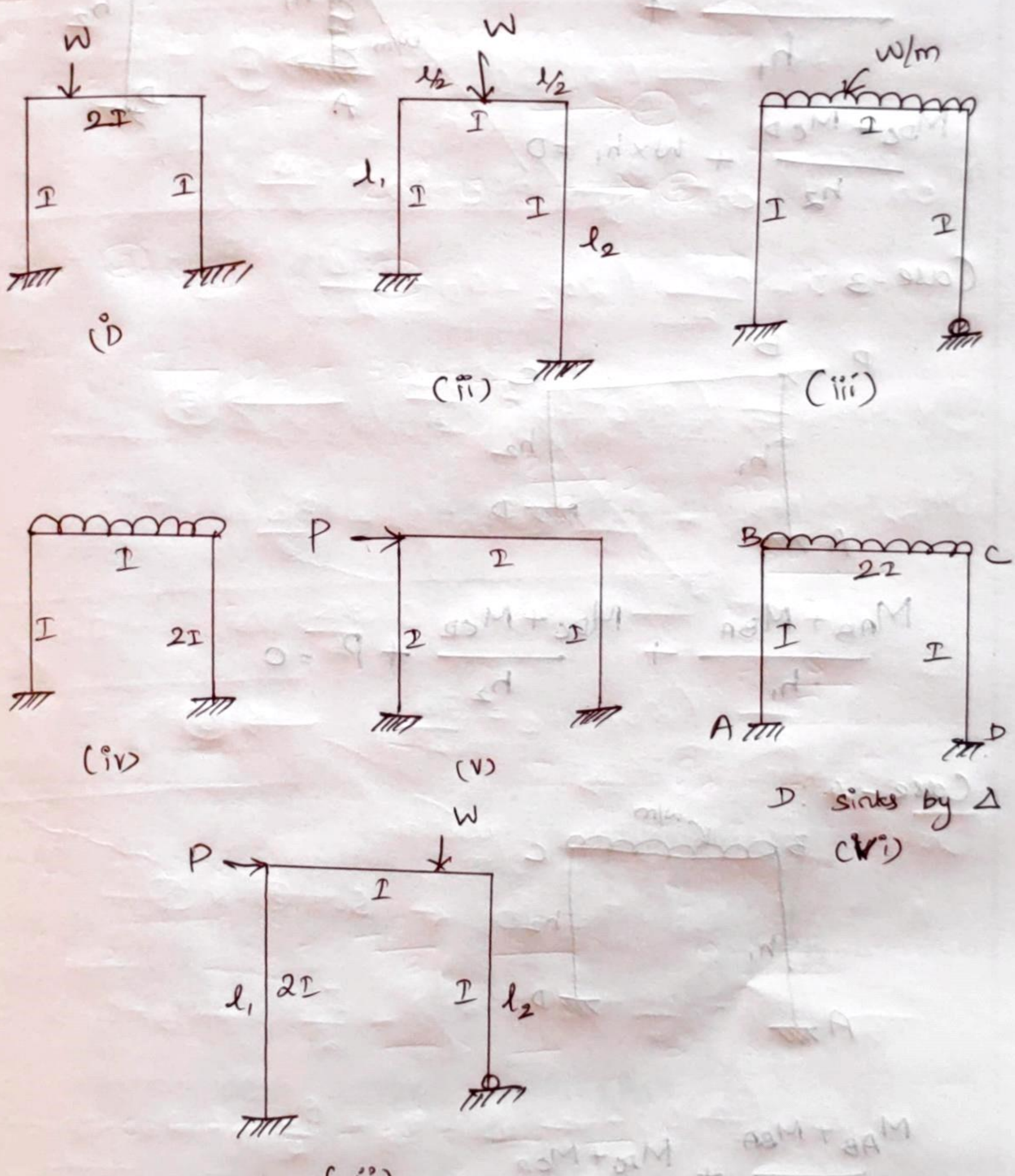
Portal Frames with Side sway:

Portal frames may sway due to one of the following reasons.

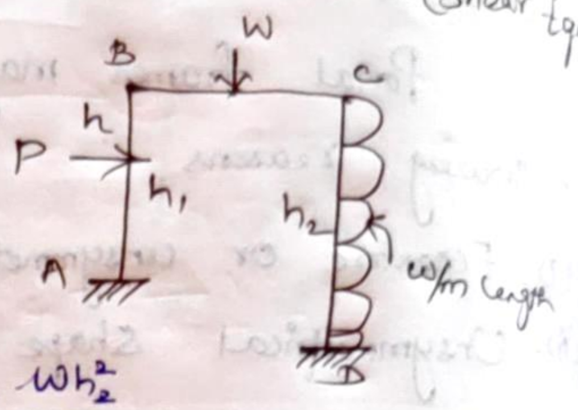
(i). Eccentric or unsymmetrical loading on the portal frames.
(ii). Unsymmetrical shape of the frame.
(iii) Different end conditions of the columns of the portal frame.
(iv) Non Uniform Section of the members of the frame.
(v) Horizontal loading on the columns of the frames.
(vi) Settlement of the supports of the frame.
(vii) A combination of the above.



Frame Sway Conditions

Genaral Equations for different conditions :(Shear Equati

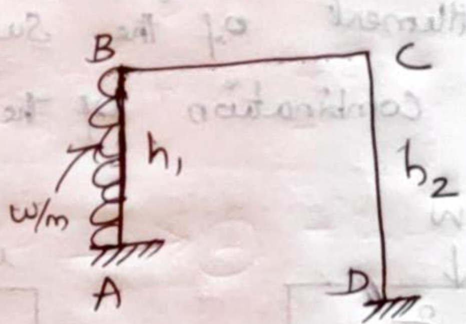
Case - 1 :



$$H_A = \frac{M_{BA} + M_{AB} - (P \times h)}{h_1}$$

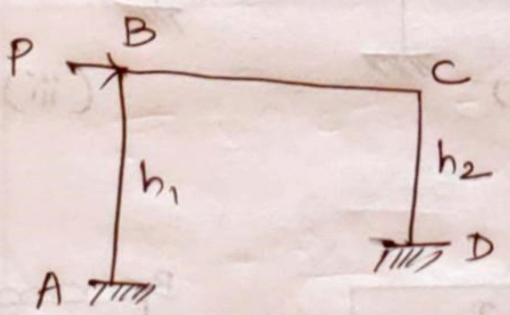
$$\frac{M_{AB} + M_{BA} - P \times h}{h_1} + \frac{M_{DC} + M_{CD} + \frac{wh_2^2}{2}}{h_2} + P - w \times h_2 = 0$$

Case - 2 :



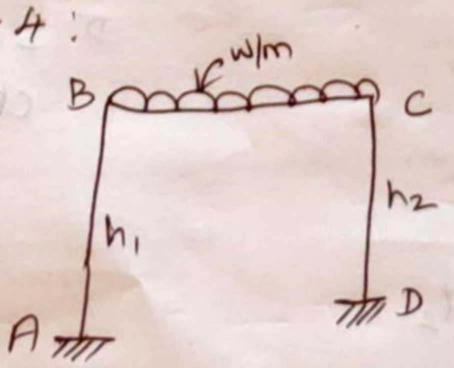
$$\frac{M_{AB} + M_{BA} - \frac{wh_1^2}{2}}{h_1} + \frac{M_{DC} + M_{CD}}{h_2} + w \times h_1 = 0$$

Case - 3 :



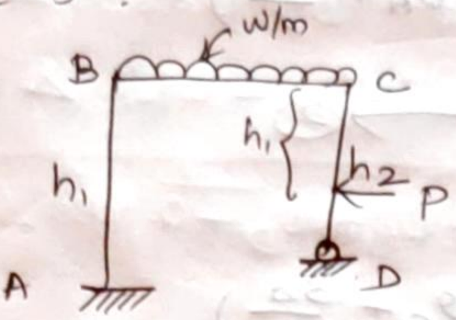
$$\frac{M_{AB} + M_{BA}}{h_1} + \frac{M_{DC} + M_{CD}}{h_2} + P = 0$$

Case - 4 :



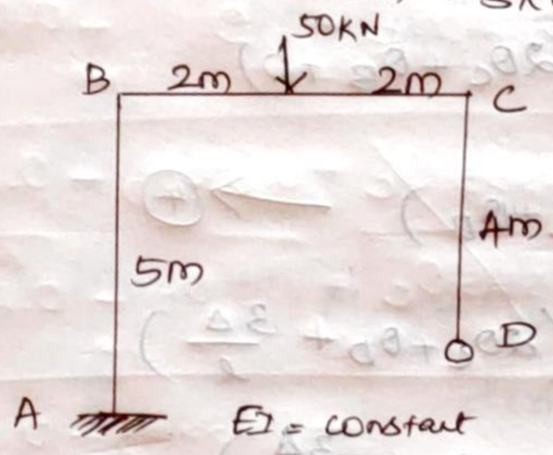
$$\frac{M_{AB} + M_{BA}}{h_1} + \frac{M_{DC} + M_{CD}}{h_2} = 0$$

Case-5 :



$$\frac{M_{AB}+M_{BA}}{h_1}+\frac{M_{CD}+Ph}{h_2}-P=0$$

6 Analyse the portal frame as shown in figure by Slope deflection method. Sketch the BMD and SFD.



Soln :

Fixed End Moments :

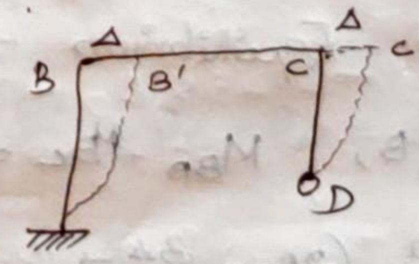
$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$

$M_{FBC} = -\frac{wl}{8} = \frac{50 \times 4}{8} = -25$ KNm

$M_{FCB} = \frac{wl}{8} = \frac{50\times4}{8} = 25$ KNm

Slope Deflection Equation :

Due to unsymmetrical shape and end conditions sway Occurs in the frame,



$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B + \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{5}\left(\theta_B - \frac{3\Delta}{5}\right) \longrightarrow ①$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A + \frac{3\Delta}{l}\right)$$

$$= \frac{2EI}{5}\left(2\theta_B - \frac{3\Delta}{5}\right) \longrightarrow (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C + \frac{3\Delta}{l}\right)$$

$$= -25 + \frac{2EI}{4}(2\theta_B + \theta_C + 0)$$

$$= -25 + \frac{EI}{2}(2\theta_B + \theta_C) \longrightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B + \frac{3\Delta}{l}\right)$$

$$= 25 + \frac{2EI}{4}(2\theta_C + \theta_B + 0)$$

$$= 25 + \frac{EI}{2}(2\theta_C + \theta_B) \longrightarrow (4)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left(2\theta_C + \theta_D + \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{4}\left(2\theta_C + \theta_D - \frac{3\Delta}{4}\right)$$

$$= \frac{EI}{2}\left(2\theta_C + \theta_D - \frac{3\Delta}{4}\right) \longrightarrow (5)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left(2\theta_D + \theta_C + \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{4}\left(2\theta_D + \theta_C - \frac{3\Delta}{4}\right)$$

$$= \frac{EI}{2}\left(2\theta_D + \theta_C - \frac{3\Delta}{4}\right) \longrightarrow (6)$$

Joint Equilibrium Equation:

At B, $M_{BA} + M_{BC} = 0 \Leftarrow$ sub (2) & (3)

$$\frac{2EI}{5}\left(2\theta_B - \frac{3\Delta}{5}\right) + (-25) + \frac{EI}{2}(2\theta_B + \theta_C) = 0$$

$$EI\left(\frac{4}{5}\theta_B - \frac{6}{25}\Delta + \theta_B + \frac{\theta_C}{2}\right) - 25 = 0$$

$$EI\left(\frac{9}{5}\theta_B - \frac{6}{25}\Delta + \theta_C/2\right) = 25$$

$$EI(1.8\theta_B + 0.5\theta_C - 0.24\Delta) = 25 \longrightarrow (7)$$

At C, $M_{CB} + M_{CD} = 0$

$$-25 + \frac{EI}{2}(2\theta_C + \theta_B) + \frac{EI}{2}\left(2\theta_C + \theta_D - \frac{3\Delta}{4}\right) = 0$$

$$\frac{EI}{2}\left(2\theta_C + \theta_B + 2\theta_C + \theta_D - \frac{3\Delta}{4}\right) = -25$$

$$EI\left(4\theta_C + \theta_B + \theta_D - \frac{3\Delta}{4}\right) = -50 \longrightarrow (8)$$

At D, $M_{DC} = 0$,

$$\frac{EI}{2}\left(2\theta_D + \theta_C - \frac{3\Delta}{4}\right) = 0$$

$$2\theta_D + \theta_C - \frac{3\Delta}{4} = 0 \longrightarrow (9)$$

Shear Equation :

$$\frac{M_{AB} + M_{BA}}{l_{AB}} + \frac{M_{CD} + M_{DC}}{l_{CD}} = 0$$

$$\frac{\frac{2EI}{5}\left(\theta_B - \frac{3\Delta}{5}\right) + \frac{2EI}{5}\left(2\theta_B - \frac{3\Delta}{5}\right)}{5} + \frac{\frac{EI}{2}\left(2\theta_C + \theta_D - \frac{3\Delta}{4}\right) + \frac{EI}{2}\left(2\theta_D + \theta_C - \frac{3\Delta}{4}\right)}{4} = 0$$

$$\frac{\frac{2EI}{5}\left(3\theta_B - \frac{6\Delta}{5}\right)}{5} + \frac{\frac{EI}{2}\left(3\theta_C + 3\theta_D - \frac{6\Delta}{4}\right)}{4} = 0$$

$$\frac{2EI}{25}\left(3\theta_B - \frac{6\Delta}{5}\right) + \frac{EI}{8}\left(3\theta_C + 3\theta_D - \frac{6\Delta}{4}\right) = 0$$

$$\frac{6EI}{25}\left(\theta_B - \frac{2\Delta}{5}\right) + \frac{3EI}{8}\left(\theta_C + \theta_D - \frac{\Delta}{2}\right) = 0$$

$$EI(0.24\theta_B - 0.096\Delta + 0.375\theta_C + 0.375\theta_D - 0.1875\Delta) = 0$$

$$EI(0.24\theta_B + 0.375\theta_C + 0.375\theta_D - 0.2835\Delta) = 0 \longrightarrow (10)$$

$$⑩ \Rightarrow 0.24\cancel{\theta_B} + 0.375\theta_C + 0.375\theta_D - 0.2835\Delta = 0$$

$$⑧\times 0.24 \Rightarrow \overset{(-)}{0.24\theta_B} \overset{(-)}{+ 0.960\theta_C} \overset{(-)}{+ 0.240\theta_D} \overset{(+)}{- 0.18\Delta} = \overset{(+)}{-12/EI}$$

$$-0.585\cancel{\theta_C} + 0.135\theta_D - 0.1035\Delta = 12/EI$$

$$9\times 0.585 \Rightarrow 0.585\theta_C + 1.17\theta_D - 0.4388\Delta = 0$$

$$1.305\theta_D - 0.5423\Delta = \frac{12}{EI} \to ⑪$$

$$⑩\times 1.8 \Rightarrow 0.432\cancel{\theta_B} + 0.675\theta_C + 0.675\theta_D - 0.510\Delta = 0$$

$$⑦\times 0.24 \Rightarrow \overset{(-)}{0.4320\theta_B} \overset{(-)}{+ 0.120\theta_C} \overset{(-)}{+ 0} \overset{(+)}{- 0.0576\Delta} = \overset{(-)}{6/EI}$$

$$0.555\cancel{\theta_C} + 0.675\theta_D - 0.4524\Delta = -6/EI$$

$$⑨\times 0.555 \Rightarrow \overset{(-)}{0.555\theta_C} \overset{(-)}{+ 1.11\theta_D} \overset{(+)}{- 0.4163\Delta} = 0$$

$$-0.435\theta_D - 0.0361\Delta = \frac{-6}{EI} \to ⑫$$

$$⑪\times 0.435 \Rightarrow 0.5677\cancel{\theta_D} - 0.2359\Delta = 5.22/EI$$

$$⑫\times 1.305 \Rightarrow -0.5677\theta_D - 0.0471\Delta = -7.83/EI$$

$$-0.283\Delta = -2.61/EI$$

$$\boxed{\Delta = 9.22/EI}$$

Sub Δ in ⑫ $\Rightarrow$

$$-0.435\theta_D - 0.0361\times\frac{9.22}{EI} = \frac{-6}{EI}$$

$$-0.435\theta_D - 0.332/EI = \frac{-6}{EI}$$

$$-0.435\theta_D = \frac{-6+0.332}{EI}$$

$$\theta_D = \frac{-5.668}{EI}\times\frac{1}{-0.435}$$

$$\boxed{\theta_D = \frac{13.02}{EI}}$$

Sub Δ and θ_D in (9) $\Rightarrow$

$$\frac{2\times 13.02}{EI} + \theta_c - \frac{3\times 9.22}{4\,EI} = 0$$

$$\frac{26.04}{EI} + \theta_c - \frac{6.915}{EI} = 0$$

$$\boxed{\theta_c = -\frac{19.13}{EI}}$$

Sub θ_c and Δ in (7) $\Rightarrow$

$$1.8\,\theta_B + 0.5\times -\frac{19.13}{EI} - 0.24\times\frac{9.22}{EI} = \frac{25}{EI}$$

$$1.8\,\theta_B - \frac{9.565}{EI} - \frac{2.21}{EI} = \frac{25}{EI}$$

$$1.8\,\theta_B = \frac{25}{EI} + \frac{9.565}{EI} + \frac{2.21}{EI}$$

$$1.8\,\theta_B = \frac{36.775}{EI}$$

$$\boxed{\theta_B = \frac{20.43}{EI}}$$

Final Moments:

$$M_{AB} = \frac{2EI}{5}\left(\theta_B - \frac{3\Delta}{5}\right)$$

$$= \frac{2EI}{5}\left(\frac{20.43}{EI} - \frac{3}{5}\left(\frac{9.22}{EI}\right)\right)$$

$$= 5.996 \text{ kNm}$$

$$M_{BA} = \frac{2EI}{5}\left(2\theta_B - \frac{3\Delta}{5}\right)$$

$$= \frac{2EI}{5}\left[2\times\frac{20.43}{EI} - \frac{3}{5}\left(\frac{9.22}{EI}\right)\right]$$

$$= 14.138 \text{ kNm}$$

$M_{BC} = -25 + \frac{EI}{2}(2\theta_B + \theta_C)$

$= -25 + \frac{EI}{2}\left[2 \times \frac{20.43}{EI} + \left(-\frac{19.13}{EI}\right)\right]$

$= -14.138$ kNm

$M_{CB} = 25 + \frac{EI}{2}(2\theta_C + \theta_B)$

$= 25 + \frac{EI}{2}\left(2 \times \left(-\frac{19.13}{EI}\right) + \frac{20.43}{EI}\right)$

$= 16.075$ kNm

$M_{CD} = \frac{EI}{2}\left(2\theta_C + \theta_D - \frac{3\Delta}{4}\right)$

$= \frac{EI}{2}\left(2 \times \frac{-19.14}{EI} + \frac{13.01}{EI} - \frac{3 \times 9.19}{4EI}\right)$

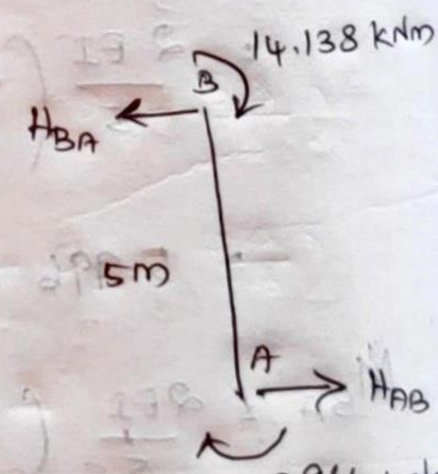
$= -16.075$ kNm

$M_{DC} = \frac{EI}{2}\left(2\theta_D + \theta_C - \frac{3\Delta}{4}\right)$

$= \frac{EI}{2}\left(2 \times \frac{13.01}{EI} + \frac{-19.14}{EI} - \frac{3 \times 9.19}{4EI}\right)$

$= 0$

Shear force:

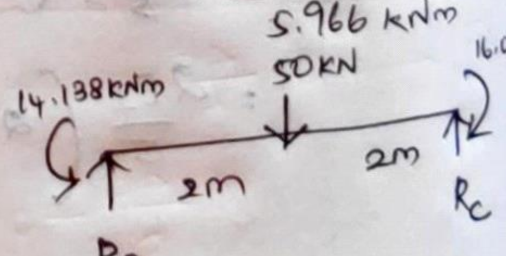


$\Sigma M_A = 0$,

$M_{AB} + M_{BA} - H_{BA} \times 5 = 0$

$5.966 + 14.138 - H_{BA} \times 5 = 0$

$H_{BA} = H_{AB} = 4.021$ kN



$\Sigma M_B = 0$,

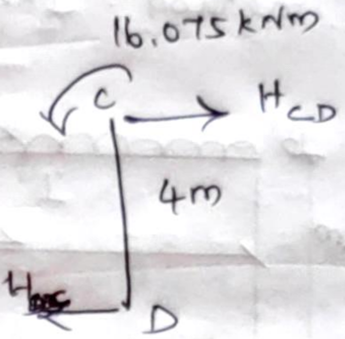
$M_{CB} + (50 \times 2) - M_{BC} - (R_C \times 4) = 0$

$16.075 + 100 - 14.138 - (R_C \times 4) = 0$

$R_C = 25.484$ kN

$R_B = 50 - 25.484$ kN

$= 24.516$ kN

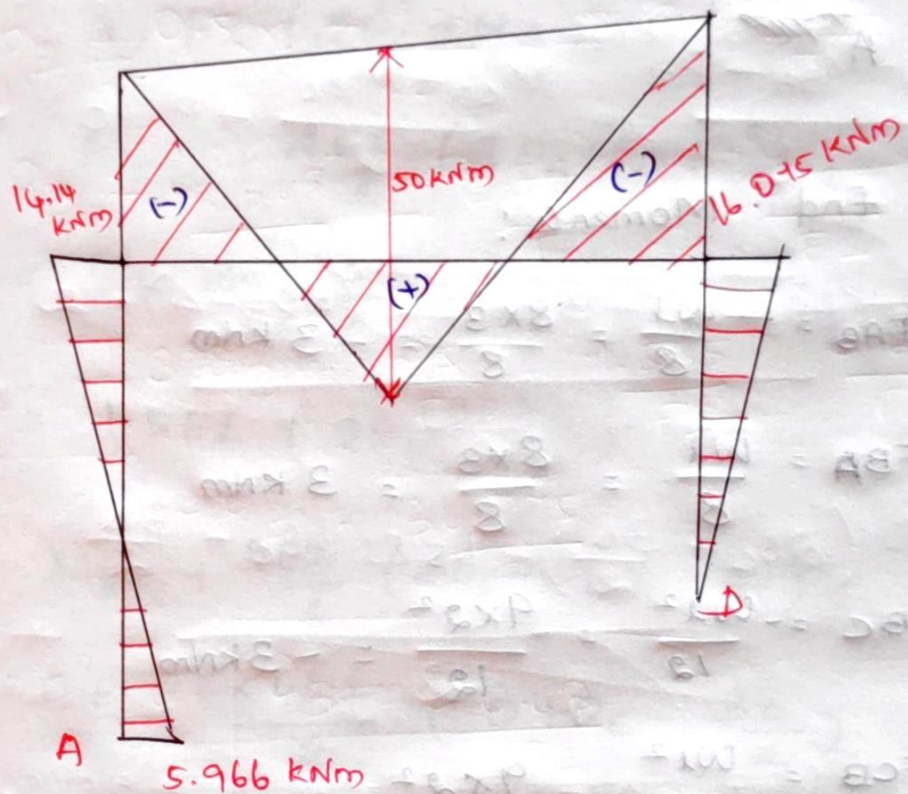


$\Sigma M_C = 0$

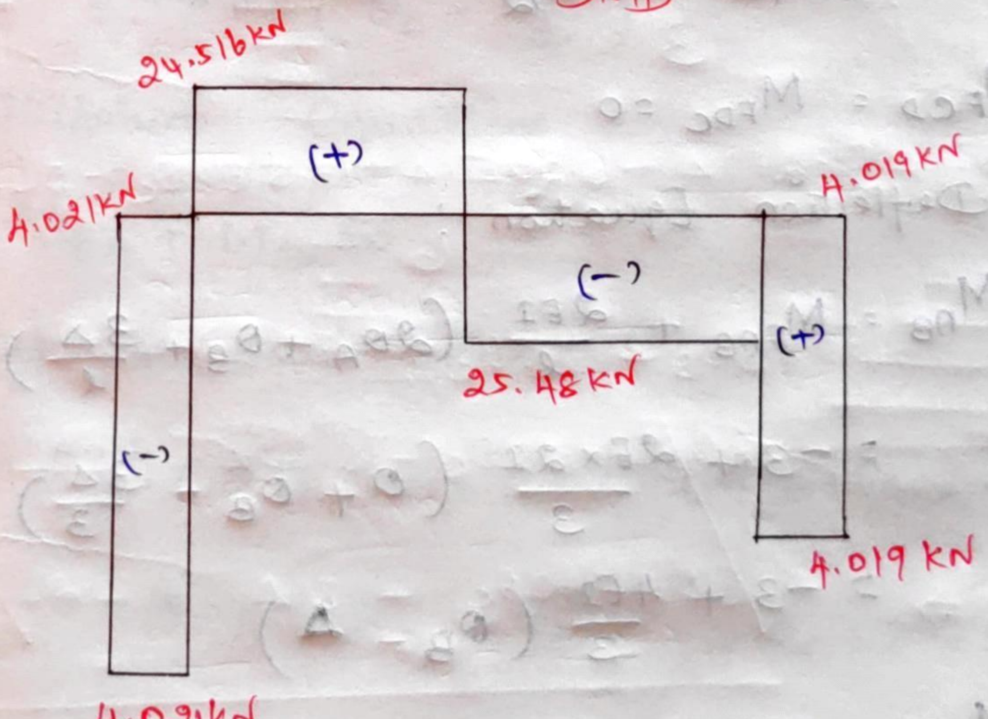
$-M_{CD} + H_{DC} \times 4 = 0$

$H_{DC} = \dfrac{16.075}{4}$

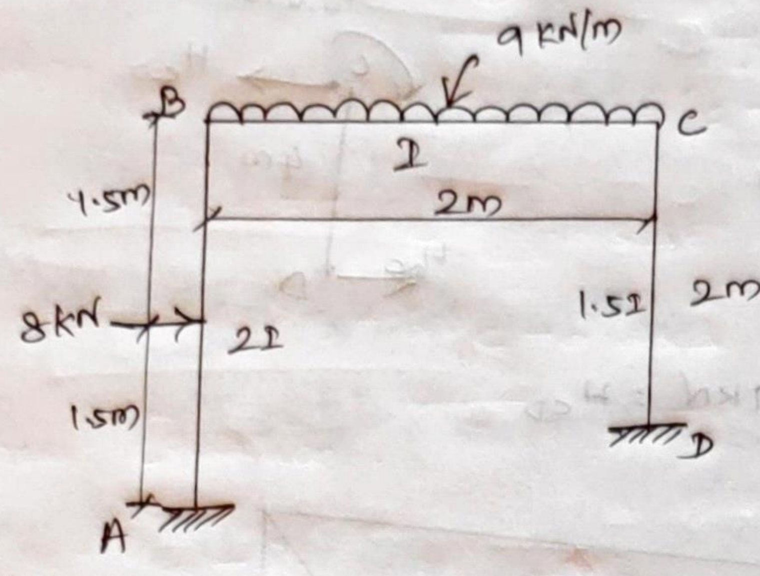
$H_{DC} = 4.019\text{ kN} = H_{CD}$



BMD



7. Analyse the frame shown in figure by the slope deflection method. Sketch the BMD.



Soln:

Fixed End Moments:

$$M_{FAB} = -\frac{Wl}{8} = \frac{8\times3}{8} = -3 \text{ kNm}$$

$$M_{FBA} = \frac{Wl}{8} = \frac{8\times3}{8} = 3 \text{ kNm}$$

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{9\times2^2}{12} = -3 \text{ kNm}$$

$$M_{FCB} = \frac{wl^2}{12} = \frac{9\times2^2}{12} = 3 \text{ kNm}$$

$$M_{FCD} = M_{FDC} = 0$$

Slope Deflection Equation:

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B + \frac{3\Delta}{l}\right)$$

$$= -3 + \frac{2E\times2I}{3}\left(0 + \theta_B - \frac{3\Delta}{3}\right)$$

$$= -3 + \frac{4EI}{3}\left(\theta_B - \Delta\right)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A + \frac{3\Delta}{l}\right)$$

$$= 3 + \frac{4EI}{3}\left(2\theta_B + 0 - \frac{3\Delta}{3}\right)$$

$$= 3 + \frac{4EI}{3}\left(2\theta_B - \Delta\right)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C + \frac{3\Delta}{l}\right)$$

$$= -3 + \frac{2EI}{2}(2\theta_B + \theta_C + 0)$$

$$= -3 + EI(2\theta_B + \theta_C)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B + \frac{3\Delta}{l}\right)$$

$$= 3 + \frac{EI \times 2}{2}(2\theta_C + \theta_B + 0)$$

$$= 3 + EI(2\theta_C + \theta_B)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left(2\theta_C + \theta_D + \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2E \times 1.5I}{2}\left(2\theta_C + 0 - \frac{3\Delta}{2}\right)$$

$$= 1.5EI\left(2\theta_C - \frac{3\Delta}{2}\right)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left(2\theta_D + \theta_C + \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2E \times 1.5I}{2}\left(0 + \theta_C - \frac{3\Delta}{2}\right)$$

$$= 1.5EI\left(\theta_C - \frac{3\Delta}{2}\right)$$

Equillibrium Equations :

$$M_{BA} + M_{BC} = 0 \longrightarrow (1)$$

Sub M_{BA} and M_{BC} in (1)

$$3 + \frac{4EI}{3}(2\theta_B - \Delta) - 3 + EI(2\theta_B + \theta_C) = 0$$

$$\frac{14\theta_B EI}{3} - \frac{4EI\Delta}{3} + \theta_C EI = 0$$

$$14\theta_B EI - 4EI\Delta + 3\theta_C EI = 0$$

$$14\theta_B - 4\Delta + 3\theta_C = 0 \longrightarrow (4)$$

$M_{CB} + M_{CD} = 0 \longrightarrow (2)$

Sub M_{CB} & M_{CD} in (2),

$3 + EI(2\theta_C + \theta_B) + 1.5\,EI\left(2\theta_C - \frac{3\Delta}{2}\right) = 0$

$EI\theta_B + 5EI\theta_C - 2.25\Delta EI = -3$

$\theta_B + 5\theta_C - 2.25\Delta = -3/EI \rightarrow (5)$

Shear Equation:

$\frac{M_{AB} + M_{BA} - Ph}{l} + \frac{M_{CD} + M_{DC}}{l} + P = 0 \longrightarrow (3)$

$$\frac{-3 + \frac{4EI}{3}(\theta_B - \Delta) + 3 + \frac{4EI}{3}(2\theta_B - \Delta) - 12}{3} +$$

$$\frac{1.5\,EI\left(2\theta_C - \frac{3\Delta}{2}\right) + 1.5\,EI\left(\theta_C - \frac{3\Delta}{2}\right)}{2} + 8 = 0$$

$$\frac{4EI\theta_B - 1.33EI\Delta - 1.33EI\Delta - 12}{3} + \frac{4.5EI\theta_C - 4.5EI\Delta}{2} + 8 = 0$$

$$\frac{8EI\theta_B - 5.33EI\Delta - 13.50EI\Delta + 13.5EI\theta_C - 24 + 48}{6} = 0$$

$EI(8\theta_B + 13.5\theta_C - 18.82\Delta) + 24 = 0$

$8\theta_B + 13.5\theta_C - 18.82\Delta = -\frac{24}{EI} \longrightarrow (6)$

(5) ×8 ⇒ $8\theta_B + 40\theta_C - 18\Delta = -24/EI$

$\quad\quad (-) \quad (-) \quad (+) \quad (+)$

$-26.5\theta_C - 0.82\Delta = 0 \longrightarrow (7)$

(4) ⇒ $14\theta_B + 3\theta_C - 48\Delta = 0$

(5)×14 ⇒ $14\theta_B + 70\theta_C - 31.5\Delta = -42/EI$

$\quad\quad (-) \quad (-) \quad (+) \quad (+)$

$-67\theta_C + 27.5\Delta = 42/EI \longrightarrow (8)$

$(7) \times -2.53 \Rightarrow \cancel{67\theta_c} - 2.07\Delta = 0$

$(8) \Rightarrow \cancel{67\theta_c} + 27.5\Delta = 42/EI$

$$-29.57\Delta = -42/EI$$

$$\Delta = \frac{1.42}{EI}$$

$(7) \Rightarrow -26.5\theta_c - 0.82 \times \frac{1.42}{EI} = 0$

$$\theta_c = -\frac{0.044}{EI}$$

$(6) \Rightarrow 8\theta_B = -\frac{24}{EI} - 13.5 \times \left(-\frac{0.044}{EI}\right) + 18.82\left(\frac{1.42}{EI}\right)$

$$\theta_B = \frac{0.4148}{EI}$$

Final Moments:

$M_{AB} = -3 + \frac{4EI}{3}\left(\frac{0.4148}{EI} - \frac{1.42}{EI}\right)$

$= -4.34$ kNm

$M_{BA} = 3 + \frac{4EI}{3}(2\theta_B - \Delta)$

$= 3 + \frac{4EI}{3}\left(2 \times \frac{0.4148}{EI} - \frac{1.42}{EI}\right)$

$= 2.21$ kNm

$M_{BC} = -3 + EI(2\theta_B + \theta_c)$

$= -3 + EI\left(2 \times \frac{0.4148}{EI} - \frac{0.044}{EI}\right)$

$= -2.21$ kNm

$M_{CB} = 3 + EI(2\theta_c + \theta_B)$

$= 3 + EI\left(2 \times -\frac{0.044}{EI} + \frac{0.4148}{EI}\right)$

$= 3.33$ kNm

$$M_{CD} = 1.5EI\left(2\theta_c - \frac{3\Delta}{2}\right)$$

$$= 1.5EI\left(2\times\frac{-0.044}{EI} - \frac{3\times1.42}{2EI}\right)$$

$$= -3.33\ kNm$$

$$M_{DC} = 1.5EI\left(\theta_c - \frac{3\Delta}{2}\right)$$

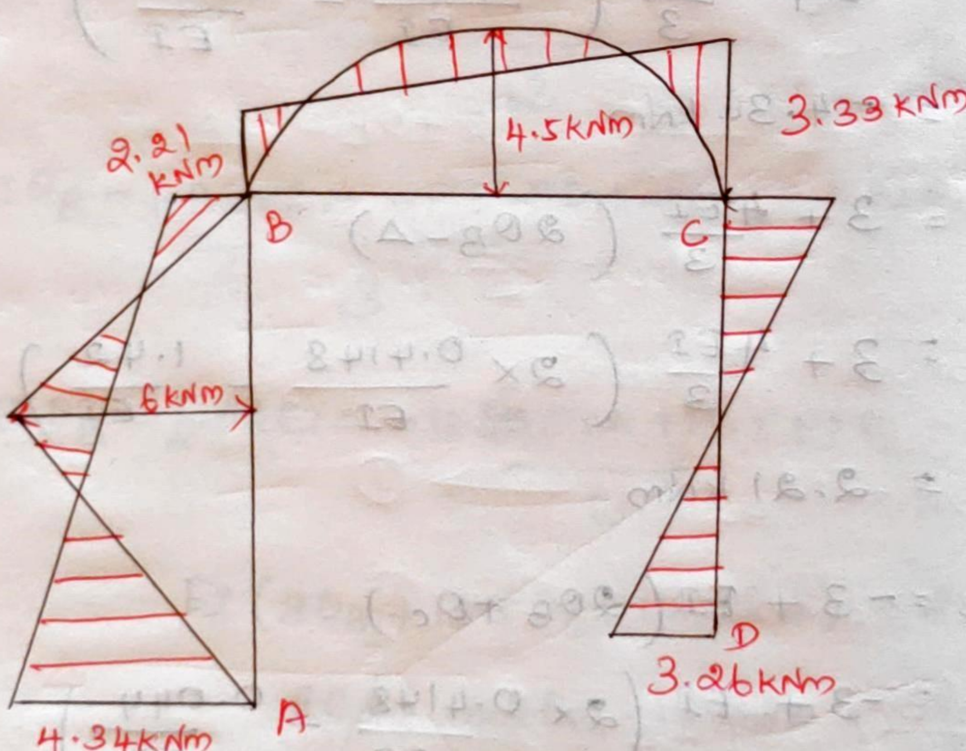
$$= 1.5EI\left(\frac{-0.044}{EI} - \frac{3}{2}\times\frac{1.42}{EI}\right)$$

$$= -3.26\ kNm$$

Maximum Bending Moment :-

$$@\ AB = \frac{Wl}{4} = \frac{8\times3}{4} = 6\ kNm$$

$$@\ BC : \frac{Wl^2}{8} = \frac{9\times2^2}{8} = 4.5\ kNm$$



BMD.