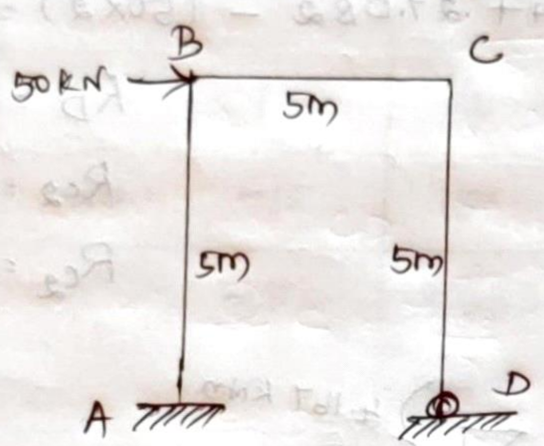
6. Analyse the portal frame loaded as shown in figure by moment distribution method. Sketch the BMD and SFD.



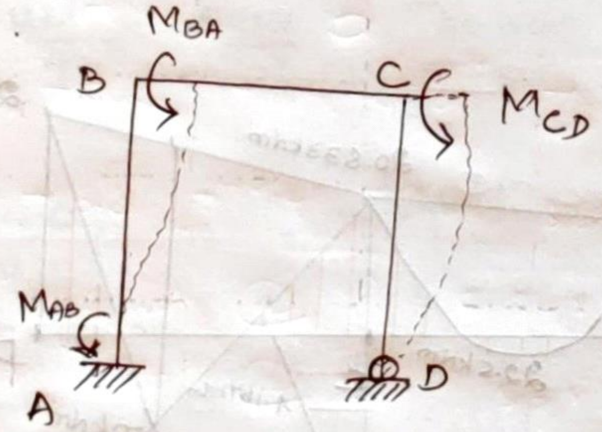
EI = constant

Soln:

Fixed End Moments:

$$M_{FAB} = M_{FBA} = M_{FBC} = M_{FCB} = M_{FCD} = M_{FDC} = 0$$

Since the load is acting at the joint, there will be no fixed end moments. However, due to side sway, moments will be induced at joints A, B and C.



Distribution factors:

Joint	Member	Relative Stiffness	Total stiffness	Distribution factor
B	BA	$I/5$	$I/5 + I/5 = \frac{2I}{5}$	$\frac{I/5}{2I/5} = 1/2$
	BC	$I/5$		$\frac{I/5}{2I/5} = 1/2$
C	CB	$I/5$	$I/5 + 3I/20 = 7I/20$	$\frac{I/5}{7I/20} = 4/7$
	CD	$\frac{3I}{4l} = \frac{3I}{4 \times 5} = \frac{3I}{20}$		$\frac{3I/20}{7I/20} = \frac{3}{7}$

Side Sway:

Under the action of 50kN load, there will be side sway to the right and the columns AB and CD will rotate in a clockwise direction. Thus negative moments will be induced at A, B and C in these columns. As the end 'A' is fixed and 'D' is hinged,

$$\frac{M_{BA}}{M_{CD}} = \frac{6EI\delta/l_1^2}{3EI\delta/l_2^2} = \frac{2}{1} = 2 \qquad M_{BA} = 2M_{CD}$$

Assume, $M_{BA} = -20\,kNm = M_{AB}$

$$M_{CD} = \frac{-20}{2} = -10\,kNm$$

Moment Distribution:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	—	1/2	1/2	4/7	3/7	—
FEM	-20	-20	—	—	-10	—
Balancing		10	10	5.71	4.29	
C/o	5		2.86	5		
Balance & C/o		-1.43	-1.43	-2.86	-2.14	
	-0.72		-1.43	-0.72		
Balance & C/o		0.72	0.72	0.41	0.31	
	0.36		0.21	0.36		
Balance & C/o		-0.11	-0.11	-0.21	-0.15	
	-0.06		-0.11	-0.06		
	+0.06	0.05	0.06	0.03	0.03	
Final Moments	-15.42	-10.76	10.77	7.66	-7.66	0

Sway Forces:

Balancing moment for AB = 10.76 + 15.42 = 26.18 kNm

Horizontal reaction at B = $\frac{26.18}{5}$ = 5.24 kN (→) = H_{BA}

@ A = 5.24 kN (←) = H_{AB}

Balancing moment for CD = 7.66 kNm

Horizontal reaction @ C = $\frac{7.66}{5}$ = 1.53 kN (→) = H_{CD}

@ D = 1.53 kN (←) = H_{DC}

Sway force due to assumed moment,

= 5.24 + 1.53 = 6.77 kN (←)

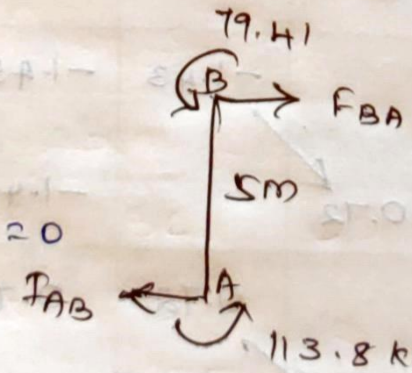
But actual sway force is 50 kN. Hence the moment will be, 50/6.77 = 7.38

Joints	A	B		C		D
H_{sway} = 6.77 kN	-15.42	-10.76	10.76	7.66	-7.66	0
H_{sway} = 50 kN	-113.8	-79.41	79.41	56.53	-56.53	0

Horizontal reaction @ A = 5.24 × 7.38 = 38.67 kN (←)

Horizontal reaction @ D = 1.53 × 7.38 = 11.29 kN (←)

Shear Force:

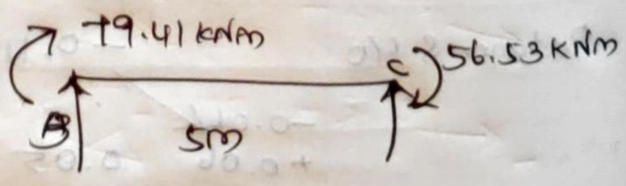


$\Sigma M_A = 0$,

$-79.41 - 113.8 + F_{BA} \times 5 = 0$

$F_{BA} = 38.64$ kN

$F_{AB} = 38.64$ kN

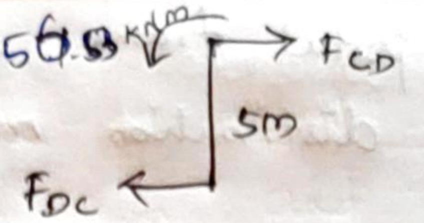


$\Sigma M_B = 0$,

$79.41 + 56.53 + F_{CB} \times 5 = 0$

$F_{CB} = -27.19$ kN $= F_{BC}$

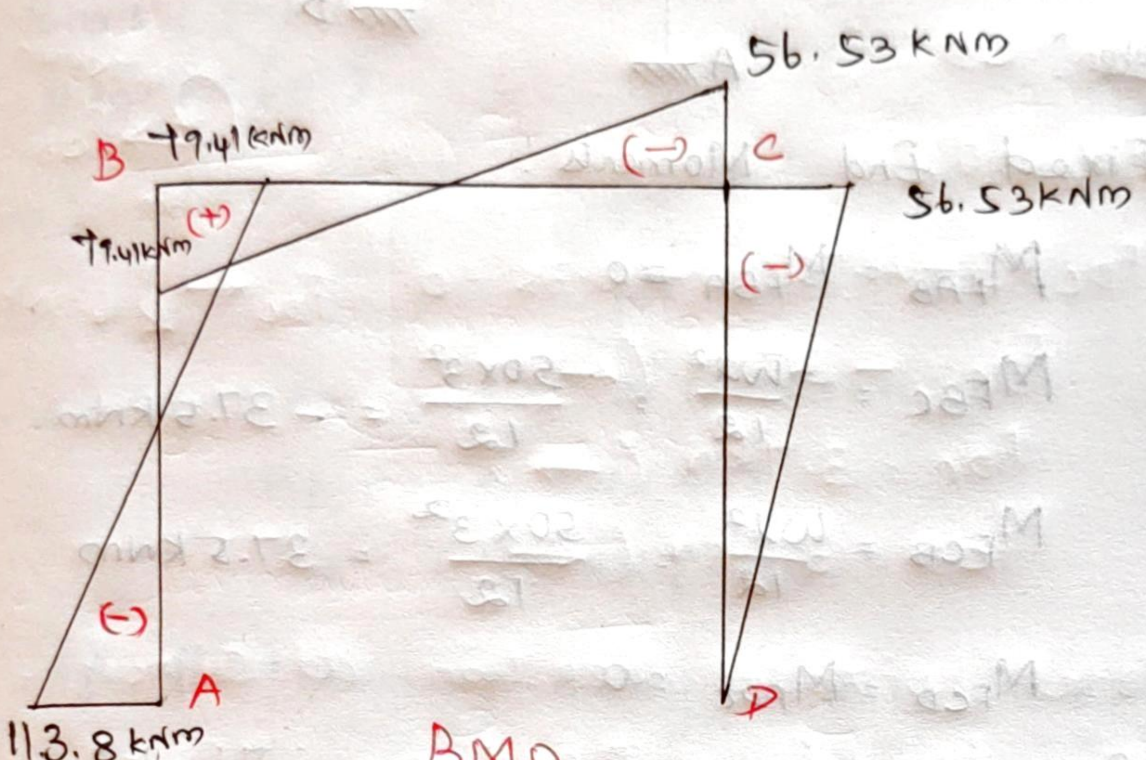
$\Sigma M_D = 0,$



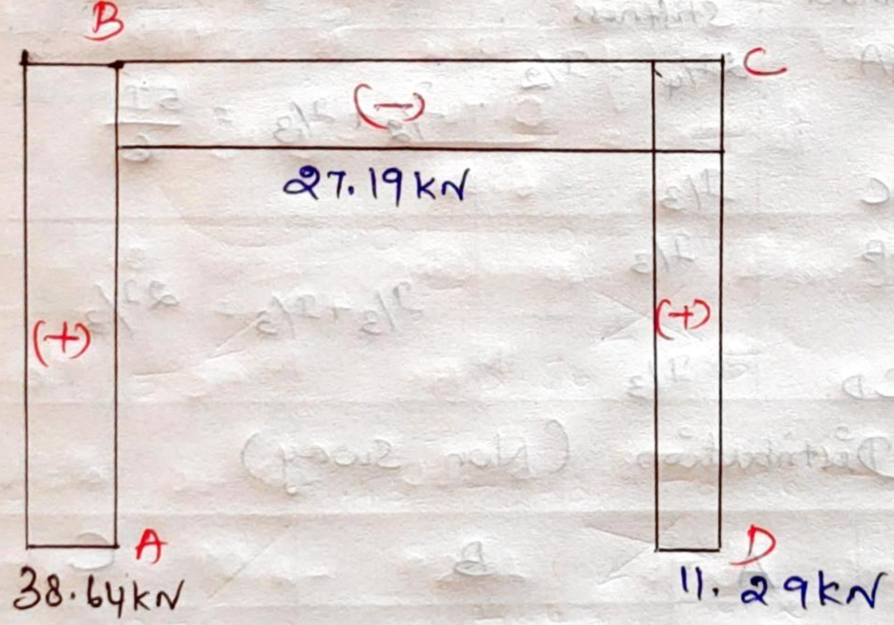
$56.52 + F_{CD} \times 5 = 0$

$F_{CD} = -11.29\ kN = 11.29\ kN\ (\leftarrow)$

$F_{DC} = 11.29\ kN\ (\rightarrow)$

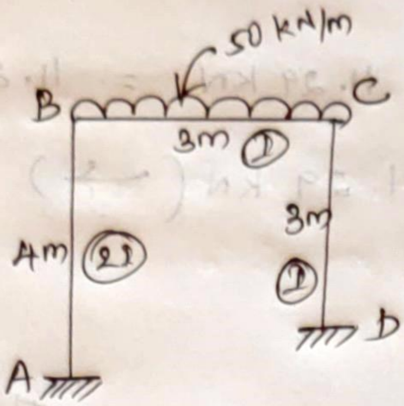


BMD



SFD

7. Analyse the portal frame loaded as shown in fig. by moment distribution method. Sketch the BMD and SFD.



Soln:

Fixed End Moments:

$$M_{FAB} = M_{FBA} = 0$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-50 \times 3^2}{12} = -37.5 \text{ kNm}$$

$$M_{FCB} = \frac{wl^2}{12} = \frac{50 \times 3^2}{12} = 37.5 \text{ kNm}$$

$$M_{FCD} = M_{FDC} = 0$$

Distribution Factors:

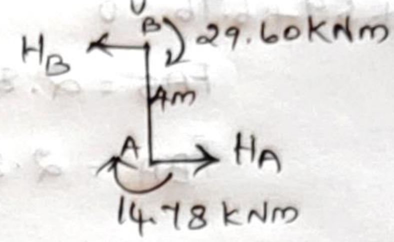
Joint	Member	Relative Stiffness	Total Stiffness	Distribution Factor
B	BA	$2I/4 = I/2$	$I/2 + I/3 = \frac{5I}{6}$	$\frac{I/2}{5I/6} = 3/5$
	BC	$I/3$		$\frac{I/3}{5I/6} = 2/5$
C	CB	$I/3$	$I/3 + I/3 = 2I/3$	$\frac{I/3}{2I/3} = 1/2$
	CD	$I/3$		$\frac{I/3}{2I/3} = 1/2$

Moment Distribution (Non sway)

Joints	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	—	3/5	2/5	1/2	1/2	—
F.E.M			-37.5	37.5		
Balance & c/o	11.25	22.5	15.0 -9.38	-18.75 7.5	-18.75	-9.38
Balance & c/o	2.82	5.63	3.75 -1.88	-3.75 1.88	-3.75	-1.88
Balance & c/o	0.57	1.13	0.75 -0.47	-0.94 0.38	-0.94	-0.47
Balance & c/o	0.14	0.28	0.19 -0.10	-0.19 0.10	-0.19	-0.10

Balance		0.06	0.04	-0.05	-0.05	
Final Moments	14.78	29.60	-29.60	23.68	-23.68	-11.83

Reaction Forces (Non sway) :



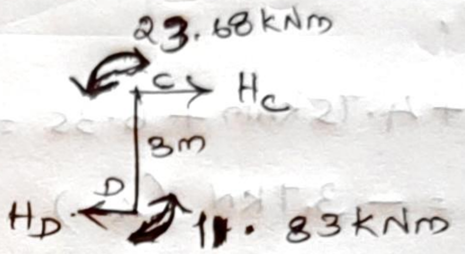
$\Sigma M_B = 0$,

$-H_A \times 4 + 29.60 + 14.78 = 0$

$H_A = 11.10$ kN $(\rightarrow)$

$H_B = 11.10$ kN $(\leftarrow)$

Span AB



Span CD :

$\Sigma M_C = 0$

$3H_D - 23.68 - 11.83 = 0$

$H_D = 11.84$ kN $(\leftarrow)$

$H_C = 11.84$ kN $(\rightarrow)$

Value of 'P' preventing sway $= 11.10 - 11.83$ kNm
$(\rightarrow)$ $(\leftarrow)$
$= 0.74$ kN $(\leftarrow)$

Side Sway :

Let a sway force of 0.74 kN $(\leftarrow)$ be applied at C. This will induce clockwise moments at A and D.

$\frac{M_{BA}}{M_{CD}} = \frac{I_1/l_1^2}{I_2/l_2^2} = \frac{2I/4^2}{I/3^2} = 9/8$

Assume, $M_{BA} = 9$ kNm $= M_{AB}$

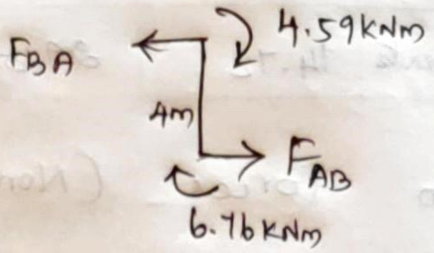
$M_{CD} = 8$ kNm $= M_{DC}$

Sway Moment Distribution :

Joints	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	–	3/5	2/5	1/2	1/2	–
FEM	9.0	9.0	–	–	8.0	8.0
Balance & C/O		-5.4	-3.6	-4.0	-4.0	
	-2.7		-2.0	-1.8		-2.0
Balance & C/O		1.2	0.8	0.9	0.9	
	0.6		0.45	0.4		0.45
Balance & C/O		-0.27	-0.18	-0.2	-0.2	
	-0.14		-0.1	-0.09		-0.1
Balance		0.06	0.04	0.05	0.05	
Final Moments	6.76	4.59	-4.59	-4.74	4.75	6.35

Sway force corresponding to the above moments :

Span AB :



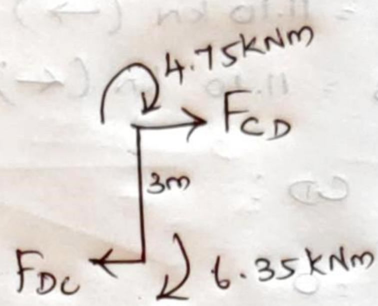
$\Sigma M_B = 0,$

$-F_{AB} \times 4 + 4.59 + 6.76 = 0$

$F_{AB} = 2.84$ kN ($\rightarrow$)

$F_{BA} = 2.84$ kN ($\leftarrow$)

Span CD :



$\Sigma M_C = 0,$

$F_{DC} \times 3 + 4.75 \text{ kNm} + 6.35 = 0$

$F_{DC} = -3.7$ kN ($\leftarrow$)

$F_{DC} = 3.7$ kN ($\rightarrow$)

$F_{CD} = 3.7$ kN ($\leftarrow$)

Total horizontal reaction at support =

$F_{AB} + F_{DC} = 2.84 (\rightarrow) + 3.7 (\rightarrow)$

$= 6.54$ kN ($\rightarrow$)

But actual sway force is 0.74 kN. Hence the

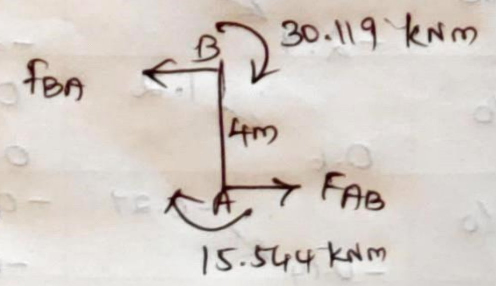
Correction factor $= \frac{0.74}{6.54} = 0.113$

Final Moments : Correction factor 0.113 to be multiplied

Moments	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}
Non Sway Moments	14.78	29.60	-29.60	23.68	-23.68	-11.83
Sway Moments	6.76	4.59	-4.59	-4.75	4.75	6.35
Sway Moments after correction	0.764	0.519	-0.519	-0.537	0.537	0.718
Final Moments	15.544	30.119	-30.119	23.143	-23.143	-11.112

Shear Force :

Span AB :



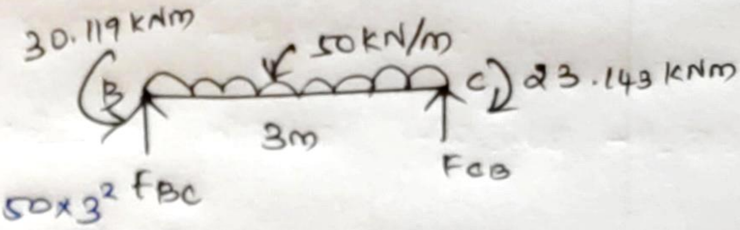
$\Sigma M_B = 0,$

$-F_{AB} \times 4 + 30.119 + 15.544 = 0$

$F_{AB} = 11.42$ kN ($\rightarrow$)

$F_{BA} = 11.42$ kN ($\leftarrow$)

Span BC :



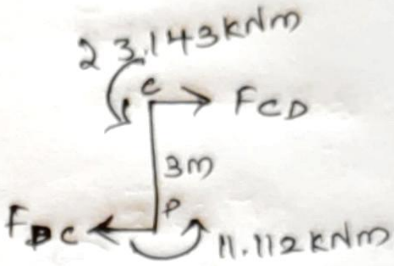
$\Sigma M_B = 0,$

$$+F_{CB} \times 3 + 23.143 - 30.119 - \frac{50 \times 3^2}{2} = 0$$

$F_{CB} = 72.67$ kN

$F_{BC} = 77.33$ kN

Span CD :



$\Sigma M_C = 0,$

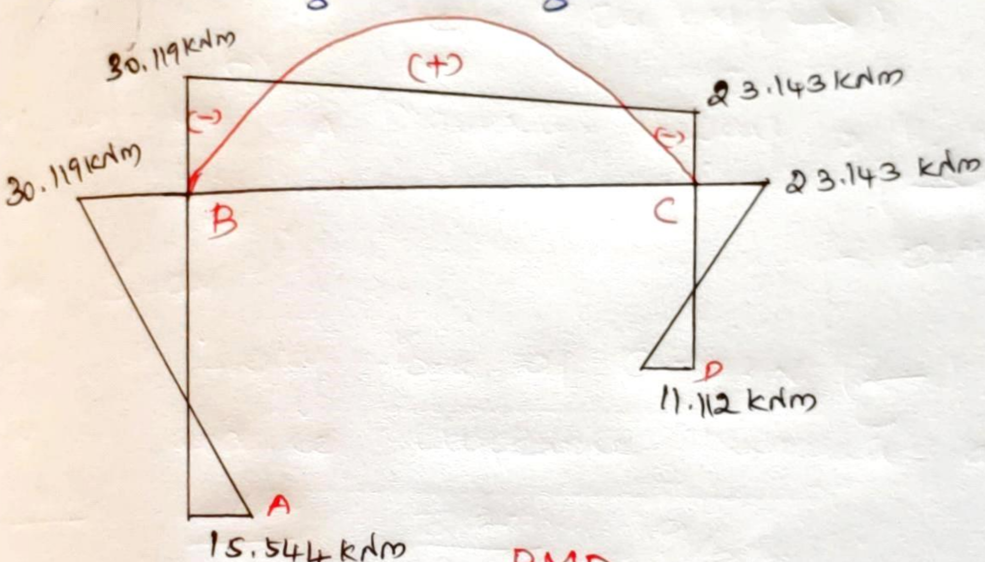
$$F_{DC} \times 3 - 23.143 - 11.112 = 0$$

$F_{DC} = 11.42$ kN

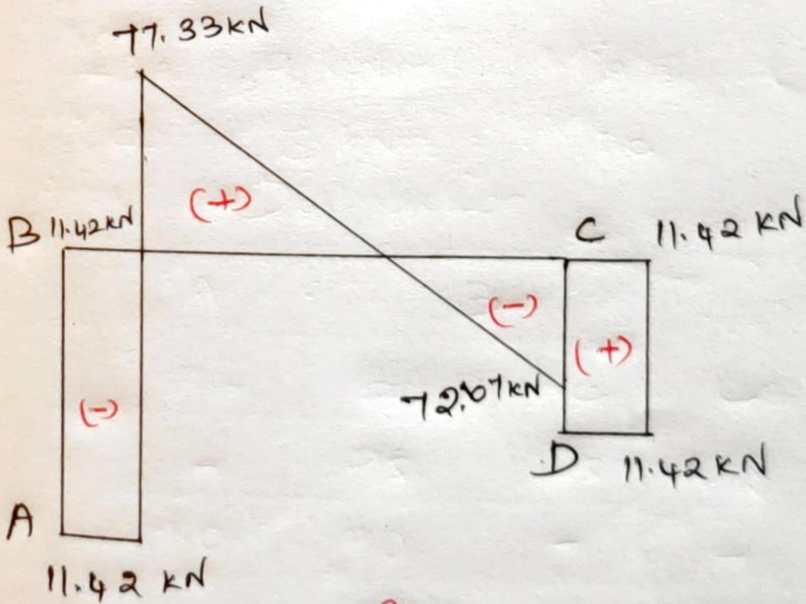
$F_{CD} = 11.42$ kN

Maximum B.M :

$$@BC = \frac{wl^2}{8} = \frac{50 \times 3^2}{8} = 56.25 \text{ kNm}$$



BMD



SFD