-1+(-1)(1)=-2	2-(-2)(1)=4
-1	-1-(-1)(1)=0	0-0(-j)=0

$x(n) = \{1, -1, 1, -1\}.$

$X(K) = \{0,0,4,0\}$

Find the DFT for the sequence x(n)={0,1,2,3}

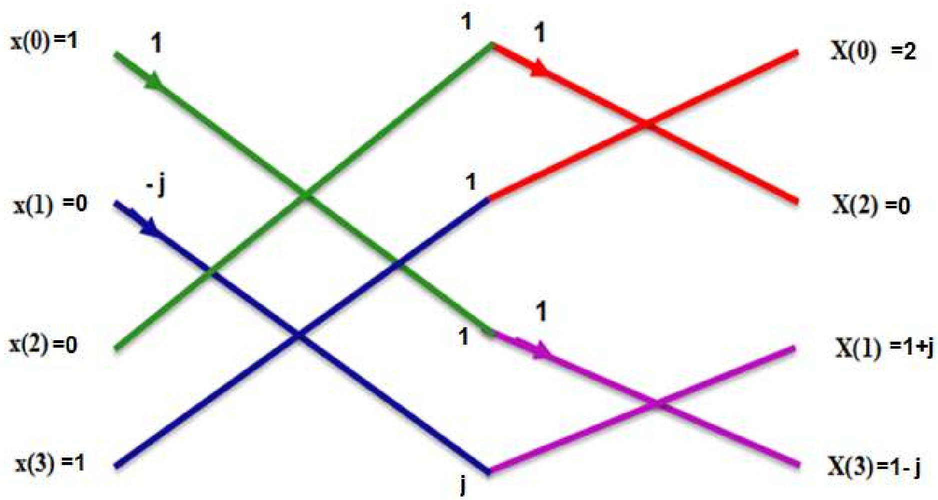


$x(n) = \{0,1,2,3\}.$

$X(k) = \{6, -2+2j, -2, -2-2j\}$

Input	S1	Output
0	0 + 2=2	2+4=6
1	1 + 3=4	(2-4)(1)=-2
2	(0 – 2)*1=-2	-2+2j
3	(1- 3)(-j)=2j	(-2-2j)(1)= -2-2j

Find the DFT for the sequence x(n)={1,0,0,1}

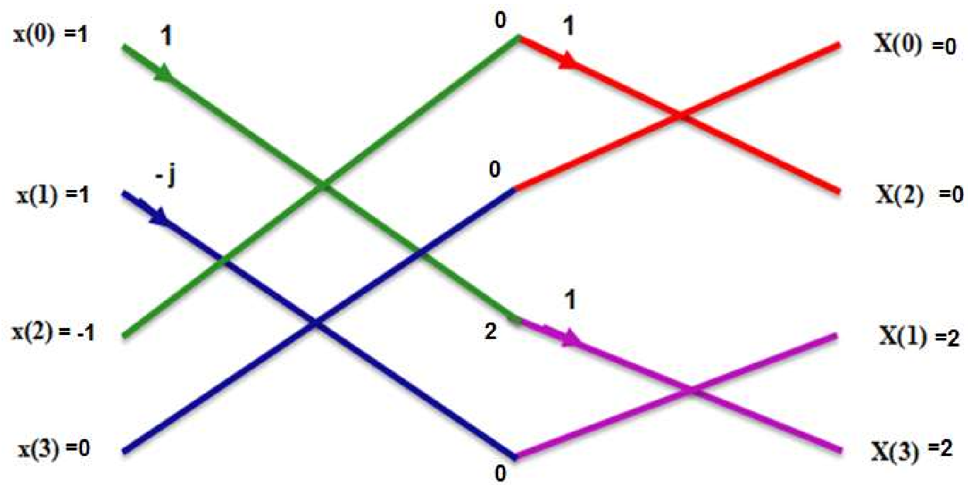


$x(n) = \{1,0,0,1\}.$

$X(k) = \{2,1+j,0,1-j\}$

Input	S1	Output
1	1 + 0 =1	2+1=2
0	0 + 1=1	(1-1)(1)=0
0	(1 – 0)*1=1	1+j
1	(0- 1)(-j)=j	(1-j)(1)= 1-j

Find the FFT for the sequence x(n)={1,0,-1,0} using DIF

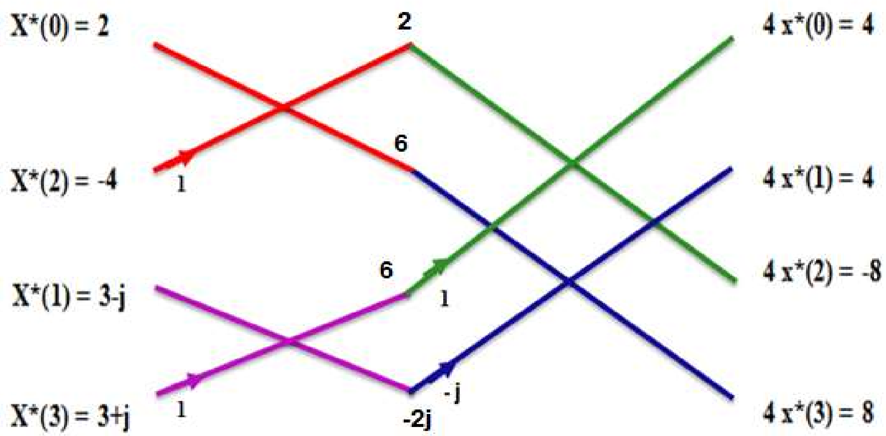


$$x(n) = \{1,0,-1,0\}.$$

$$X(k) = \{0,2,0,2\}$$

Input	S1	Output
1	1 - 1 =0	0+0=0
0	0 + 0=0	(0-0)(1)=0
-1	(1 +1)*1=2	2+0=2
0	(0- 0)(-j)=0	(2-0)(1)= 2

Find the IDFT for the sequence X(k)={2,3+j,-4,3-j}



Input	S1 Output	Output
2	2+(-4)=-2	(-2)+(6)=4
-4	2-(-4)=6	6 + (-2j)(-j)=4
3-j	3-j +(3+j)=6	(-2)-(6)=-8
3+j	3-j -(3+j)=-2j	6 - (-2j)(-j)=8

$$X(k) = \{2, 3 + j, -4, 3 - j\}$$

$$x(n) = \{1,1, -2,2\}$$

Thank You!