23GET275 & VQAR – 1

Unit-1 QUANTITAVE ABILITY – I

Topic 3 : Square root and Cube Root, Decimal & Fraction

1. Square Roots

Definition:

The square root of a number xxx is a number yyy such that y2=xy^2 = xy2=x. It's denoted as x\sqrt{x}x.

Calculating Square Roots

1. **Simple Square Roots:

- **Perfect Squares:** Numbers like 1, 4, 9, 16, 25, etc., have exact square roots.
  - **Example:** 16=4\sqrt{16} = 416=4

2. **Non-Perfect Squares:

- Use approximation methods or a calculator.
  - **Example:** To find 50\sqrt{50}50:
    - We know 49=7\sqrt{49} = 749=7 and 64=8\sqrt{64} = 864=8.
    - Thus, 50\sqrt{50}50 is slightly more than 7, approximately 7.077.077.07.

3. **Using Approximation for Non-Perfect Squares:

- **Method:**
  - Find the two nearest perfect squares.
  - Use linear interpolation for an approximate value.
  - **Example:** 30\sqrt{30}30:
    - Nearest perfect squares are 25 and 36.
    - Approximate 30\sqrt{30}30 is between 555 and 666, closer to 5.5.

4. **Square Root Shortcuts:

- For numbers ending in 5: (10k+5)2=100k(k+1)+25(10k + 5)^2 = 100k(k + 1) + 25(10k+5)2=100k(k+1)+25
  - **Example:** 352=122535^2 = 1225352=1225
  - Therefore, 1225=35\sqrt{1225} = 351225=35.

Calculating Square Roots Manually:

1. **Prime Factorization Method:

- **Example:** Find 72\sqrt{72}72:
  - Factorize 72: 72=23×3272 = 2^3 \times 3^272=23×32
  - Square root: 72=23×32=23/2×32/2=22×3=62\sqrt{72} = \sqrt{2^3 \times 3^2} = 2^{3/2} \times 3^{2/2} = 2 \sqrt{2} \times 3 = 6 \sqrt{2}72=23×32=23/2×32/2=22×3=62

2. **Long Division Method:

- **Steps:**
  - Group digits in pairs from right to left.
  - Find the largest number whose square is less than or equal to the group.
  - Subtract and bring down the next pair of digits.
  - Repeat the process.
- **Example:** To find 529\sqrt{529}529:
  - Group digits: (5)(29)
  - Largest square less than 52 is 7 (since 72=497^2 = 4972=49)
  - Continue the division process to find that 529=23\sqrt{529} = 23529=23.

---

2. Cube Roots

Definition:

The cube root of a number xxx is a number yyy such that y3=xy^3 = xy3=x. It's denoted as x3\sqrt[3]{x}3x.

Calculating Cube Roots

1. **Simple Cube Roots:

- **Perfect Cubes:** Numbers like 1, 8, 27, 64, etc., have exact cube roots.
  - **Example:** 643=4\sqrt[3]{64} = 4364=4

2. **Non-Perfect Cubes:

- Use approximation methods or a calculator.
  - **Example:** To find 503\sqrt[3]{50}350:
    - We know 273=3\sqrt[3]{27} = 3327=3 and 643=4\sqrt[3]{64} = 4364=4.
    - Thus, 503\sqrt[3]{50}350 is slightly more than 3, approximately 3.683.683.68.

3. **Using Approximation for Non-Perfect Cubes:

- **Method:**
  - Find the two nearest perfect cubes.
  - Use interpolation for an approximate value.
  - **Example:** 303\sqrt[3]{30}330:
    - Nearest perfect cubes are 27 and 64.
    - Approximate 303\sqrt[3]{30}330 is between 333 and 444, closer to 3.1.

4. **Cube Root Shortcuts:

- For numbers with a known cube: (a+b)3=a3+3a2b+3ab2+b3(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3(a+b)3=a3+3a2b+3ab2+b3

- **Example:** (2+3)3=23+3·22·3+3·2·32+33(2 + 3)^3 = 2^3 + 3 \cdot 2^2 \cdot 3 + 3 \cdot 2 \cdot 3^2 + 3^3(2+3)3=23+3·22·3+3·2·32+33

Calculating Cube Roots Manually:

1. **Prime Factorization Method:

- **Example:** Find 723\sqrt[3]{72}372:
- Factorize 72: 72=23×3272 = 2^3 \times 3^272=23×32
- Cube root: 723=23×323=2×323≈2×1.44=2.88\sqrt[3]{72} = \sqrt[3]{2^3 \times 3^2} = 2 \times \sqrt[3]{3^2} \approx 2 \times 1.44 = 2.88372=323×32=2×332≈2×1.44=2.88

2. **Cube Root by Estimation:

- **Example:** To estimate 1003\sqrt[3]{100}3100:
- Between 643=4\sqrt[3]{64} = 4364=4 and 1253=5\sqrt[3]{125} = 53125=5.
- Approximate 1003≈4.64\sqrt[3]{100} \approx 4.643100≈4.64.

---

3. Decimal and Fraction Conversions

Decimal to Fraction

1. **Simple Decimals:

- **Example:** Convert 0.75 to a fraction:
- 0.75 = 75/100
- Simplify by dividing both numerator and denominator by their GCD, which is 25: 75÷25100÷25=34\frac{75 \div 25}{100 \div 25} = \frac{3}{4}100÷2575÷25=43

2. **Recurring Decimals:

- **Example:** Convert 0.666… to a fraction:
- Let x=0.666…x = 0.666\ldotsx=0.666…
- 10x=6.666…10x = 6.666\ldots10x=6.666…
- Subtract xxx from 10x10x10x: 10x−x=610x - x = 610x−x=6
- 9x=69x = 69x=6
- x=69=23x = \frac{6}{9} = \frac{2}{3}x=96=32

Fraction to Decimal

1. **Simple Fractions:

- **Example:** Convert 3/4 to a decimal:
- Divide 3 by 4: 3÷4=0.753 \div 4 = 0.753÷4=0.75

2. **Recurring Fractions:

- **Example:** Convert 7/6 to a decimal:
- Divide 7 by 6: 7÷6=1.1666…=1.16¯7 \div 6 = 1.1666\ldots = 1.1\overline{6}7÷6=1.1666…=1.16

Conversions for Mixed Numbers

1. **Mixed Number to Fraction:

- **Example:** Convert 2 1/3 to a fraction:
  - 213=2×3+13=732 \frac{1}{3} = \frac{2 \times 3 + 1}{3} = \frac{7}{3}231=32×3+1=37

2. **Fraction to Mixed Number:

- **Example:** Convert 11/4 to a mixed number:
  - Divide 11 by 4: Quotient = 2, Remainder = 3
  - 114=234\frac{11}{4} = 2 \frac{3}{4}411=243