



## DEPARTMENT OF MATHEMATICS

### PUZZLE-2

**1. If you pick five numbers from the integers 1 to 8, then two of them must add up to nine.**

Every number can be paired with another to sum to nine. In all, there are four such pairs: the numbers 1 and 8, 2 and 7, 3 and 6, and lastly 4 and 5.

Each of the five numbers belongs to one of those four pairs. By the pigeonhole principle, two of the numbers must be from the same pair—which by construction sums to 9.

**2.If you draw five points on the surface of an orange in permanent marker, then there is a way to cut the orange in half so that four of the points will lie on the same hemisphere (suppose a point exactly on the cut belongs to both hemispheres).**

Two points determine a great circle on a sphere, so for any two points, cut the orange into half. The remaining three points can be on either one of the two resulting hemispheres. By the pigeonhole principle, at least two of them belong to the same hemisphere, bringing the total to 4 points.

**3.Imagine a certain college has 6,000 American students, at least one from each of the 50 states. Then there must be a group of 120 students coming from same state.**

Again, we invoke the second version that “the maximum must at least be the average.”

The average is  $6,000 / 50 = 120$  students per state. The maximum must at least be the average, so there must be a state where 120 students share in common.