



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)



GRADIENT, DIVERGENCE & CURL



Vector differential operator: ∇



- $\nabla \equiv \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z}$
- The operator ∇ is known as '*Nabla*'



The Gradient: $\nabla\phi$

- Let $\phi(x, y, z)$ be defined and differentiable at each point (x, y, z) in a certain region of space. Then ***the gradient of ϕ*** , is defined by

$$\nabla\phi \equiv \left(\frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right) \phi = \frac{\partial\phi}{\partial x} \mathbf{i} + \frac{\partial\phi}{\partial y} \mathbf{j} + \frac{\partial\phi}{\partial z} \mathbf{k}$$



The Divergence: $\nabla \cdot \mathbf{V}$

- Let $\mathbf{V} = V_1\mathbf{i} + V_2\mathbf{j} + V_3\mathbf{k}$ be defined and differentiable at each point (x, y, z) in a certain region of space.
- Then ***the divergence of \mathbf{V}*** , is defined by

$$\nabla \cdot \mathbf{V} \equiv \left(\frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right) \cdot (V_1\mathbf{i} + V_2\mathbf{j} + V_3\mathbf{k}) = \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}$$

N.B.: $\nabla \cdot \mathbf{V}$ defines a scalar Field



The Curl: $\nabla \times \mathbf{V}$

- Let $\mathbf{V} = V_1\mathbf{i} + V_2\mathbf{j} + V_3\mathbf{k}$ be defined and differentiable at each point (x, y, z) in a certain region of space. Then **the curl of \mathbf{V}** , is defined by

$$\begin{aligned}\nabla \times \mathbf{V} &\equiv \left(\frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right) \times (V_1\mathbf{i} + V_2\mathbf{j} + V_3\mathbf{k}) \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_1 & V_2 & V_3 \end{vmatrix}\end{aligned}$$

N.B.: $\nabla \times \mathbf{V}$ defines a vector field



Some properties



$$1. \nabla \times \mathbf{V} \equiv \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = \left(\frac{\partial V_3}{\partial y} - \frac{\partial V_2}{\partial z} \right) \mathbf{i} - \left(\frac{\partial V_3}{\partial x} - \frac{\partial V_1}{\partial z} \right) \mathbf{j} + \left(\frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y} \right) \mathbf{k}$$

$$2. \nabla \times (\mathbf{A} + \mathbf{B}) = \nabla \times \mathbf{A} + \nabla \times \mathbf{B}$$

$$3. \nabla \cdot (\phi \mathbf{A}) = (\nabla \phi) \cdot \mathbf{A} + \phi (\nabla \cdot \mathbf{A})$$

$$4. \nabla \times (\phi \mathbf{A}) = (\nabla \phi) \times \mathbf{A} + \phi (\nabla \times \mathbf{A})$$

$$5. \nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$6. \nabla \cdot (\nabla \phi) \equiv \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$

$$7. \nabla \times (\nabla \cdot \mathbf{A}) \equiv \mathbf{0}$$

$$8. \nabla \cdot (\nabla \times \mathbf{A}) \equiv \mathbf{0}$$



Problem 1: If $\phi(x, y, z) = 3x^2y - y^3z^2$, find $\nabla\phi$ at the point $(1, -2, -1)$.



$$\text{Solution: } \nabla\phi = \left(\frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right) (3x^2y - y^3z^2)$$

$$= \mathbf{i} \frac{\partial}{\partial x} (3x^2y - y^3z^2) + \mathbf{j} \frac{\partial}{\partial y} (3x^2y - y^3z^2) +$$

$$\mathbf{k} \frac{\partial}{\partial z} (3x^2y - y^3z^2)$$

$$= 6xy \mathbf{i} + (3x^2 - 3y^2z^2) \mathbf{j} - 2y^3z \mathbf{k}$$

$$\nabla\phi \Big|_{(1, -2, -1)} = 6(1)(-2) \mathbf{i} + \{3(1)^2 - 3(-2)^2(-1)^2\} \mathbf{j}$$

$$- 2(-2)^3(-1) \mathbf{k}$$

$$= -12 \mathbf{i} - 9 \mathbf{j} - 16 \mathbf{k}$$



Problem 2: If $A = x^2z \mathbf{i} - 2y^3z^2 \mathbf{j} + xy^2z \mathbf{k}$,
find $\nabla \cdot A$ at the point $(1, -1, 1)$.

Solution:

$$\nabla \cdot A = \left(\frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right) \cdot (x^2z \mathbf{i} - 2y^3z^2 \mathbf{j} + xy^2z \mathbf{k})$$

$$= \frac{\partial}{\partial x} (x^2z) + \frac{\partial}{\partial y} (-2y^3z^2) + \frac{\partial}{\partial z} (xy^2z)$$

$$= 2xz - 6y^2z^2 + xy^2$$

$$\begin{aligned} \nabla \cdot A \Big|_{(1, -1, 1)} &= 2(1)(1) - 6(-1)^2(1)^2 + (1)(-1)^2 \\ &= -3 \end{aligned}$$





Problem 3: If $\phi = 2x^3y^2z^4$, find $\nabla \cdot \nabla\phi$.

$$\text{Solution: } \nabla\phi = \left(\frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right) (2x^3y^2z^4)$$

$$= \mathbf{i} \frac{\partial}{\partial x} (2x^3y^2z^4) + \mathbf{j} \frac{\partial}{\partial y} (2x^3y^2z^4) + \mathbf{k} \frac{\partial}{\partial z} (2x^3y^2z^4)$$

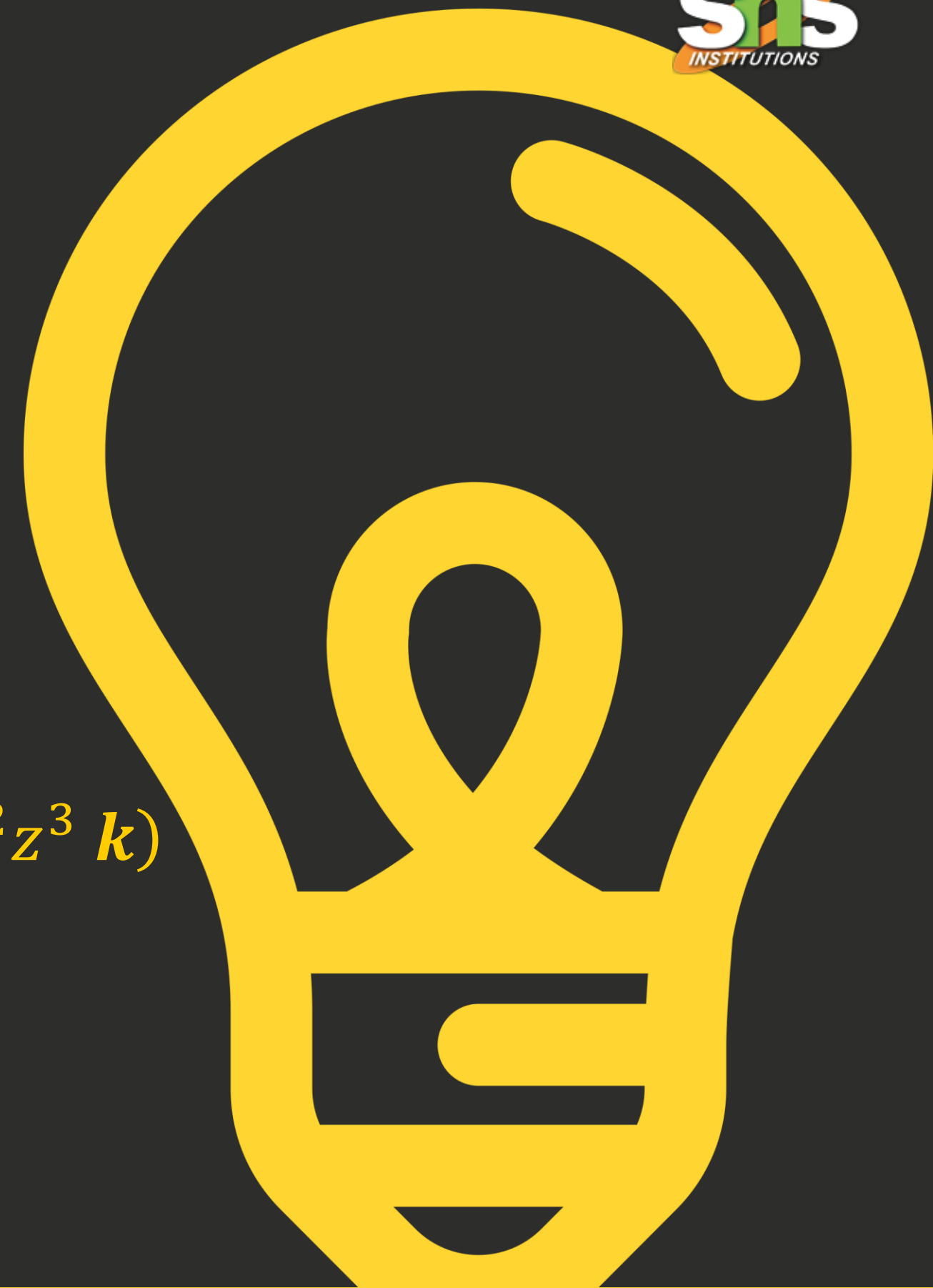
$$= 6x^2y^2z \mathbf{i} + 4x^3yz^4 \mathbf{j} + 8x^3y^2z^3 \mathbf{k}$$

Then,

$$\nabla \cdot \nabla\phi = \left(\frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \right) \cdot (6x^2y^2z \mathbf{i} + 4x^3yz^4 \mathbf{j} + 8x^3y^2z^3 \mathbf{k})$$

$$= \frac{\partial}{\partial x} (6x^2y^2z) + \frac{\partial}{\partial y} (4x^3yz^4) + \frac{\partial}{\partial z} (8x^3y^2z^3)$$

$$= 12xy^2z^4 + 4x^3z^4 + 24x^3y^2z^2$$





THANK YOU