23MCT203 - Theory of Control Engineering

MASON'S GAIN FORMULA

INTRODUCTION

Mason's gain formula provides a graph-theoretic method to calculate the gain of a control system. The system block diagram is redrawn as a signal-flow graph and Mason's formula is then applied. This technique is useful for any block diagram but is especially useful when complicated block diagrams would involve many reduction steps.

Signal-flow diagrams are composed of nodes and arcs. Summation junctions and pickoff points become nodes in the graph. Signals are branches connecting the nodes. The signal gain is the branch weight. Forward and feedback gain paths will be identifiable from the signal flow graph. These gain paths become terms in Mason's equation.

The learning objectives for this EE371 technical note are:

1. **Define** the key terms describing signal-flow graphs.
2. **Draw** signal flow graphs for a block diagram.
3. **Draw** a block diagram from a signal-flow graph.
4. **State** Mason's gain formula.
5. **List** the steps in the process used to solve a system using Mason's gain formula.
6. **Apply** Mason's gain formula to systems in block-diagrams or signal-graph form.

DEFINITIONS

Forward paths: Forward paths are continuous paths through the graph from input to output. No node is passed more than once.

Feedback loops: Feedback loops are continuous paths through the graph that start and end at the same node.

Path gain: Path gain is the produce of signal gains encountered on the path.

Loop gain: Loop gain is the product of signal gains encountered in a feedback loop.

Source node: Source nodes are nodes with only outgoing branches.

Sink nodes: Sink nodes are nodes with only incoming branches.

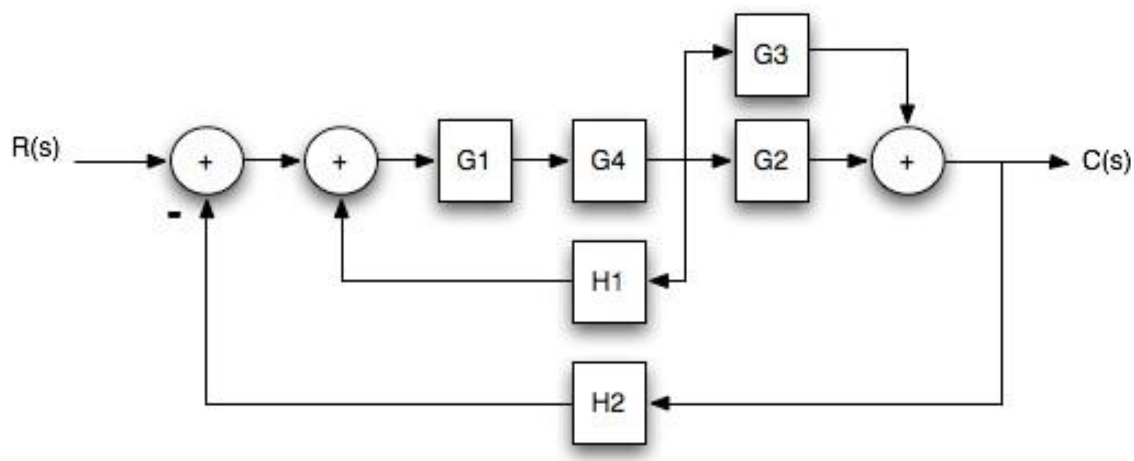
PROCESS FOR CREATING A SIGNAL-FLOW GRAPH

1. **Add** an input source node.
2. **Add** an output sink node.
3. **Add** a node for each summation node.
4. **Add** a node for each pickoff point.
5. **Interconnect** the nodes with branches of appropriate weight. **Use** the gain of a block sitting between the nodes as the branch weight. **Use** unity gain (1) if there is no block sitting between the nodes because the nodes are simply interconnected with a wire.

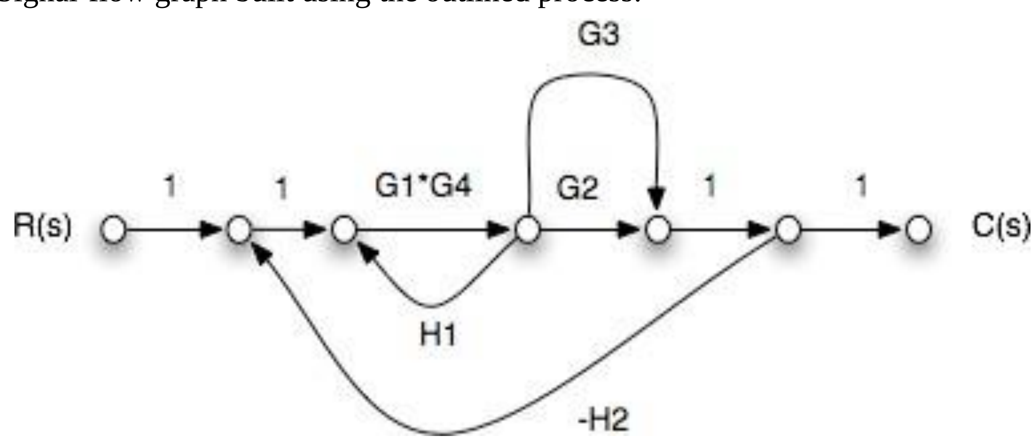
Note that this process can be reversed to generate the block diagram from a signal-flow graph.

EXAMPLE SIGNAL-FLOW GRAPHS

Original block diagram[1]:



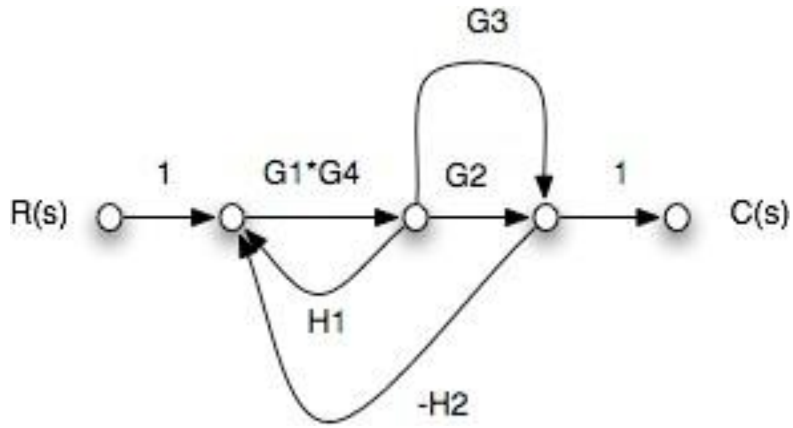
Signal-flow graph built using the outlined process:



---

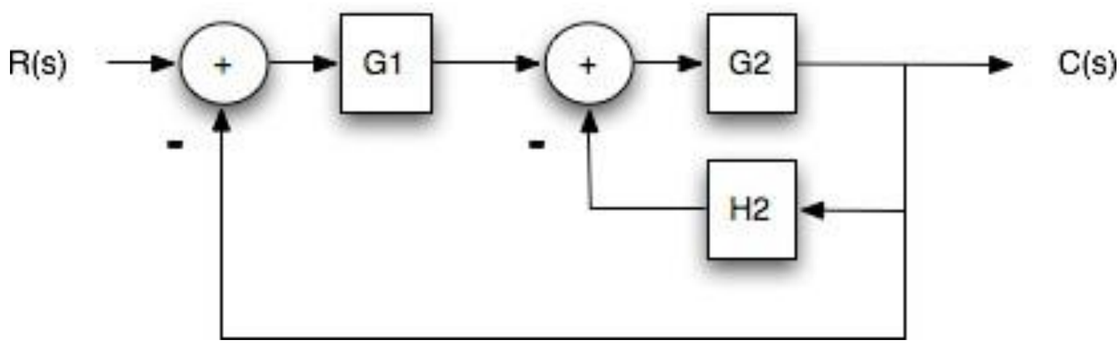
[1] This diagram was taken from a textbook reference.

Reduced signal-flow graph created by removing branches of unity gain:

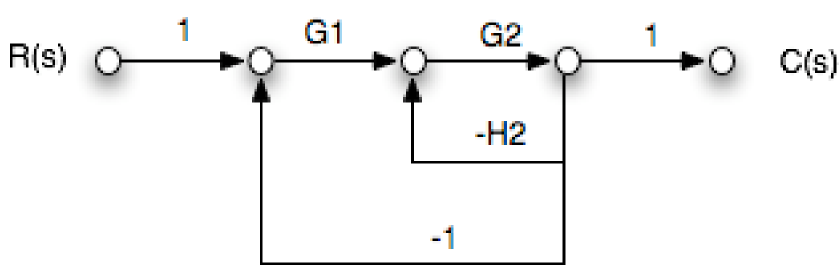


Note that the instructor always leaves the source and sink nodes separated from the system by unity gain branches even though they could be reduced.

Original block diagram:



Signal-flow graph built using the outlined process:



Note the path of unity gain and how it is represented on the signal-flow graph. The weight is -1 because it is negative feedback in the block diagram (- sign at the summation node).

MASON’S GAIN FORMULA

Mason’s gain formula is applied to a signal-flow graph to generate the transfer function. The formula is:

$$T(s) = \frac{C(s)}{R(s)} = \frac{\sum_k P_k \Delta_k}{\Delta}$$

where:

- P_K represents the path gain of the kth forward path
- $\Delta = 1 - \sum loop\ gains + \sum nontouching\ loop\ gains\ taken\ 2\ at\ a\ time$
  $- \sum nontouching\ loop\ gains\ taken\ 3\ at\ a\ time + \ldots$
- $\Delta_k = \Delta - \sum loop\ gains\ in\ \Delta\ that\ touch\ forward\ path\ k$

This equation seems very complicated to use at first glance but actually is straightforward if a systematic procedure is applied to help identify all terms.

1. **List** all forward paths
2. **List** all feedback loops
3. **List** all feedback loops that don’t touch each other
4. **List** all feedback loops that touch each path
5. **Calculate** the values of delta and delta-k and solve the equation.