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SNS INSTITUTIONS

DEPARTMENT OF AEROSPACE ENGINEERING

Faculty Name	:	**Dr.M.Subramanian, Prof & Head/ Aerospace**	Academic Year	:	**2024-2025 (Odd)**
Year & Branch	:	**III Aerospace**	Semester	:	**V**
Course	:	**19ASB302 – Finite Element Method for Aerospace**			

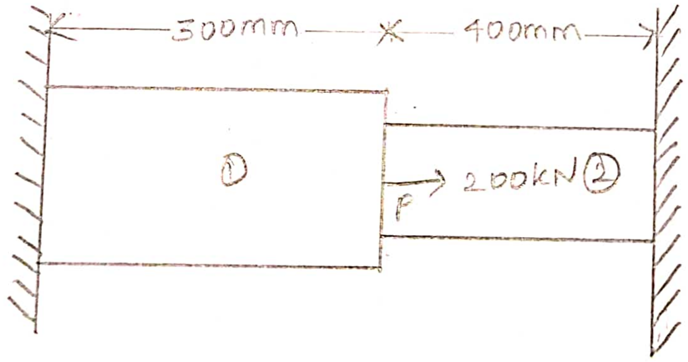
① Consider a bar as shown in fig.(i). An axial load of 200kN is applied at point p. Take $A_1 = 2400\,mm^2$; $E_1 = 70\times10^9\ N/m^2$, $A_2 = 600\,mm^2$, $E_2 = 200\times10^9\ N/m^2$.

Calculate the following:

(a) The nodal displacement at point p.

(b) Stress in each material

(c) Reaction force.



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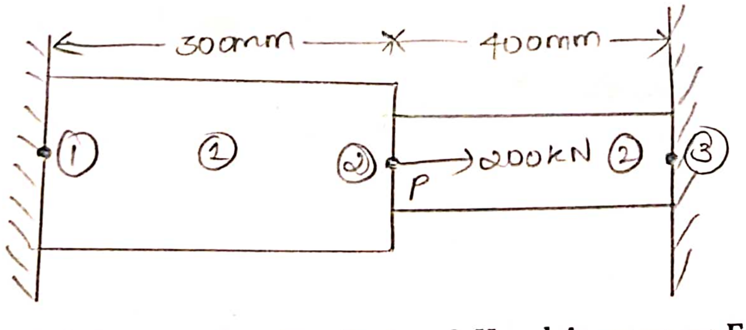


Area of element (1), $A_1 = 2400 mm^2$

Area of element (2), $A_2 = 600 mm^2$

Length of element (1), $l_1 = 300 mm$

Length of element (2), $l_2 = 400 mm$

Young's modulus of element (1), $E_1 = 70 \times 10^9 N/m^2$

$= 70 \times 10^3 N/mm^2$

Young's modulus of element (2), $E_2 = 200 \times 10^9 N/m^2$

$= 200 \times 10^3 N/mm^2$

Point load, $p = 200 kN = 200 \times 10^3 N$

TO FIND:

(i) Nodal displacement at point, p i.e., u_2.

(ii) Stress in each material, σ_1 and σ_2

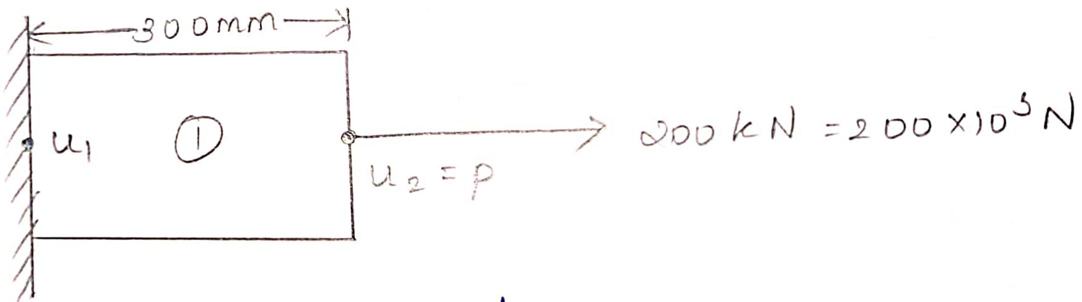
(iii) Reaction forces, R_1, R_2.

SOLUTION:

Finite element equation for one dimensional two noded bar element is given by,

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

For ~~example 1~~ element 1:



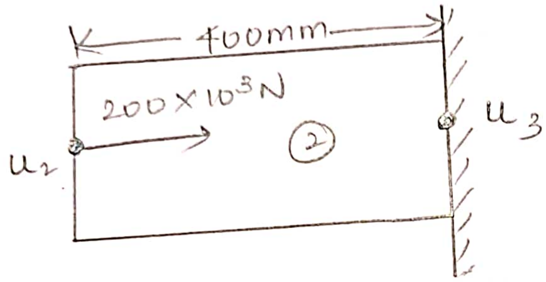
Finite element equation is,

$$\frac{A_1 E_1}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$\Rightarrow \frac{2400 \times 70 \times 10^3}{300} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$\Rightarrow 1 \times 10^5 \begin{bmatrix} \overset{a_{11}}{5.6} & \overset{a_{12}}{-5.6} \\ \overset{a_{21}}{-5.6} & \overset{a_{22}}{5.6} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad \ldots\ldots (1)$$

For element 2: Finite element equation is,



$$\frac{A_2 E_2}{l_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow \frac{600\times 200\times 10^3}{400}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} \quad \text{------} (2)$$

Assemble the finite elements. i.e., assemble the finite element equations (1) and (2).

$$\Rightarrow 1\times 10^5 \begin{bmatrix} \overset{a_{11}}{5.6} & \overset{a_{12}}{-5.6} & \overset{a_{13}}{0} \\ \overset{a_{21}}{-5.6} & \overset{a_{22}}{5.6+3} & \overset{a_{23}}{-3} \\ \overset{a_{31}}{0} & \overset{a_{32}}{-3} & \overset{a_{33}}{3} \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow 1\times10^5 \underbrace{\begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix}}_{[K]} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \quad \text{---} (3)$$

NOTE: The bar has 3 nodes. Each node has single degree of freedom. So, the global stiffness matrix [K] size is 3×3, The properties of the stiffness matrix are also satisfied.

(i) [K] matrix is symmetric.

(ii) The sum of elements in any column is equal to zero.

Applying boundary conditions:

Displacement at node 1 and node 3 are zero. So, $u_1 = u_3 = 0$. A Load of 200×10^3 N is acting at node 2. So, $F_2 = 200\times10^3$ N. Self-weight is neglected .i.e., $F_1 = F_3 = 0$. Substitute

u_1, u_3 and F_1, F_2 and F_3 values in equation (3).

$$(3) \Rightarrow 1\times 10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.8 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2\times 10^5 \\ 0 \end{Bmatrix}$$

In the above equation, $u_1 = 0$. So, neglect first row and first column of [k] matrix. $u_3 = 0$, so, neglect third row and third column of [k] matrix. The final reduced equation is,

$$1\times 10^5 [8.6] \{u_2\} = \{2\times 10^5\}$$

$$\Rightarrow 8.6\times 10^5 u_2 = 2\times 10^5$$

$$8.6 u_2 = 2$$

$$\boxed{u_2 = 0.2325 \text{ mm}}$$

Stress in each element:

We know that, stress, $\sigma = E \dfrac{du}{dx}$

For element (1), stress, $\sigma_1 = E_1 \times \dfrac{u_2 - u_1}{l_1} = 70\times 10^3 \times \dfrac{(0.2325 - 0)}{300}$

$$\boxed{\sigma_1 = 54.25\ N/mm^2}$$

For element(2), Stress, $\sigma_2 = E_2 \times \dfrac{u_3 - u_2}{l_2}$

$$= 200 \times 10^3 \times \frac{(0 - 0.2325)}{400}$$

$\Rightarrow \sigma_2 = -116.25\ N/mm^2$ [compressive stress is acting]

Reaction force:

We know that,

Reaction force, $\{R\} = [k]\{u^*\} - \{F\}$

$$\Rightarrow \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1 \times 10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} - \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1 \times 10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.2325 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 2 \times 10^5 \\ 0 \end{Bmatrix}$$

$$= 1 \times 10^5 \begin{bmatrix} -1.302 \\ 2 \\ -0.6975 \end{bmatrix} - \begin{Bmatrix} 0 \\ 2 \times 10^5 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 1.302 \times 10^5 \\ 2 \times 10^5 \\ -0.6975 \times 10^5 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 2 \times 10^5 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = \begin{Bmatrix} -1.302 \times 10^5 \\ 0 \\ -0.6975 \times 10^5 \end{Bmatrix}$$

$\Rightarrow R_1 = -1.302 \times 10^5$ N

$R_2 = 0$ N

$R_3 = -0.6975 \times 10^5$ N

We know that, Reaction force is equivalent and opposite to the applied force.

Verification:

$R_1 + R_2 + R_3 = -1.302 \times 10^5 + 0 - 0.6975 \times 10^5$

$= -200 \times 10^3$ N (Applied force)

Result: (i) Nodal displacement at point, p,

i.e., $u_2 = 0.2325$ mm.

(ii) Stress in each material,

$\sigma_1 = 54.25\ N/mm^2$ (tensile)

$\sigma_2 = -116.25\ N/mm^2$ (compressive)

(iii) Reaction forces,

$R_1 = -1.302 \times 10^5\ N$; $R_2 = 0$

$R_3 = -0.695 \times 10^5\ N$