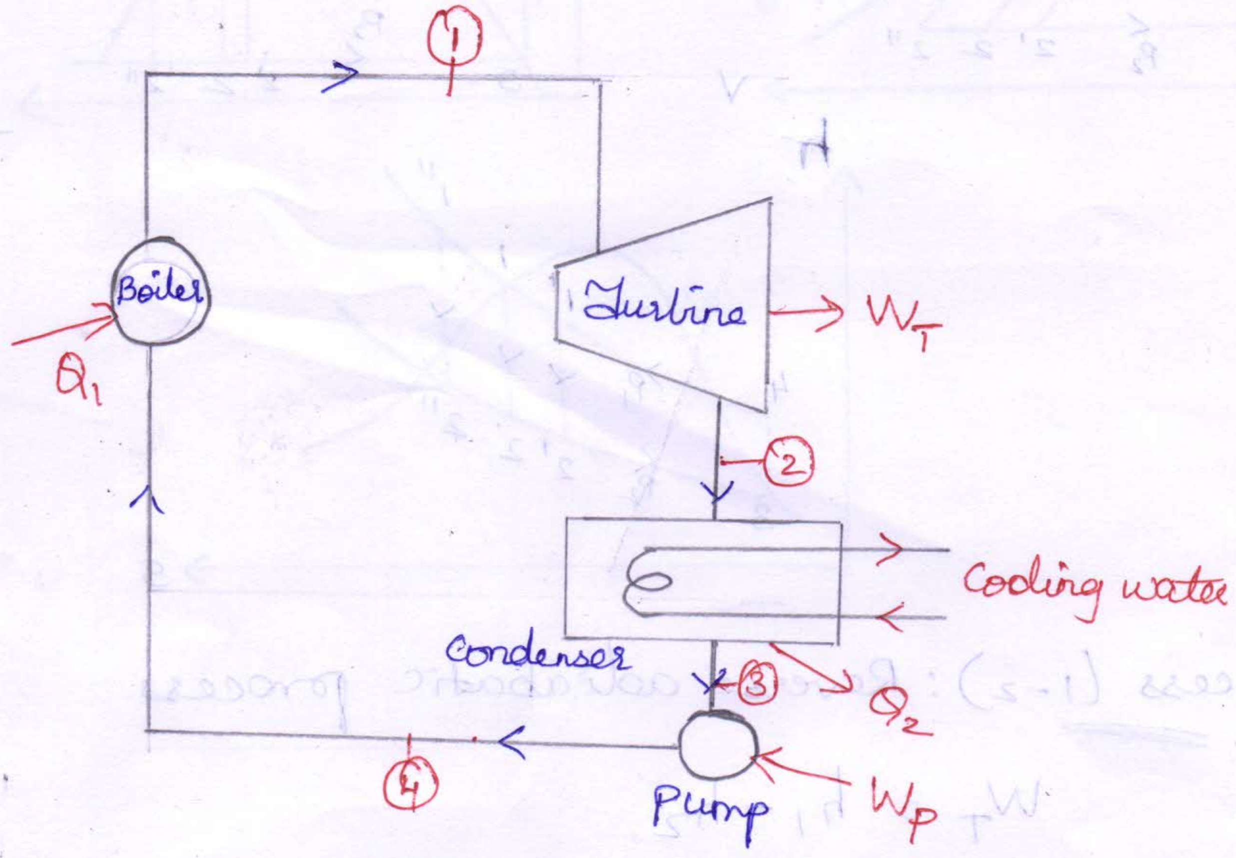
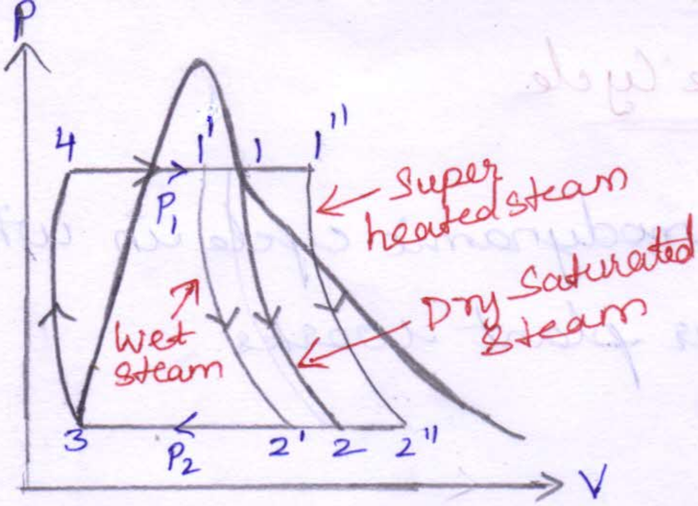
Rankine Cycle

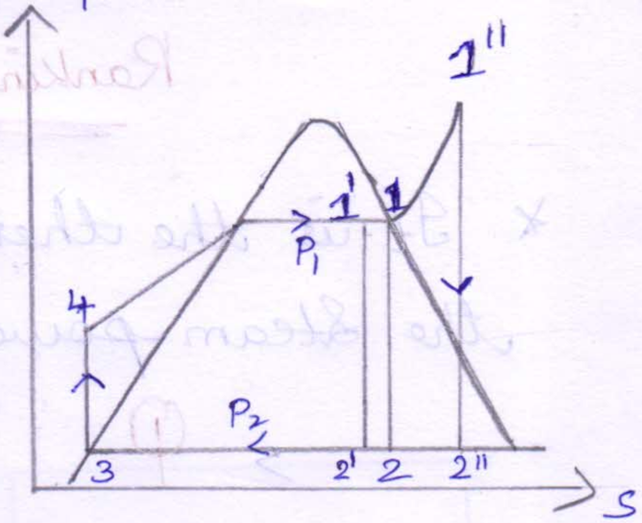
* It is the thermodynamic cycle in which the Steam power plant works.

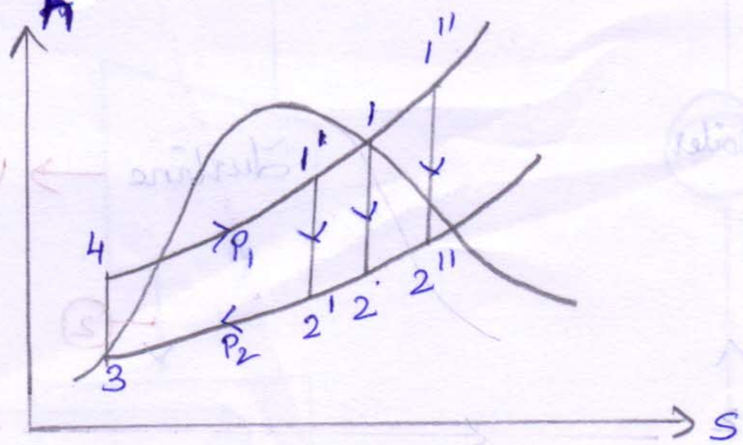


Process Involved in Rankine cycle

* Reversible adiabatic expansion in turbine
* Constant pressure transfer of heat in condenser
* Reversible adiabatic pumping process in feed pump
* Constant pressure transfer of heat in boiler







Process (1-2): Reverse adiabatic process

$$W_T = h_1 - h_2$$

Process (2-3): Constant Pressure [Heat Rejection]

$$Q_2 = h_2 - h_3$$

Process (3-4): Reversible adiabatic pumping process

$$W_P = h_4 - h_3$$

Process (4-1): Constant pressure [Heat Added]

$$Q_1 = h_1 - h_4.$$

$$\eta_{rankine} = \frac{W_{net}}{Q_1} = \frac{W_T - W_P}{Q_1}$$

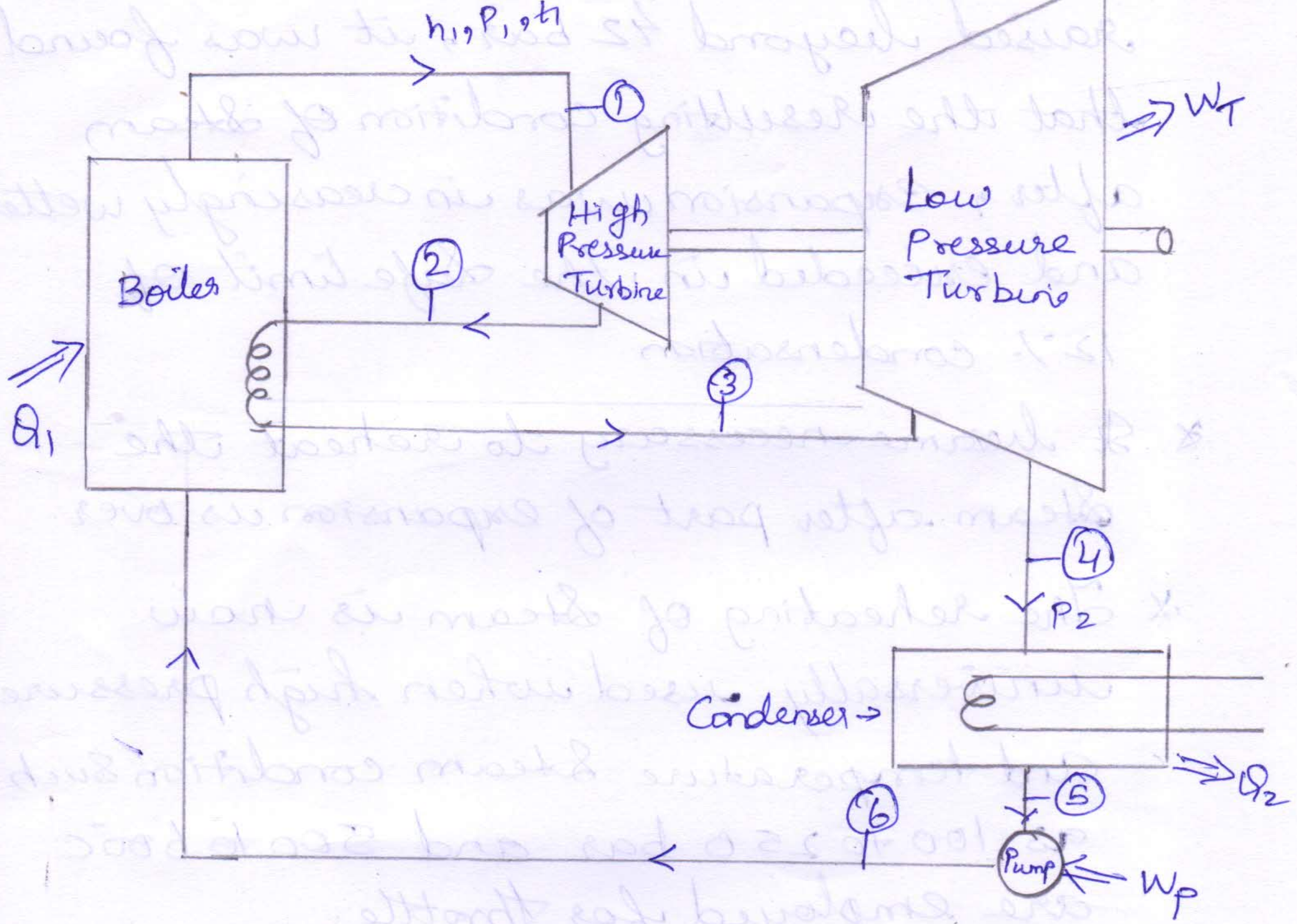
$$= \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4}$$

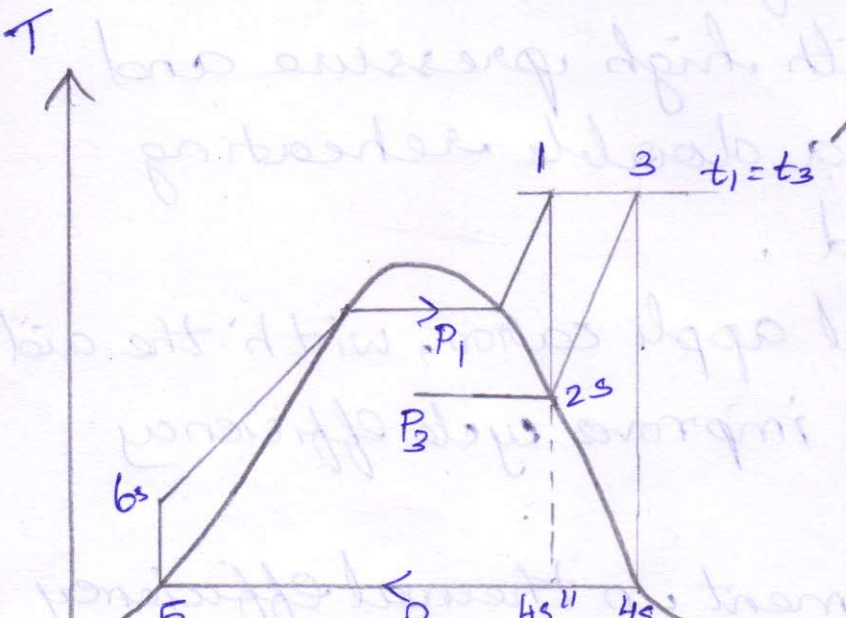
Since, the value of the W_P is small, so it can be neglected

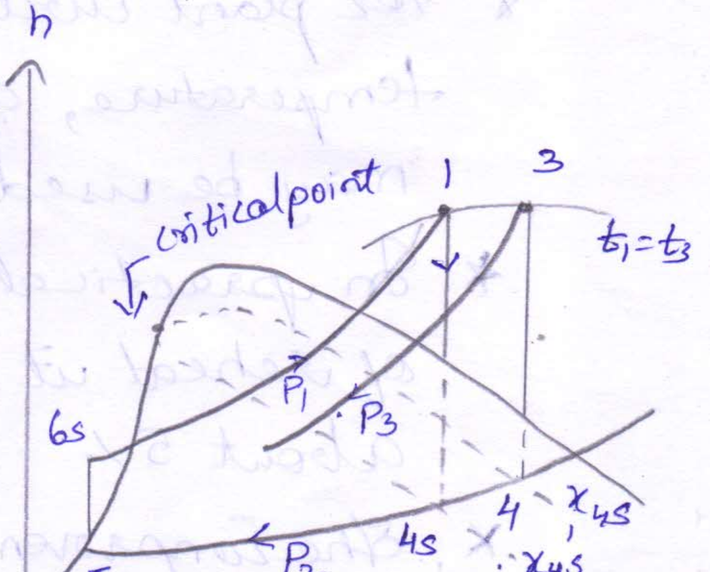
$$\eta_{rankine} = \frac{(h_1 - h_2) - \cancel{(h_4 - h_3)}^{\,0}}{(h_1 - h_4)}$$

$$\boxed{\eta_{rankine} = \frac{h_1 - h_2}{h_1 - h_4}}$$

Reheat cycle







* For attaining greater thermal efficiencies when the initial pressure of steam was raised beyond 42 bar, it was found that the resulting condition of steam after expansion was increasingly wetter and exceeded in the safe limit of 12% condensation
* It became necessary to reheat the steam after part of expansion is over.
* The reheating of steam is now universally used when high pressure and temperature steam condition such as 100 to 250 bar and 500 to 600°C are employed for throttle.
* For plant with high pressure and temperature, a double reheating may be used.
* In practical application, with the aid of reheat it improves cycle efficiency about 5%.
* The improvement in thermal efficiency due to reheat is greatly dependent upon the reheat pressure with respect to the original pressure of steam.

Workdone by Turbine

$$W_T = (h_1 - h_2) + (h_3 - h_4)$$

Workdone by Pump

$$W_P = h_6 - h_5$$

Heat Supplied to boiler

$$Q_1 = (h_1 - h_6) + (h_3 - h_2)$$

Heat rejected

$$Q_2 = h_4 - h_5$$

$$\eta_{rankine\text{-}reheat} = \frac{W_{net}}{Q_1} = \frac{W_T - W_P}{Q_1}$$

$$= \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}$$

neglecting pump work we get

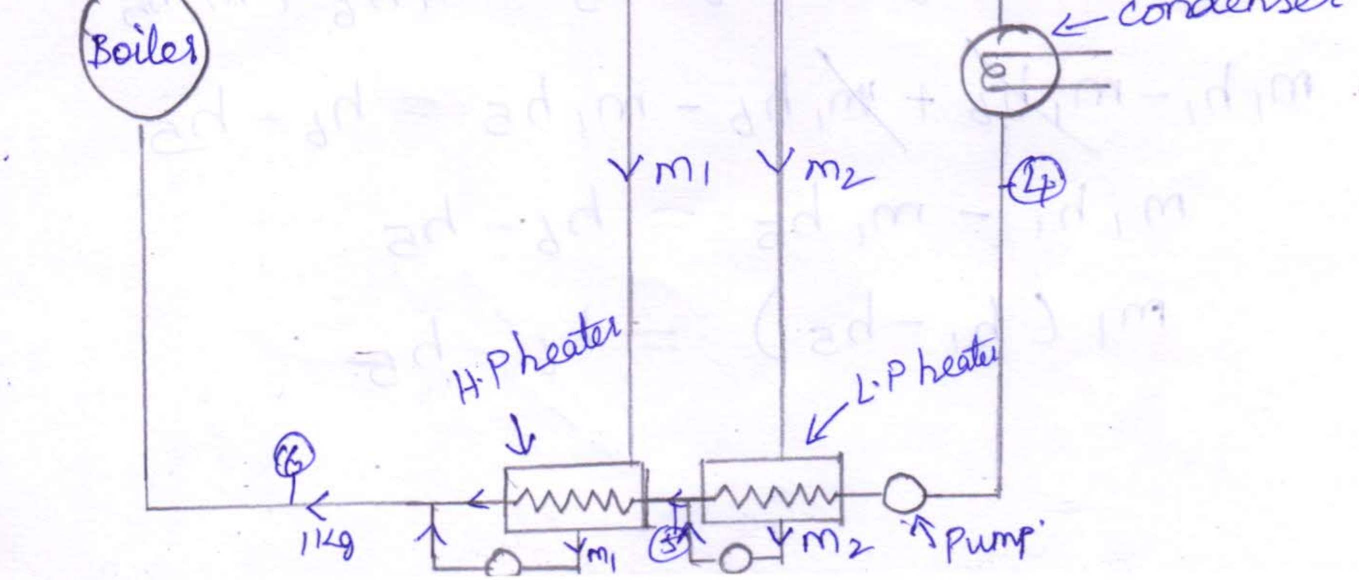
$$\boxed{\eta_{reheat} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_6) + (h_3 - h_2)}}$$

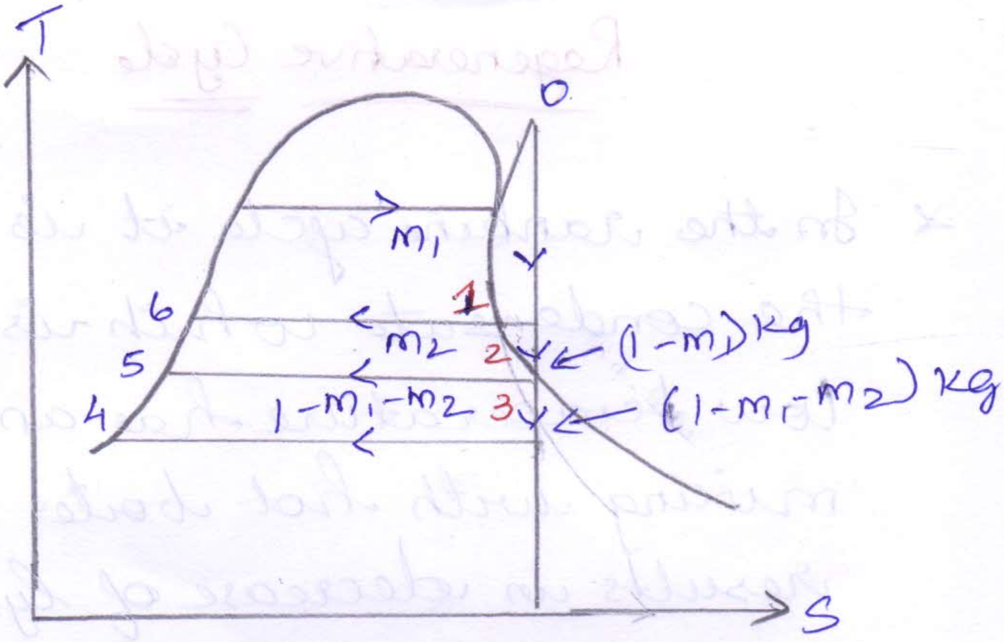
Regenerative Cycle

* In the rankine cycle it is observed that the condensate which is fairly at low temperature has an irreversible mixing with hot boiler and this results in decrease of lycle efficiency.

* Methods adopted to heat the feed water from the hot well of condenser irreversably by interchange of heat with in the system and thus improving cycle efficiency.

* This method is known as regenerative feed heat and the cycle is regenerative cycle.





Let, $m_1 \to$ kg of high pressure Steam per kg of Steam of flow

$m_2 \to$ kg of low pressure Steam extracted per kg of Steam flow

$1-m_1-m_2 \to$ kg of Steam entering the condenser per kg of Steam flow.

Energy balance equation for H.P heater

$$m_1(h_1-h_6) = (1-m_1)(h_6-h_5)$$

$$m_1h_1 - m_1h_6 = h_6 - h_5 - m_1h_6 + m_1h_5$$

$$m_1h_1 - \cancel{m_1h_6} + \cancel{m_1}h_6 - m_1h_5 = h_6 - h_5$$

$$m_1h_1 - m_1h_5 = h_6 - h_5$$

$$m_1(h_1 - h_5) = h_6 - h_5.$$

$$\boxed{m_1 = \frac{h_6 - h_5}{h_1 - h_5}}$$

Energy balance equation for L.P heater

$$m_2(h_2 - h_5) = (1 - m_1 - m_2)(h_5 - h_3)$$

$$m_2 h_2 - m_2 h_5 = h_5 - h_3 - m_1 h_5 + m_1 h_3 - m_2 h_5 + m_2 h_3$$

$$m_2 h_2 - \cancel{m_2 h_5} + m_1 h_5 - m_1 h_3 + \cancel{m_2 h_5} - m_2 h_3 = h_5 - h_3$$

$$m_2(h_2 - h_3) + m_1(h_5 - h_3) = (h_5 - h_6)$$

$$m_2(h_2 - h_3) = h_5 - h_6 - m_1(h_5 - h_3)$$

$$\boxed{m_2 = \frac{(h_5 - h_6) - m_1(h_5 - h_3)}{h_2 - h_3}}$$

Heat Supplied

$$Q_1 = h_0 - h_6$$

Workdone $W_{net} = W_T - W_P$

$$\eta_{regenerative} = \boxed{\frac{W_{net}}{Q_1}}$$

$W_{net} = W_T - W_P$ [Pump work is neglected]

$W_{net} = W_T$

$$= m_1(h_0 - h_1) + m_2(h_0 - h_2) + (1 - m_1 - m_2)(h_0 - h_3)$$

$$\eta_{regenerative} = \frac{m_1(h_0 - h_1) + m_2(h_0 - h_2) + (1 - m_1 - m_2)(h_0 - h_3)}{h_0 - h_6}$$

$$\boxed{\eta_{regenerative} = \frac{m_1(h_0 - h_1) + m_2(h_0 - h_2) + (1 - m_1 - m_2)(h_0 - h_3)}{h_0 - h_6}}$$

Problems Related to Diesel Cycle

1) A diesel engine has a compression ratio of 15 and heat addition at constant pressure takes place at 6% of stroke. Find the air standard efficiency of the engine. Take γ for air as 1.4

Given data:

$$r = 15$$

$$\gamma = 1.4$$

$$V_3 - V_2 = 0.06\, V_s$$

To find:

$$\eta_{diesel}$$

Solution:

$$\eta_{diesel} = 1 - \frac{1}{\gamma (r)^{\gamma-1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1} \right]$$

We know

$$\rho = \frac{V_3}{V_2}$$

$$r = \frac{V_1}{V_2} = 15 \Rightarrow \frac{V_1}{V_2} = 15, \quad \boxed{V_1 = 15 V_2}$$

From given data, $V_3 - V_2 = 0.06\, V_s$

$$V_3 - V_2 = 0.06 (V_1 - V_2)$$

$$V_3 - V_2 = 0.06 (15 V_2 - V_2)$$

$$V_3 - V_2 = 0.06 (14 V_2)$$

$$V_3 = 0.84 V_2 + V_2$$

$$\boxed{V_3 = 1.84\, V_2}$$

$$\rho = \frac{V_3}{V_2} = \frac{1.84\, \cancel{V_2}}{\cancel{V_2}}$$

$$\boxed{\rho = 1.84}$$

$$\eta_{diesel} = 1 - \frac{1}{\gamma (r)^{\gamma-1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1} \right]$$

$$= 1 - \frac{1}{1.4 (15)^{1.4-1}} \left[\frac{1.84^{1.4} - 1}{1.84 - 1} \right]$$

$$= 1 - 0.2418 \left[1.605 \right]$$

$$= 1 - 0.3881$$

$$= 0.612$$

$$\boxed{\eta_{diesel} = 61.2\%}$$

Result:

$$\eta_{diesel} = 61.2\%$$

2. An engine with 200 mm cylinder diameter and 300 mm Stroke works on theoretical diesel cycle. The Initial pressure and temperature of air used are 1 bar and 27°c. The cut off is 8% of the Stroke. Determine

(i) Pressure and Temperature at all Salient Points

(ii) Theoretical air Standard efficiency

(iii) Mean effective pressure

(iv) Power of the engine if the working cycle per minutes are 380.

Assume that compression ratio is 15 and working fluid is air. Consider all conditions to be ideal.

Given data:

$D = 200\,mm = 0.2\,m$

$L = 300\,mm = 0.3\,m$

$P_1 = 1\,bar = 1 \times 10^5\,N/m^2$

$T_1 = 27°C = 27 + 273 = 300\,K.$

$\rho = 8\%$ of V_s

$\rho = 0.08\,V_s$

$r = 15.$

Working cycle per minute = 30

To find:

(i) $P_2, P_3, P_4, T_2, T_3, T_4$

(ii) η_{diesel}

(iii) P_m

(iv) P

Solution:

$$V_1 = V_s + V_c$$

$$V_s = \frac{\pi}{4} D^2 \times L = \frac{\pi}{4} \times 0.2^2 \times 0.3$$

$$\boxed{V_s = 0.00942 \, m^3}$$

$$r = \frac{V_s + V_c}{V_c} \Rightarrow 15 = \frac{0.00942 + V_c}{V_c}$$

$$15 V_c = 0.00942 + V_c$$

$$14 V_c = 0.00942$$

$$\boxed{V_c = 0.0006732 \, m^3}$$

$$V_1 = V_s + V_c$$

$$= 0.00942 + 0.0006732$$

$$\boxed{V_1 = 0.01001 \, m^3}$$

~~we know~~

$$\cancel{\rho = \frac{V_3}{V_2}}$$

~~from given data~~ $\cancel{\rho = 0.08\, V_s}$

$$\cancel{\rho = 0.08 \times 0.0006732}$$

$$\cancel{\rho =}$$

$$r = \frac{V_1}{V_2} \Rightarrow 15 = \frac{0.01001}{V_2}$$

$$\boxed{V_2 = 0.000667\, m^3}$$

$$V_3 = 0.08\, V_s + V_c$$

$$= 0.08 \times 0.00942 + 0.0006732$$

$$\boxed{V_3 = 0.001426\, m^3}$$

$$\rho = \frac{V_3}{V_2} = \frac{0.001426}{0.000667}$$

$$\boxed{\rho = 2.14}$$

Process 1 → 2 (Reverse adiabatic)

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\frac{P_2}{P_1} = \frac{V_1^{\gamma}}{V_2^{\gamma}} \Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$\frac{P_2}{P_1} = (r)^{\gamma}$$

$$\frac{P_2}{1 \times 10^5} = (15)^{1.4}$$

$$\boxed{P_2 = 44.31 \times 10^5 \ N/m^2}$$

Since process 2 → 3 is constant pressure

so, $P_2 = P_3$

$$\boxed{P_3 = 44.31 \times 10^5 \ N/m^2}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = (r)^{\gamma-1}$$

$$\frac{T_2}{300} = (15)^{1.4-1}$$

$$\boxed{T_2 = 886.2K}$$

From process (2→3) Constant Pressure

$$PV = mRT$$

$$P = \frac{mRT}{V}$$

$$P_2 = P_3$$

$$\frac{\cancel{m}RT_2}{V_2} = \frac{\cancel{m}RT_3}{V_3}$$

$$\frac{T_2}{V_2} = \frac{T_3}{V_3}$$

$$\frac{886}{0.000667} = \frac{T_3}{0.001426}$$

$$\boxed{T_3 = 1894.2\,K}$$

From process (3→4) Reverse adiabatic

$$P_3 V_3^{\gamma} = P_4 V_4^{\gamma}$$

$$P_4 = P_3 \times \frac{V_3^{\gamma}}{V_4^{\gamma}}$$

$$P_4 = P_3 \times \left(\frac{V_3}{V_4}\right)^{\gamma}$$

$$P_4 = P_3 \times \left(\frac{\rho}{r}\right)^{\gamma}$$

$$\frac{V_3}{V_4} = \frac{V_3}{V_1}$$

$$= \frac{V_3}{V_1} \times \frac{V_2}{V_2}$$

$$\frac{V_3}{V_2} \times \frac{V_2}{V_1}$$

$$= \rho \times \frac{1}{r} \Rightarrow \frac{\rho}{r}$$

$$= 44.31 \times 10^5 \times \left(\frac{2.14}{15}\right)^{1.4}$$

$$\boxed{P_4 = 2.9 \times 10^5\,N/m^2}$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$\frac{T_4}{1894.2} = \left(\frac{\rho}{r}\right)^{\gamma-1}$$

$$\frac{T_4}{1894.2} = \left(\frac{2.14}{15}\right)^{1.4-1}$$

$$\boxed{T_4 = 869.27\,K}$$

(ii) $$\eta_{diesel} = 1 - \frac{1}{\gamma (r)^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\rho-1}\right]$$

$$= 1 - \frac{1}{1.4(15)^{1.4-1}}\left[\frac{(2.14)^{1.4}-1}{2.14-1}\right]$$

$$= 1 - \frac{1}{4.14}(1.67)$$

$$= 1 - 0.404$$

$$= 0.596$$

$$\boxed{\eta_{diesel} = 59.6\%}$$

~~(iii) $P_m = P_1 (r)^{\gamma} [\gamma(\rho-1) - r^{1-\gamma}$~~

(iii) $P_m = P_1 r \left[\dfrac{\gamma (r)^{\gamma-1} (\rho-1) - (\rho^{\gamma}-1)}{(\gamma-1)(r-1)} \right]$

$= 1\times10^5 \times 15 \left[\dfrac{1.4 (15)^{1.4-1} (2.14-1) - (2.14^{1.4}-1)}{(1.4-1)(15-1)} \right]$

$= 1\times10^5 \times 15 \,(0.5024)$

$P_m = 7.54 \times 10^5 \; N/m^2$

$\boxed{P_m = 7.54 \text{ bar}}$

(iv) Power of engine

Workdone per cycle $P = P_m \times V_s = 7.54\times10^5 \times 0.00942$

$P = 7102.68 \; N\,m$

$= 7102.68 \times \dfrac{1}{1000} \; kJ$ $\qquad 1\,Nm = \dfrac{1}{1000}\,kJ$

$= 7.102 \; kJ$

Workdone per second $= 7.102 \times \dfrac{380}{60}$

$= 44.98 \; kJ/s = 44.98 \; kW$

Power of engine = 44.27 kW

Result:

(i) $P_2 = 44.31$ bar

$T_2 = 886.2$ K

$P_3 = 44.31$ bar

$T_3 = 1894.2$ K.

$P_4 = 2.7 \times 10^5$

$T_4 = 869.27$ K.

(ii) $\eta_{diesel} = 59.6\%$

(iii) $P_m = 7.54$ bar

(iv) Power of engine = 44.27 kW

Problems related to Otto cycle

1) The efficiency of an Otto cycle is 60% and $\gamma = 1.5$. What is the compression ratio?

Given data:

$$\eta = 60\% = 0.6$$

$$\gamma = 1.5$$

To find:

r

Solution:

$$\eta_{otto} = 1 - \frac{1}{(r)^{\gamma - 1}}$$

$$0.6 = 1 - \frac{1}{(r)^{1.5-1}}$$

$$\frac{1}{(r)^{0.5}} = 1 - 0.6$$

$$(r)^{0.5} = \frac{1}{0.4}$$

$$\boxed{r = 6.25}$$

Result:

$r = 6.25$

2) An engine of 250 mm bore and 375 mm stroke works on Otto cycle. The clearance volume is 0.00263 m^3. The initial pressure and temperature are 1 bar and 50°C. If the maximum pressure is limited to 25 bar, Find the following:

(i) Air Standard efficiency of the cycle

(ii) The mean effective pressure for the cycle.

Assume the ideal conditions.

Given data:

$D = 250\,mm = 0.25\,m$

$L = 375\,mm = 0.375\,m$

$V_c = 0.00263\,m^3$

$P_1 = 1\,bar = 1 \times 10^5\,N/m^2$

$T_1 = 50°C$

$P_3 = 25\,bar = 25 \times 10^5\,N/m^2$

To find:

(i) η_{otto}

(ii) P_m

Solution:

(i) $\eta_{otto} = 1 - \dfrac{1}{(r)^{\gamma-1}}$

$r = \dfrac{V_s + V_c}{V_c}$; $V_s = \dfrac{\pi}{4} \times D^2 \times L$

$V_s = \dfrac{\pi}{4} \times (0.25)^2 \times 0.375$

$\boxed{V_s = 0.0184\ m^3}$

$r = \dfrac{0.0184 + 0.00263}{0.00263}$

$r = 7.99 \simeq 8$

$\boxed{r = 8}$

$\boxed{\gamma = 1.4 \text{ Standard Value}}$

$\eta_{otto} = 1 - \dfrac{1}{(8)^{1.4-1}}$

$\eta_{otto} = 0.565$

$\boxed{\eta_{otto} = 56.5\%}$

(ii) $P_m = \dfrac{P_1 r\left[(r_p - 1)\left((r)^{\gamma-1} - 1\right)\right]}{(\gamma - 1)(r - 1)}$

$r_p = \dfrac{P_3}{P_2}$,

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$P_2 = \dfrac{P_1 V_1^{\gamma}}{V_2^{\gamma}}$

$P_2 = P_1 \left(\dfrac{V_1}{V_2}\right)^{\gamma}$

$P_2 = P_1 \left(\dfrac{V_1}{V_2}\right)^{\gamma}$

$P_2 = 1 \times 10^5 \left(8\right)^{1.4}$

$\boxed{P_2 = 18.38 \times 10^5 \ N/m^2}$ (or) $\boxed{P_2 = 18.38 \text{ bar}}$

$r_p = \dfrac{25 \times 10^5}{18.38 \times 10^5}$

$\boxed{r_p = 1.36}$

$P_m = \dfrac{1 \times 10^5 \times 8 \left[(1.36 - 1)\left((8)^{1.4-1} - 1\right)\right]}{(1.4 - 1)(8 - 1)}$

$= \dfrac{1 \times 10^5 \times 8 \times \left[(0.36) \times (1.29)\right]}{0.4 \times 7}$

$\boxed{P_m = 1.334 \times 10^5 \ N/m^2}$ (or) $\boxed{P_m = 1.334 \text{ bar}}$

Result:

(i) $\eta_{otto} = 56.5\%$

(ii) $P_m = 1.334$ bar

3) The minimum pressure and temperature in an otto cycle are 100 kPa and 27°C. The amount of heat added to the air per cycle is 1500 kJ/kg.

(i) Determine the pressure and temperature at all points of the air standard otto cycle.

(ii) Also calculate the specific work and thermal efficiency of the cycle for a compression ratio of 8:1.

Take for air $C_v = 0.72\ kJ/kg\,K$, $\gamma = 1.4$.

<u>Given data:</u>

$P_1 = 100\ kPa = 100 \times 10^3\ pa$

$\Rightarrow 10^5\ pa \Rightarrow 10^5\ N/m^2 \Rightarrow 1\ bar$

$T_1 = 27 + 273 = 300\ k.$

$Q_s = 1500\ kJ/kg = 1500 \times 10^3\ J/kg$

$r = 8:1 = 8$

$C_v = 0.72\ kJ/kg\,K = 0.72 \times 10^3\ J/kg\,K$

$\gamma = 1.4.$

<u>To find:</u>

(i) $P_2, P_3, P_4, T_2, T_3, T_4$

(ii) W, η_{otto}.

Solution:

$$r = \frac{V_1}{V_2}$$

Process 1→2 (Adiabatic compression)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = (r)^{\gamma-1}$$

$$\frac{T_2}{300} = (8)^{1.4-1}$$

$$\boxed{T_2 = 689.2\,K}$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \Rightarrow \frac{P_2}{P_1} = (r)^{\gamma}$$

$$\frac{P_2}{1\times10^5} = (8)^{1.4}$$

$$\boxed{P_2 = 18.38\times10^5\ N/m^2}$$

or

$$\boxed{P_2 = 18.38\ bar}$$

Process 2→3 (Heat addition → constant Volume)

$$Q_S = m\,c_v\,(T_3 - T_2)$$

$$1500\times10^3 = 1\times0.72\times10^3\,(T_3 - 689.2)$$

$$\boxed{T_3 = 2772.5\,K}$$

Since it is constant Volume process, so

$$V_2 = V_3$$

$$PV = mRT$$

$$V = \frac{mRT}{P}$$

$$\frac{\cancel{mR}T_2}{P_2} = \frac{\cancel{mR}T_3}{P_3}$$

$$\frac{T_2}{P_2} = \frac{T_3}{P_3}$$

$$\frac{689.2}{18.38 \times 10^5} = \frac{2772.5}{P_3}$$

$$\boxed{P_3 = 73.94 \times 10^5 \ N/m^2}$$

(or)

$$\boxed{P_3 = 73.94 \ bar}$$

Adiabatic process (3→4)

$$\boxed{\frac{V_1}{V_2} = \frac{V_4}{V_3} = r}$$

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma - 1}$$

$$\frac{T_3}{T_4} = (r)^{\gamma - 1}$$

$$\frac{2772.5}{T_4} = (8)^{1.4 - 1}$$

$$\boxed{T_4 = 1206.8 \ K}$$

$$P_3 V_3^{\gamma} = P_4 V_4^{\gamma} \qquad \frac{V_3}{V_4} = \frac{1}{r}$$

$$P_4 = \frac{P_3 \times V_3^{\gamma}}{V_4^{\gamma}}$$

$$= P_3 \times \left(\frac{V_3}{V_4}\right)^{\gamma}$$

$$P_4 = P_3 \times \left(\frac{1}{r}\right)^{\gamma}$$

$$= 73.94 \times 10^5 \left(\frac{1}{8}\right)^{1.4}$$

$$\boxed{P_4 = 4.02 \times 10^5 \ N/m^2}$$

(or)

$$\boxed{P_4 = 4.02 \text{ bar}}$$

(ii) $W = Q_S - Q_R$

$$= m C_v (T_3 - T_2) - m C_v (T_4 - T_1)$$

$$= m C_v [T_3 - T_2 - T_4 + T_1]$$

$$= 1 \times 0.72 \times 10^3 [2772.5 - 689.2 - 1206.8 + 300]$$

~~W = 847 KJ/kg~~

$$= 847 \times 10^3$$

$$\boxed{W = 847 \ KJ/kg}$$

$$\eta_{otto} = 1 - \frac{1}{(r)^{\gamma-1}}$$

$$= 1 - \frac{1}{(8)^{1.4-1}}$$

$$\boxed{\eta_{otto} = 56.47\%}$$

Result:

(i) $T_2 = 689.2K$

$P_2 = 18.38$ bar

$T_3 = 2772.5$ K.

$P_3 = 73.94$ bar

$T_4 = 1206.8$ K.

$P_4 = 4.02$ bar

(ii) $W = 847$ kJ/kg.

$\eta_{otto} = 56.47\%$.

Problems Related to Rankine Cycle

1) A simple rankine cycle works between pressure 28 bar and 0.06 bar, the initial condition of steam being dry saturated. Calculate the cycle efficiency, work ratio and specific steam consumption.

Given

$P_1 = 28$ bar

$P_2 = 0.06$ bar

To find:

(i) η_{cycle}

(ii) Work ratio

(iii) Specific steam consumption

Solution:

$$\eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{W_T - W_P}{Q_1}$$

from steam table, For $P_1 = 28$ bar.

$h_1 = 2802$ kJ/kg, $S_1 = 6.211$ kJ/kg K from Steam table, For $P_2 = 0.06$ bar

$h_{f_2} = 151.5$ KJ/kg, $h_{fg_2} = 2416$ KJ/kg

$h_g = 2507.5$ KJ/kg, $S_{f_2} = 0.521$ KJ/kgK

$S_{fg_2} = 7.81$ KJ/kg K.

We know that

$S_1 = S_2$

$\boxed{S = S_f + x S_{fg}}$

$6.211 = S_{f_2} + x S_{fg_2}$

$6.211 = 0.521 + x(7.81)$

$\boxed{x = 0.728}$

$h_2 = h_{f_2} + x h_{fg_2}$

$= 151.5 + 0.728(2416)$

$\boxed{h_2 = 1910.3 \text{ KJ/kg}}$

$W_T = h_1 - h_2$

$= 2802 - 1910.3$

$\boxed{W_T = 891.7 \text{ KJ/kg}}$

$$W_p = h_{f_4} - h_{f_3} = V_{f_2}(P_1 - P_2)$$

$[V_{f_2} = 0.001\ m^3/kg]$ from Steam table $P = 0.06$ bar.

$$W_p = 0.001\left(28 \times 10^5 - 0.06 \times 10^5\right)$$

$$= 2794\ Nm/kg$$

$$= \frac{2794}{1000}\ kJ/kg$$

$\boxed{1\,Nm = \frac{1}{1000}\ kJ}$

$$\boxed{W_p = 2.794\ kJ/kg}$$

$$W_{net} = W_T - W_p$$

$$= 891.70 - 2.794$$

$$\boxed{W_{net} = 888.906\ kJ/kg}$$

$$Q_1 = h_1 - h_{f_4}.$$

$h_{f_4} = ?$

$$W_{pump} = h_{f_4} - h_{f_3}$$

$$2.794 = h_{f_4} - 151.5$$

Since, $h_{f_3} = h_{f_2}$

$h_{f_3} = 151.5$ kJ/kg

$$\boxed{h_{f_4} = 154.294\ kJ/kg}$$

$$Q_1 = 2802 - 154.294$$

$$\boxed{Q_1 = 2647.706 \text{ KJ/kg}}$$

$$\eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{888.906}{2647.706}$$

$$\eta_{cycle} = 0.335$$

$$\boxed{\eta_{cycle} = 33.5\%}$$

(ii) $\text{Workratio} = \frac{W_{net}}{W_T} = \frac{888.906}{891.70}$

$$\boxed{\text{Workratio} = 0.9968}$$

(iii) $\text{Specific Steam consumption} = \frac{3600}{W_{net}}$

$$= \frac{3600}{888.906} = 4.05$$

$$\boxed{\text{Specific Steam consumption} = 4.05 \text{ Kg/Kw-h}}$$

Result:

(i) $\eta_{cycle} = 33.5\%$

(ii) Workratio = 0.9968

(iii) Specific Steam Consumption = 4.05 kg/kw-h

Problems Related to Reheat Cycle

1) A steam powerplant operates on a theoretical reheat cycle. Steam at boiler at 150 bar, 550°C expands through the high pressure turbine. It is reheated at a constant pressure of 40 bar, 550°C and expands through the low pressure turbine to a condenser at 0.1 bar. Draw T-s and H-s diagram. Find
(i) Quality of steam at turbine exhaust
(ii) Cycle efficiency, (iii) Steam rate in kg/kW-h

Given data:

$P_1 = 150$ bar, $T_1 = 550°C$

$P_2 = P_3 = 40$ bar, $T_3 = 550°C$

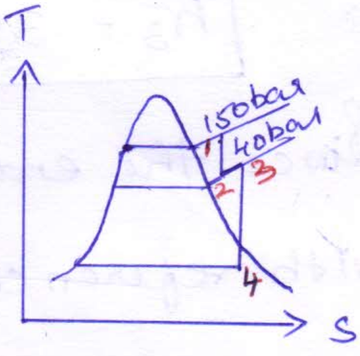
$P_4 = 0.1$ bar

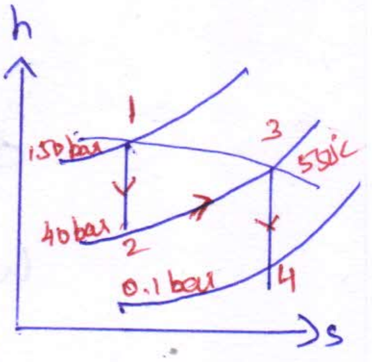
To find:

(i) x_4

(ii) η_{cycle}

(iii) Steam rate





Solution:

From mollier chart,

For $P_1 = 150$ bar & $T_1 = 550°C$ we get

$$\boxed{h_1 = 3450 \text{ kJ/kg}}$$ $S_1 = 6.48$ kJ/kg K

Since, the entropy is equal, so $S_1 = S_2$

Since $S_1 = S_2$, With reference to S_2 & 40 bar we get

$$\boxed{h_2 = 3050 \text{ kJ/kg}}$$

From mollier chart,

For $P_3 = 40$ bar, $T_3 = 550°C$

$$\boxed{h_3 = 3560 \text{ kJ/kg}}$$ $S_3 = 7.2$ kJ/kgK

Since the entropy $S_3 = S_4$.

With reference to $S_4 = 7.2$ kJ/kg K &

$P_4 = 0.1$ bar.

$$\boxed{h_4 = 2300 \text{ kJ/kg}}$$

Since, $h_6 = h_{f_4}$

So, from Steam table at 0.1 bar

$$\boxed{h_{f_4} = 191.8 \text{ KJ/kg}}$$

We know that

$$\eta_{cycle} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_{f_4}) + (h_3 - h_2)}$$

$$= \frac{(3450 - 3050) + (3560 - 2300)}{(3450 - 191.8) + (3560 - 3050)}$$

$$\eta_{cycle} = 0.4405$$

$$\boxed{\eta_{cycle} = 44.05\%}$$

Steam rate $= \dfrac{3600}{(h_1 - h_2) + (h_3 - h_4)}$

$$= \frac{3600}{(3450 - 3050) + (3560 - 2300)}$$

$$\boxed{\text{Steam rate} = 2.17 \text{ kg/kW-h}}$$

~~Result:~~

~~(i) $\eta_{cycle} = 44.05\%$~~

~~(ii) Steam rate = 2.17 kg/kW h.~~

For finding quality of Steam "x_4"

from mollier diagram

With reference to $h_4 = 2300$ kJ/kg and

$P_4 = 0.1$ bar

$$\boxed{x_4 = 0.88}$$

Result:- (i) $x_4 = 0.88$

(ii) $\eta_{cycle} = 44.05\%$.

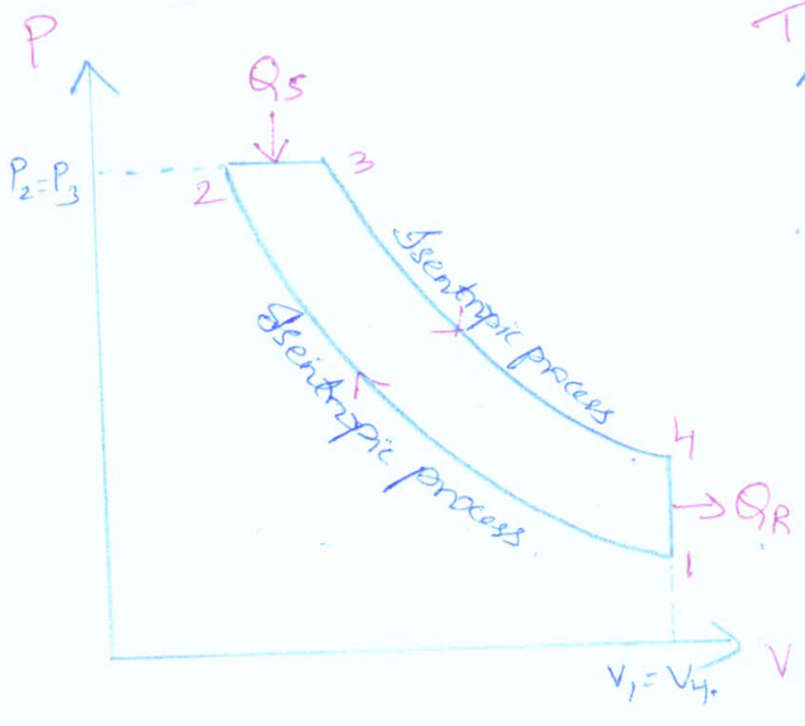
(iii) Steam rate = 2.17 kg/kW h.

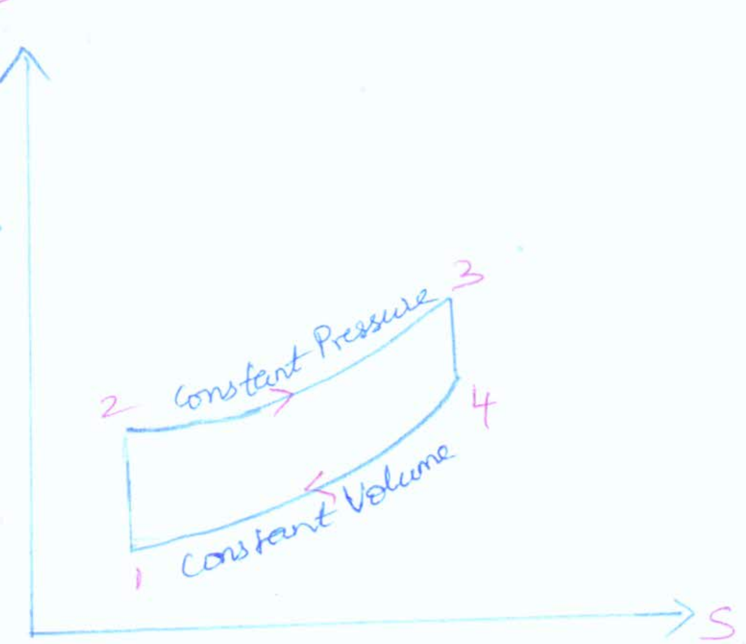
Diesel Cycle

* Diesel cycle was Introduced by Dr. Rudolph Diesel in 1892.
* It is air standard cycle for compression Ignition engine.
* It differs from Otto cycle by the heat addition at constant pressure.

Process:

* 1 → 2 Isentropic compression
* 2 → 3 Constant pressure heat addition
* 3 → 4 Isentropic expansion
* 4 → 1 Constant Volume heat rejection





Air Standard efficiency

$$\eta_{diesel} = \frac{Q_S - Q_R}{Q_S} \rightarrow (1)$$

From process 2 → 3 [Heat Supplied] we get,

$$Q_s = m c_p (T_3 - T_2) \rightarrow (2)$$

From process 4 → 1 [Heat Rejected] we get,

$$Q_R = m C_v (T_4 - T_1) \rightarrow (3)$$

Sub (3) & (2) in (1)

$$\eta_{diesel} = \frac{m C_p (T_3 - T_2) - m C_v (T_4 - T_1)}{m C_p (T_3 - T_2)}$$

$$= 1 - \frac{\cancel{m} C_v (T_4 - T_1)}{\cancel{m} C_p (T_3 - T_2)}$$

$$\boxed{\gamma = \frac{C_p}{C_v}}$$

$$= 1 - \frac{(T_4 - T_1)}{\gamma (T_3 - T_2)}$$

$$\boxed{\eta_{diesel} = 1 - \frac{1}{\gamma} \left[\frac{T_4 - T_1}{T_3 - T_2} \right]} \rightarrow (4)$$

we know,

$$\text{compression ratio} = \frac{V_1}{V_2} = r \rightarrow (5)$$

$$\text{cut off ratio} = \frac{V_3}{V_2} = \rho \rightarrow (6)$$

$$\text{Expansion ratio} = \frac{V_4}{V_3} = \frac{V_1}{V_3} = r_1 \rightarrow (7)$$

from equation (7),

$$\frac{V_1}{V_3} = r_1$$

multiply both sides by V_2

$$\frac{V_1}{V_3} \times V_2 = r_1 \times V_2$$

$$r_1 = \frac{V_1 \times V_2}{V_3 \times V_2}$$

$$r_1 = \frac{V_1}{V_2} \times \frac{V_2}{V_3}$$

$$= r \times \frac{1}{\rho}$$

$$\boxed{r_1 = \frac{r}{\rho}} \rightarrow (8)$$

From process 2 → 3 [Constant Pressure]

$$\frac{V_2}{T_2} = \frac{V_3}{T_3}$$

$$T_2 = \frac{V_2}{V_3} \times T_3$$

$$T_2 = \frac{1}{\rho} \times T_3$$

$$\boxed{T_3 = \rho \cdot T_2} \rightarrow (9)$$

From process 1 → 2 [Reverse adiabatic]

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$

$$\frac{T_2}{T_1} = (r)^{\gamma - 1}$$

$$\boxed{T_2 = (r)^{\gamma-1} \times T_1} \rightarrow (10)$$

Sub (10) in (9)

$$\boxed{T_3 = \rho \times (r)^{\gamma-1} T_1} \rightarrow (11)$$

From Process 3 ⇒ 4 [Reverse adiabatic].

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$\frac{T_4}{T_3} = \left(\frac{1}{\gamma_1}\right)^{\gamma-1}$$

$$\boxed{\gamma_1 = \frac{\gamma}{\rho}}$$

$$\frac{T_4}{T_3} = \left(\frac{\rho}{\gamma}\right)^{\gamma-1}$$

$$T_4 = \left(\frac{\rho}{\gamma}\right)^{\gamma-1} \times T_3$$

$$= \left(\frac{\rho}{\gamma}\right)^{\gamma-1} \times \rho\,(\gamma)^{\gamma-1}\,T_1$$

$$= \frac{(\rho)^{\gamma-1}}{\cancel{(\gamma)^{\gamma-1}}} \times \rho \times \cancel{(\gamma)^{\gamma-1}} \times T_1$$

$$T_4 = (\rho)^{\gamma-1} \times \rho \times T_1$$

$$T_4 = \rho^{\gamma} \times \cancel{\rho^{-1}} \times \cancel{\rho} \times T_1$$

$$\boxed{T_4 = \rho^{\gamma}\,T_1} \rightarrow (12)$$

Sub (12) & (11) & (10) in (4)

$$\eta_{diesel} = 1 - \frac{1}{\gamma}\left[\frac{T_4 - T_1}{T_3 - T_2}\right]$$

$$= 1 - \frac{1}{\gamma}\left[\frac{\rho^{\gamma} T_1 - T_1}{\rho (r)^{\gamma-1} T_1 - (r)^{\gamma-1} T_1}\right]$$

$$= 1 - \frac{1}{\gamma}\left[\frac{(\rho^{\gamma} - 1)\cancel{T_1}}{(r)^{\gamma-1}\cancel{T_1}(\rho - 1)}\right]$$

$$= 1 - \frac{1}{\gamma \times (r)^{\gamma-1}}\left[\frac{(\rho^{\gamma} - 1)}{(\rho - 1)}\right]$$

$$= 1 - \frac{1}{r^{\gamma-1}}\left[\frac{(\rho^{\gamma} - 1)}{\gamma(\rho - 1)}\right]$$

$$\boxed{\eta_{diesel} = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{(\rho^{\gamma} - 1)}{\gamma(\rho - 1)}\right]} \rightarrow (13)$$

Equation (13) is the air standard efficiency for diesel cycle.

Mean effective pressure

$$P_m = \frac{\text{Workdone}}{\text{Swept Volume}} = \frac{Q_S - Q_R}{V_1 - V_2} \longrightarrow (14)$$

$$\text{Swept Volume} = (V_1 - V_2) = V_1\left[1 - \frac{V_2}{V_1}\right]$$

$$= V_1\left[1 - \frac{1}{r}\right]$$

$$\boxed{R = C_v(\gamma - 1)}$$

$$= \frac{mRT_1}{P_1}\left[\frac{r-1}{r}\right]$$

$$\boxed{V_1 - V_2 = \frac{m C_v(\gamma-1) T_1}{P_1}\left[\frac{r-1}{r}\right]} \longrightarrow (15)$$

$$\text{Work done} = Q_S - Q_R$$

$$= m C_p (T_3 - T_2) - m C_v (T_4 - T_1) \longrightarrow (16)$$

Sub (15) & (16) in (14)

$$P_m = \frac{m C_p (T_3 - T_2) - m C_v (T_4 - T_1)}{\frac{m C_v (\gamma - 1) T_1}{P_1}\left[\frac{r-1}{r}\right]}$$

$$P_m = \frac{mc_p(T_3-T_2) - mc_v(T_4-T_1)}{\frac{(\gamma-1)}{P_1} \times mc_vT_1\left[\frac{r-1}{r}\right]}$$

$$= \frac{P_1}{(\gamma-1)} \times \frac{r}{r-1} \times \left[\frac{mc_p(T_3-T_2) - mc_v(T_4-T_1)}{mc_vT_1}\right]$$

$$= \frac{P_1}{\gamma-1} \times \frac{r}{r-1} \times \left[\frac{\cancel{m}\left[c_p(T_3-T_2) - c_v(T_4-T_1)\right]}{\cancel{m}c_vT_1}\right]$$

$$= \frac{P_1}{\gamma-1} \times \frac{r}{r-1} \times \left(\frac{c_p(T_3-T_2)}{c_v(T_1)} - \frac{\cancel{c_v}(T_4-T_1)}{\cancel{c_v}(T_1)}\right)$$

$$P_m = \frac{P_1}{\gamma-1} \times \frac{r}{r-1} \times \left[\frac{\gamma(T_3-T_2)}{T_1} - \frac{T_4-T_1}{T_1}\right]$$

Sub equation (10) & (11) & (12) in above equation

$$P_m = \frac{P_1}{\gamma-1} \times \frac{r}{r-1}\left[\frac{\gamma\left[\rho(r)^{\gamma-1}T_1 - (r)^{\gamma-1}T_1\right]}{T_1} - \frac{(\rho^{\gamma}T_1 - T_1)}{T_1}\right]$$

$$= \frac{P_1}{\gamma-1} \times \frac{r}{r-1}\left[\frac{\gamma(r)^{\gamma-1}\cancel{T_1}\left[\rho-1\right]}{\cancel{T_1}} - \frac{\cancel{T_1}(\rho^{\gamma}-1)}{\cancel{T_1}}\right]$$

$$= \frac{P_1}{\gamma-1} \times \frac{r}{r-1}\left[\gamma(r)^{\gamma-1}(\rho-1) - (\rho^{\gamma}-1)\right]$$

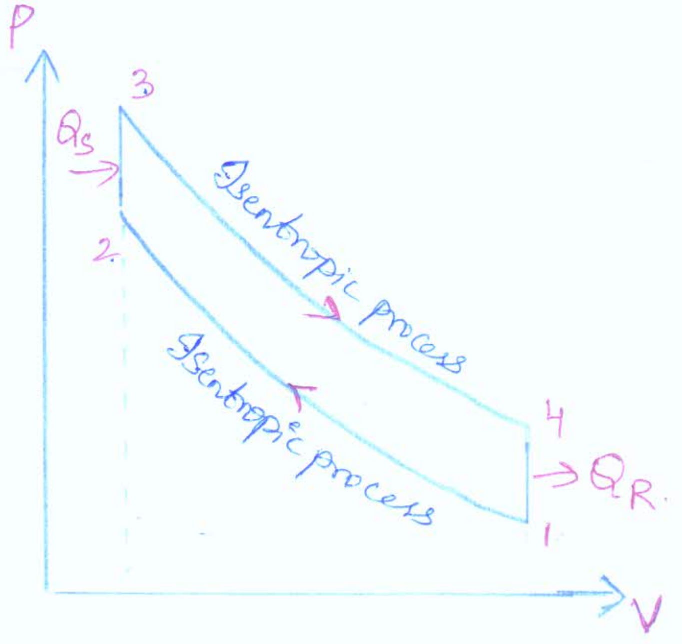
$$\boxed{P_m = \frac{P_1 r\left[\gamma(r)^{\gamma-1}(\rho-1) - (\rho^{\gamma}-1)\right]}{(\gamma-1)(r-1)}}$$

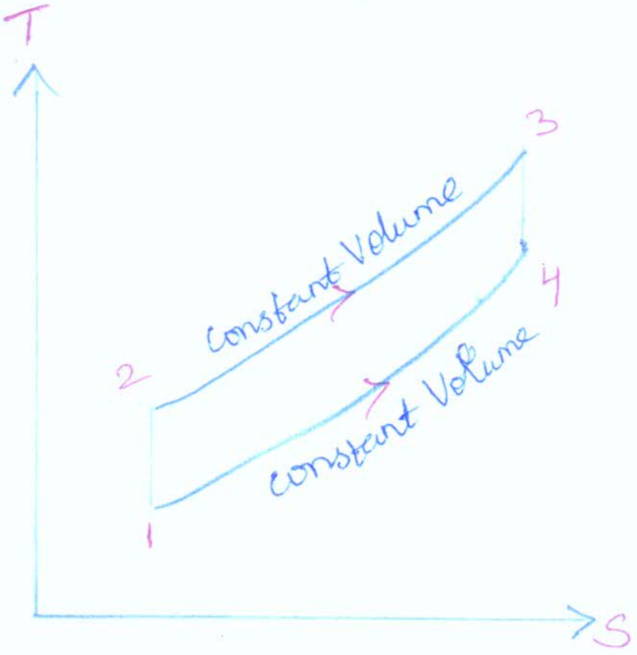
* Otto cycle is named after Dr. Nikolus. A. Otto a German Scientist, who built a Successful four Stroke engine in 1876.

* Otto cycle is the ideal cycle for Spark Ignition Reciprocating engine
* Petrol and Gas engine is the main application of Otto cycle.

Process:

* (1 – 2) Isentropic compression
* (2 - 3) Constant Volume heat addition
* (3 - 4) Isentropic expansion
* (4 - 1) Constant Volume heat rejection



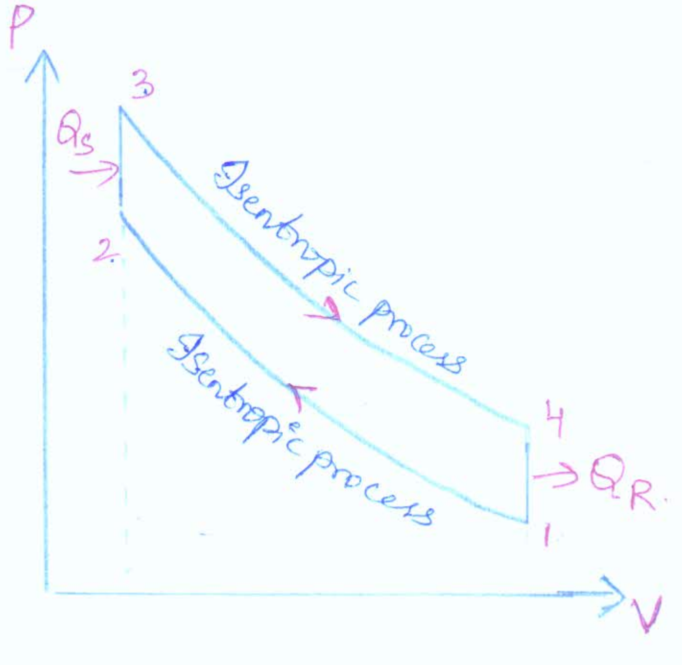


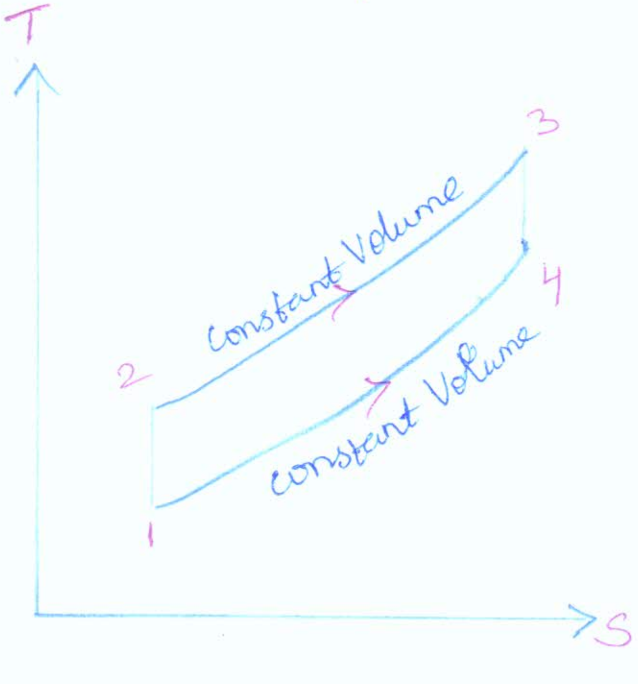
Ottocycle

* Ottocycle is named after Dr. Nikolus. A. Otto a German Scientist, who built a Successful four Stroke engine in 1876.
* Ottocycle is the ideal cycle for Spark Ignition Reciprocating engine
* Petrol and Gas engine is the main application of Ottocycle.

Process:

* (1-2) Isentropic compression
* (2-3) Constant Volume heat addition
* (3-4) Isentropic expansion
* (4-1) Constant Volume heat rejection





We know that,

$$\eta_{otto} = \frac{Q_S - Q_R}{Q_S} \rightarrow (1)$$

From process $2 \rightarrow 3$ we get,

$$Q_S = m c_v (T_3 - T_2) \rightarrow (2)$$

From process $4 \rightarrow 1$ we get,

$$Q_R = m c_v (T_4 - T_1) \rightarrow (3)$$

Sub (2) & (3) in (1)

$$\eta_{otto} = \frac{m c_v (T_3 - T_2) - m c_v (T_4 - T_1)}{m c_v (T_3 - T_2)}$$

$$= \frac{\cancel{m} \cancel{c_v} [T_3 - T_2 - T_4 + T_1]}{\cancel{m} \cancel{c_v} (T_3 - T_2)}$$

$$= \frac{T_3 - T_2 - T_4 + T_1}{T_3 - T_2}$$

$$= \frac{\cancel{T_3 - T_2}}{\cancel{T_3 - T_2}}^{1} - \frac{(T_4 - T_1)}{T_3 - T_2}$$

$$\boxed{\eta_{otto} = 1 - \frac{T_4 - T_1}{T_3 - T_2}} \rightarrow (4)$$

From process $1 \rightarrow 2$ we get,
(Isentropic process)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} \rightarrow (5)$$

From process $3 \rightarrow 4$ we get,
(Isentropic process)

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma - 1} \rightarrow (6)$$

But, $$\frac{V_1}{V_2} = \frac{V_4}{V_3} = r \rightarrow (7)$$

Sub (7) in (6)

$$\frac{T_3}{T_4} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} \rightarrow (8)$$

Compare (5) & (8)

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \Rightarrow \boxed{T_4 = \frac{T_3 \times T_1}{T_2}} \rightarrow (9)$$

Sub (9) in (4)

$$\eta_{otto} = 1 - \frac{(T_4 - T_1)}{T_3 - T_2}$$

$$= 1 - \frac{\left(\frac{T_1 \times T_3}{T_2} - T_1\right)}{T_3 - T_2}$$

$$= 1 - \left[\frac{\frac{T_1 T_3 - T_2 T_1}{T_2}}{T_3 - T_2}\right]$$

$$= 1 - \left[\frac{\frac{T_1 (T_3 - T_2)}{T_2}}{T_3 - T_2}\right]$$

$$= 1 - \left[\frac{T_1 \cancel{(T_3 - T_2)}}{T_2} \times \frac{1}{\cancel{(T_3 - T_2)}}\right]$$

$$\boxed{\eta_{otto} = 1 - \frac{T_1}{T_2}} \rightarrow (10)$$

from equation (5)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma - 1} \rightarrow (11)$$

Sub (11) in (10)

$$\eta_{otto} = 1 - \frac{\cancel{T_1}}{\cancel{T_1}\left(\frac{V_1}{V_2}\right)^{\gamma-1}}$$

$$\eta_{otto} = 1 - \frac{1}{\left(\frac{V_1}{V_2}\right)^{\gamma-1}}$$

$$\boxed{\eta_{otto} = 1 - \frac{1}{(r)^{\gamma-1}}} \rightarrow (12)$$

Equation (12) is Airstandard efficiency of Otto cycle

Mean effective pressure

It is defined as the average force acting on the piston during the entire power stroke that would produce same amount of net work output during actual cycle

$$\boxed{M.E.P}\ P_m = \frac{\text{Workdone}}{\text{Swept Volume}} = \frac{Q_S - Q_R}{V_1 - V_2} \rightarrow (13)$$

$$\text{Swept Volume} = (V_1 - V_2) = V_1\left(1 - \frac{V_2}{V_1}\right)$$

$$= V_1\left(1 - \frac{1}{r}\right)$$

$$\boxed{\frac{V_1}{V_2} = r}$$

$$= \frac{mRT_1}{P_1}\left(\frac{r-1}{r}\right)$$

$$\boxed{V_1 - V_2 = \frac{m\,C_v(\gamma-1)T_1}{P_1}\left(\frac{r-1}{r}\right)}$$

$$\boxed{R = C_v(\gamma-1)}$$

↳ (14)

Work done = $Q_s - Q_R = m\,C_v(T_3 - T_2) - [m\,C_v(T_4 - T_1)]$

$$\boxed{Q_s - Q_R = m\,C_v(T_3 - T_2) - [m\,C_v(T_4 - T_1)]}$$

↳ (15)

Sub (15) & (14) in (13)

$$P_m = \frac{\cancel{m}\,\cancel{C_v}\,[(T_3 - T_2) - (T_4 - T_1)]}{\frac{\cancel{m}\,\cancel{C_v}(\gamma-1)T_1}{P_1}\left(\frac{\gamma-1}{\gamma}\right)}$$

$$P_m = \frac{[(T_3 - T_2) - (T_4 - T_1)]}{\frac{T_1(\gamma-1)(\gamma-1)}{P_1\,r}}$$

$$\boxed{P_m = \frac{P_1\,r\,[T_3 - T_2 - T_4 + T_1]}{T_1(\gamma-1)(r-1)}}$$

↳ (16)

We know, from process 1 → 2 (Isentropic)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\frac{T_2}{T_1} = (r)^{\gamma-1}$$

$$\boxed{T_2 = T_1 (r)^{\gamma-1}} \rightarrow (17)$$

From process 2 → 3

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} = r_p$$

$$\boxed{T_3 = r_p \cdot T_2} \Rightarrow \boxed{T_3 = r_p (r)^{\gamma-1} \times T_1} \rightarrow (18)$$

From process 3 – 4,

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$$

$$\frac{T_3}{T_4} = (r)^{\gamma-1}$$

$$T_4 = \frac{T_3}{(r)^{\gamma-1}}$$

$$= \frac{r_p \cancel{(r)^{\gamma-1}} \times T_1}{\cancel{(r)^{\gamma-1}}}$$

$$\boxed{T_4 = r_p T_1} \rightarrow (19)$$

Sub (19), (18) & (17) in (16)

$$P_m = \frac{P_1 r \left[r_p (r)^{\gamma-1} T_1 - T_1 (r)^{\gamma-1} - r_p T_1 + T_1 \right]}{T_1 (\gamma-1)(r-1)}$$

$$= \frac{P_1 r \cancel{T_1} \left[r_p (r)^{\gamma-1} - (r)^{\gamma-1} - r_p + 1 \right]}{\cancel{T_1} (\gamma-1)(r-1)}$$

$$= \frac{P_1 r \left[r_p \left[(r)^{\gamma-1} - 1 \right] - 1 \left[(r)^{\gamma-1} - 1 \right] \right]}{(\gamma-1)(r-1)}$$

$$\boxed{P_m = \frac{P_1 r \left[(r_p - 1) \left((r)^{\gamma-1} - 1 \right) \right]}{(\gamma-1)(r-1)}} \rightarrow (20)$$

Equation (20) is Mean effective pressure of otto cycle.