SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
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COIMBATORE-641 035, TAMIL NADU

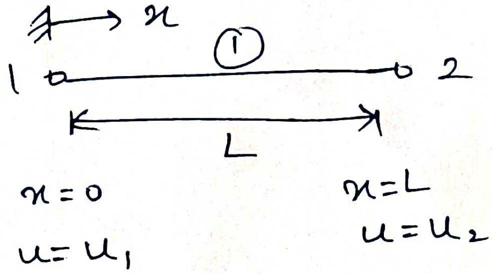
DEPARTMENT OF AEROSPACE ENGINEERING

DERIVATION (DISPLACEMENT FUNCTION) STEP FUNCTION)

Faculty Name	:	**Dr.M.Subramanian, Prof & Head/ Aerospace**	Academic Year	:	**2024-2025 (Odd)**
Year & Branch	:	**III Aerospace**	Semester	:	**V**
Course	:	**19ASB302 – Finite Element Method for Aerospace**			

Trial function equation

$u = a_0 + a_1 x$



In matrix form,

$$u = \begin{bmatrix} 1 & x \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \quad \text{—} (1)$$

Apply Boundary condition BC →

$u_1 = a_0 + a_1(0)$ $\qquad u_2 = a_0 + a_1 L$

$u_1 = a_0$ — (2) $\qquad u_2 = a_0 + a_1 L$ — (3)

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & L \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & L \end{bmatrix}^{-1} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{1}{L} \begin{bmatrix} L & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \text{—} (4)$$

(4) → (1)

$$U = \begin{bmatrix} 1 & x \end{bmatrix} \frac{1}{L} \begin{bmatrix} L & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$= \frac{1}{L}\begin{bmatrix}1 & x\end{bmatrix}\begin{bmatrix}L & 0\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$U = \frac{1}{L}\begin{bmatrix}L-x & x\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$= \begin{bmatrix}\frac{L-x}{L} & \frac{x}{L}\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$U = \frac{L-x}{L}u_1 + \frac{x}{L}u_2 = N_1u_1 + N_2u_2$$

$$N_1 = \frac{L-x}{L} \; ; \; N_2 = \frac{x}{L}$$

BC^1 $x = 0$ (Boundary condition)

$$N_1 = \frac{L-0}{L} = 1 \qquad N_2 = \frac{0}{L} = 0$$

BC^2: $x = L$

$$N_1 = \frac{L-L}{L} = 0 \qquad N_2 = \frac{L}{L} = 1$$

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Shape function of 1-D linear bar element

$$U = N_1 u_1 + N_2 u_2$$

$$U = \left(\frac{L-x}{L}\right) u_1 + \left(\frac{x}{L}\right) u_2$$

Stiffness matrix 1D bar element.

$$[K] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K][U] = [F] \longrightarrow \text{Load}$$

displacement

Stress on element.

$$\sigma = E \cdot e$$

$$= E \cdot \frac{du}{dx} = E\,\frac{u_2 - u_1}{x}$$

Force vector / Load vector for 1-D

$$[F] = \frac{\rho A L}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Reaction force,

$$[R] = [K][u] - [F]$$