Problems related to Steady State Heat conduction at Hollow cylinder

1) A pipe carrying steam at 230°C has an internal diameter of 12 cm and the pipe thickness is 7.5 mm. The conductivity of the pipe material is 49 W/mK, the convective heat transfer coefficient on the inside is 85 W/m²K. The pipe is insulated by two layers of insulation one of 5 cm thickness of conductivity 0.15 W/mK. and over it another 5 cm thickness of conductivity 0.48 W/mK. The outside is exposed to air at 35°C with a convection coefficient of 18 W/m²K. Determine the heat loss for 5m length. Also determine the interface temperatures and the overall heat transfer coefficient based on inside and outside area.

Given data:

$T_a = 230°C$

$d_1 = 12\,cm = 0.12\,cm$

$r_1 = 0.06\,m$

$t_1 = 7.5\,mm = 0.0075\,m$

$k_1 = 49\,W/mK = 49\,W/m°C$

$h_a = 85\ W/m^2 K$

$= 85\ W/m^{2\,\circ}C$

$t_2 = 5cm$

$= 0.05m$

$K_2 = 0.15\ W/mK$

$= 0.15\ W/m^\circ C$

$t_3 = 5cm$

$= 0.05m$

$K_3 = 0.48\ W/mK$

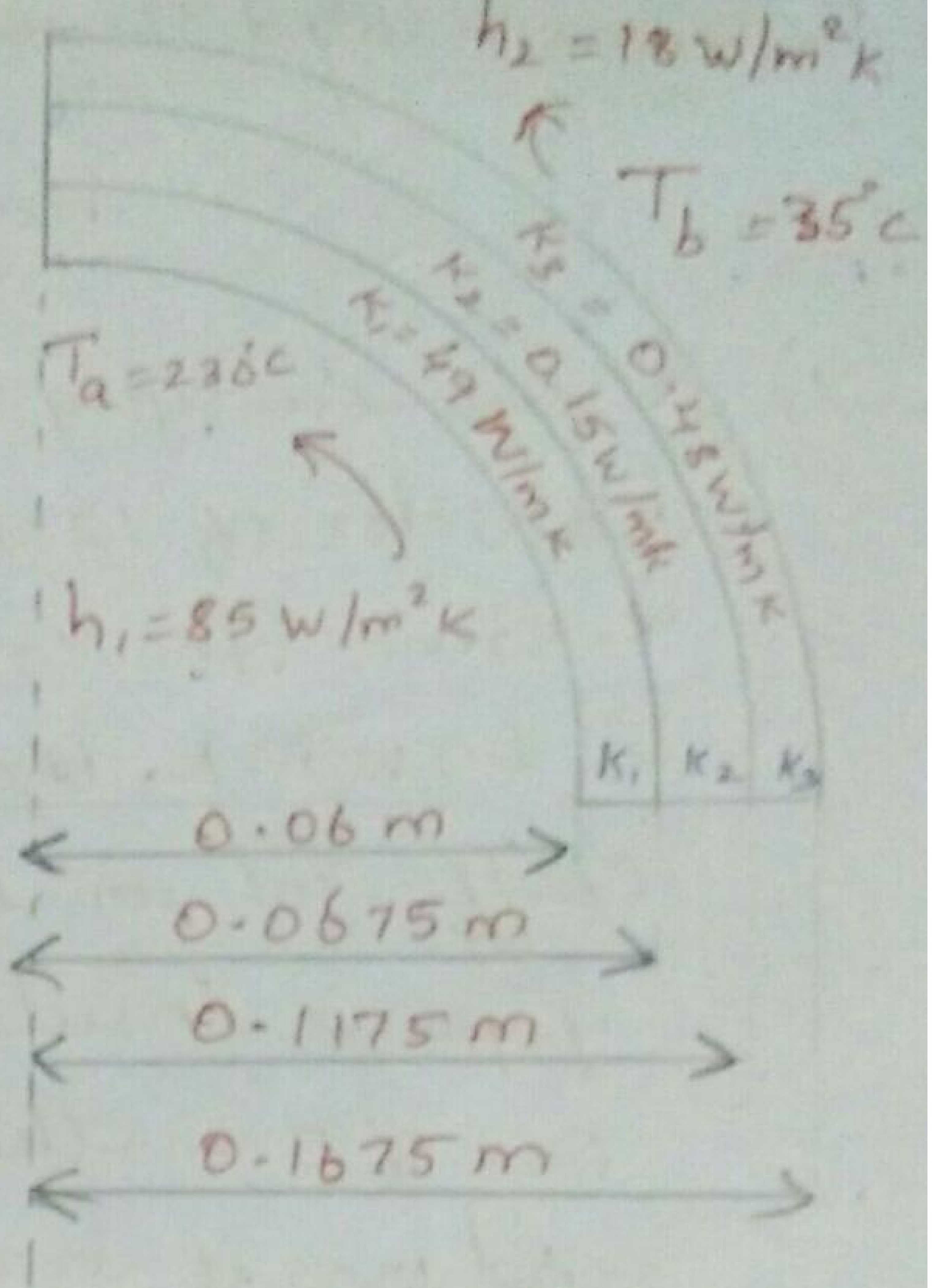
$= 0.48\ W/m^\circ C$

$T_b = 35^\circ C$

$h_b = 18\ W/m^2 K$

$= 18\ W/m^2\ ^\circ C$

$L = 5m$



To find:

(i) Q

(ii) T_1, T_2, T_3, T_4.

(iii) U_i, U_o

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Solution:

$$Q = \frac{\Delta T}{R}$$

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2}\ln\left(\frac{r_3}{r_2}\right) + \frac{1}{k_3}\ln\left(\frac{r_4}{r_3}\right) + \frac{1}{h_b r_4}\right]$$

$$R = \frac{1}{2\pi \times 5}\left[\frac{1}{85 \times 0.06} + \frac{1}{49}\ln\left(\frac{0.0675}{0.06}\right) + \frac{1}{0.15}\ln\left[\frac{0.1175}{0.0675}\right] + \frac{1}{0.48}\ln\left(\frac{0.1675}{0.1175}\right) + \frac{1}{18 \times 0.1675}\right]$$

$$= \frac{1}{2\pi \times 5}\left[0.1961 + 2.404 \times 10^{-3} + 3.695 + 0.7386 + 0.3317\right]$$

$$\boxed{R = 0.158}$$

$$\Delta T = T_a - T_b = 230 - 35$$

$$\boxed{\Delta T = 195^\circ C}$$

$$Q = \frac{195}{\cancel{0.10375}\ 0.158}$$

$$\boxed{Q = 1234.16\ W}$$

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Overall heat transfer Coefficient

Based on Inside area.

$$Q = U_i A_i \Delta T$$

$$U_i = \frac{Q}{A_i \Delta T} = \frac{Q}{2\pi r_i l \times (T_a - T_b)}$$

$$= \frac{1234.16}{2\pi \times 0.06 \times 5 \times (230-35)}$$

$$\boxed{U_i = 3.3576 \ W/m^2 K}$$

Based on Outside area

$$Q = U_o A_o \Delta T$$

$$U_o = \frac{Q}{A_o \Delta T} = \frac{Q}{2\pi r_o l \times (T_a - T_b)}$$

$$= \frac{1234.16}{2\pi \times 0.1675 \times 5 \times (230-35)}$$

$$\boxed{U_o = 1.203 \ W/m^2 K}$$

Interface temperature

$$Q = \frac{\Delta T}{R} = \frac{T_a - T_1}{\frac{1}{2\pi L}\left[\frac{1}{h_a \times r_1}\right]}$$

$$Q = \frac{T_a - T_1}{\frac{1}{2\pi L}\left[\frac{1}{h_a \times r_1}\right]} \Rightarrow 1234.16 = \frac{230 - T_1}{\frac{1}{2\pi \times 5}\left[\frac{1}{85 \times 0.06}\right]}$$

$$\boxed{T_1 = 222.3\,^\circ C}$$

$$Q = \frac{\Delta T}{R} = \frac{T_1 - T_2}{\frac{1}{2\pi L k_1}\left[\ln\left(\frac{r_2}{r_1}\right)\right]}$$

$$1234.16 = \frac{222.3 - T_2}{\frac{1}{2\pi \times 5 \times 49}\left[\ln\left(\frac{0.0675}{0.06}\right)\right]}$$

$$\boxed{T_2 = 222.2\,^\circ C}$$

$$Q = \frac{\Delta T}{R} = \frac{T_2 - T_3}{\frac{1}{2\pi L k_2}\left[\ln\left(\frac{r_3}{r_2}\right)\right]}$$

$$1234.16 = \frac{222.2 - T_3}{\frac{1}{2\pi \times 5 \times 0.15}\left[\ln\left(\frac{0.1175}{0.0675}\right)\right]}$$

$$\boxed{T_3 = 77.04^\circ C}$$

$$Q = \frac{\Delta T}{R} = \frac{T_3 - T_4}{\frac{1}{2\pi L k_3}\left[\ln\left(\frac{r_4}{r_3}\right)\right]}$$

$$1234.16 = \frac{77.04 - T_4}{\frac{1}{2\pi \times 5 \times 0.48}\left[\ln\left(\frac{0.1675}{0.1175}\right)\right]}$$

$$\boxed{T_4 = 48.02^\circ C}$$

Result:

(i) $Q = 1234.16$ W

(ii) $T_1 = 222.3^\circ C$

$T_2 = 222.2^\circ C$

$T_3 = 77.04^\circ C$

$T_4 = 48.02^\circ C$

(iii) $U_i = 3.3576\ W/m^2K$

$U_o = 1.203\ W/m^2K.$

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Problems related to Fins

1) A long rod 5cm diameter its base is connected to a furnace wall at 150°c, while the end is projecting into the room at 20°c. The temperature of the rod at distance of 20cm apart from its base is 60°c. The conductivity of the wall is 200 W/mK. Determine Convective heat transfer coefficient.

Given data:

$d = 5cm = 0.05 m$

$T_b = 150°C = 150 + 273 = 423 K.$

$T_\infty = 20°C = 20 + 273 = 293 K$

$x = 20 cm = 0.2 m$

$T = 60°C = 60 + 273 = 333 K.$

$K = 200 W/mK$

To find:

h

Solution:

We know for Infinite long rod

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx}$$

$$\frac{333 - 293}{423 - 293} = e^{-m \times 0.2}$$

$$0.3077 = e^{-m \times 0.2}$$

$$\ln(0.3077) = -m \times 0.2$$

$$-1.17865 = -m \times 0.2$$

$$\boxed{m = 5.89}$$

$$m = \sqrt{hP/KA}$$

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0.05^2$$

$$\boxed{A = 1.96 \times 10^{-3}\ m^2}$$

$$P = \pi \times d = \pi \times 0.05$$

$$\boxed{P = 0.157\ m}$$

$$m = \sqrt{\frac{hP}{KA}}$$

$$5.89 = \sqrt{\frac{h \times 0.157}{200 \times 1.96 \times 10^{-3}}}$$

$$\boxed{h = 86.62 \ W/m^2K}$$

Result: $h = 86.62 \ W/m^2K$.

2) An Aluminium wall fin of 7mm thick and 50mm long protruted from a wall, which is maintained at 120°C. The ambient air temperature is 22°C. The heat transfer coefficient and conductivity of the fin wall are $140 W/m^2K$ & $55 W/m^\circ K$. Determine

1) Temperature at end of fin
2) Temperature at the middle of fin
3) Total heat dissipated by fin

Given data:

$t = 7 mm = 0.007 m$

$L = 50 mm = 0.05 m$

$T_b = 120^\circ C = 120 + 273 = 393 K$

$T_\infty = 22^\circ c = 22 + 273 = 295 K$

$h = 140 \ W/m^2 k$

$k = 55 \ W/mk.$

To find:

(i) T

(ii) T_{mid}

(iii) Q.

Solution

(i) Temperature at end of film

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m[L - x]}{\cosh (mL)}$$

$x = L$

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m[L - L]}{\cosh (mL)}$$

$$\frac{T - 295}{393 - 295} = \frac{\cosh (0)}{\cosh (mL)}$$

$$\frac{T-295}{98} = \frac{1}{\cosh[mL]}$$

$$m = \sqrt{\frac{hP}{KA}}$$

$$P = 2 \times L = 2 \times 0.05$$

$$\boxed{P = 0.1\ m}$$

$$A = L \times t = 0.05 \times 0.007$$

$$\boxed{A = 3.5 \times 10^{-4}\ m^2}$$

$$m = \sqrt{\frac{140 \times 0.1}{55 \times 3.5 \times 10^{-4}}}$$

$$\boxed{m = 26.96}$$

$$\frac{T-295}{98} = \frac{1}{\cosh[26.96 \times 0.05)}$$

$$\frac{T-295}{98} = \frac{1}{2.055}$$

$$\boxed{T = 342.8\ k}$$

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(ii) Temperature at middle of fins.

$$x = \frac{L}{2}$$

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m\left[L - L/2\right]}{\cosh (mL)}$$

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m\left[L/2\right]}{\cosh (mL)}$$

$$\frac{T - 295}{393 - 295} = \frac{\cosh 26.96 \left(\frac{0.05}{2}\right)}{\cosh (26.96 \times 0.05)}$$

$$\boxed{T = 354\,K}$$

(iii) Total Heat dissipated

$$Q = (hPKA)^{1/2} (T_b - T_\infty) \tanh (mL)$$

$$= (140 \times 0.1 \times 55 \times 3.5 \times 10^{-4})^{1/2} (393 - 295) \times \tanh (26.96 \times 0.05)$$

$$\boxed{Q = 44.4\,W}$$

Result:

(i) $T_{end} = 342.8\,K$

(ii) $T_{mid} = 354\,K$.

(iii) $Q = 44.4\,W$

3. A Stainless Steel cylindrical rod fin of 1·2 cm diameter and 6 cm height with thermal conductivity of 25 W/mK is exposed to Surroundings with a temperature of 60°C. The heat transfer coefficient is 45 W/m²K and the temperature at the base of the fin is 100°C. Determine Fin efficiency, Temperature at the edge of the rod, Heat dissipation, Fin effectiveness. Assume fin end is Insulated.

Given data:

$d = 1.2\,cm = 0.012\,m$

$L = 6\,cm = 0.06\,m$

$k = 25\,W/mk$

$T_\infty = 60°C = 60 + 273 = 333\,k.$

$h = 45\,W/m^2k.$

$T_b = 100°C = 100 + 273 = 373\,K$

To find: (i) η_{fin}, (ii) T, (iii) Q, (iv) ε

Solution:

(i) $\eta_{fin} = \dfrac{\tanh(mL)}{mL}$

$m = \sqrt{\dfrac{hP}{KA}}$

$P = \pi d = \pi \times 0.012 = 0.0377\ m$

$A = \dfrac{\pi}{4} \times d^2 = \dfrac{\pi \times 0.012^2}{4}$

$\boxed{A = 1.13 \times 10^{-4}\ m^2}$

$m = \sqrt{\dfrac{45 \times 0.0377}{25 \times 1.13 \times 10^{-4}}}$

$\boxed{m = 24.5}$

$\eta_{fin} = \dfrac{\tanh(24.5 \times 0.06)}{24.5 \times 0.06}$

$= 0.6119$

$\boxed{\eta_{fin} = 61.2\%}$

(ii) Temperature at edge (T)

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x)}{\cosh (mL)}$$

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - L)}{\cosh (mL)}$$

$$\frac{T - 333}{373 - 333} = \frac{\cosh m(0)}{\cosh (mL)}$$

$$\frac{T - 333}{40} = \frac{1}{\cosh (24.5 \times 0.06)}$$

$$\boxed{T = 350.47 K}$$

(iii) Heat Dissipation (Q)

$$Q = (hPkA)^{1/2} (T_b - T_\infty) \tanh (mL)$$

$$= (45 \times 0.0377 \times 25 \times 1.13 \times 10^{-4})^{1/2} \times (373 - 333) \times \tanh (24.5 \times 0.06)$$

$$\boxed{Q = 2.49 W}$$

(iv) Fin effectiveness (ε)

$$\varepsilon = \frac{\tanh(mL)}{\sqrt{\frac{hA}{KP}}}$$

$$= \frac{\tanh(24.5 \times 0.06)}{\sqrt{\frac{45 \times 1.13 \times 10^{-4}}{25 \times 0.0377}}} = \frac{0.8996}{0.0735}$$

$$\boxed{\varepsilon = 12.2}$$

Result:

(i) $\eta_{fin} = 61.2\%$

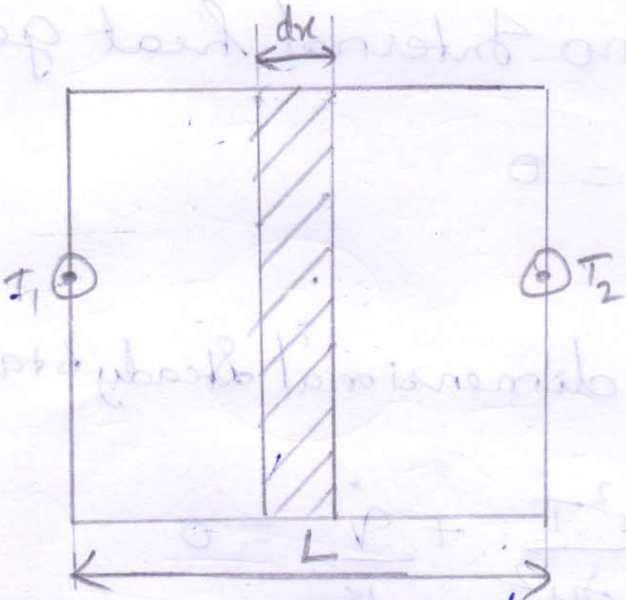
(ii) $T = 350.47\,K$

(iii) $Q = 2.49\,W$

(iv) $\varepsilon = 12.2$

One Dimensional Steady State Heat Conduction

Conduction of heat through a Slab (or) plane wall



Let K → Uniform thermal conductivity

T_1 → Inner temperature

T_2 → Outer temperature

L → Thickness

dx → Small element area of thickness

From fourier law of heat conduction

$$Q = -KA\frac{dT}{dx}.$$

$$Q.dx = -KA\,dT \rightarrow (1)$$

Integrating equation ① with the limits

0 to L, T_2 to T_1

$$\int_0^L Q\,dx = -KA\int_{T_1}^{T_2} dT$$

$$Q\,[x]_0^L = -KA\,[T]_{T_1}^{T_2}$$

$$Q\,[L-0] = -KA\,[T_2 - T_1]$$

$$Q\,[L] = KA\,[T_1 - T_2]$$

$$Q = \frac{KA}{L}\,[T_1 - T_2]$$

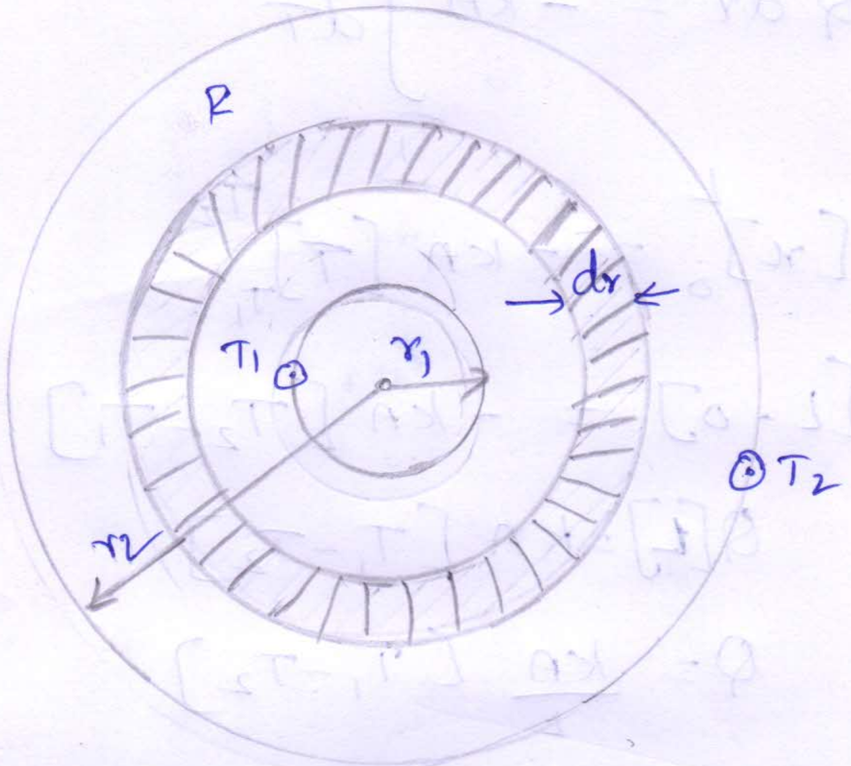
$$Q = \frac{T_1 - T_2}{L/KA}$$

$$Q = \frac{T_1 - T_2}{R} \qquad R = \frac{L}{KA}$$

R → Thermal resistance

$$\boxed{Q = \frac{\Delta T}{R}} \qquad \Delta T = T_1 - T_2$$

Conduction of heat through Hollow Cylinder



K → Thermal conductivity

r_1 → Inner Radius

r_2 → Outer Radius

T_1 → Inter temperature

T_2 → Outer temperature

dr → Small element area of thickness

From fourier law of heat conduction

$$Q = -KA \frac{dT}{dx}$$

Area of cylinder $A = 2\pi r L$

$$Q = -K \times 2\pi r L \times \frac{dT}{dx} \rightarrow ①$$

from ①

$$Q \times \frac{dr}{r} = -K \times 2\pi L \times dT \rightarrow ②$$

Integrating equation ② we get

$$Q \int_{r_1}^{r_2} \frac{dr}{r} = -K \times 2\pi L \int_{T_1}^{T_2} dT$$

$$Q \left[\ln r\right]_{r_1}^{r_2} = -K \times 2\pi L \left[T_2 - T_1\right]$$

$$\boxed{\begin{array}{l} Q \cancel{\ln \left[\frac{r_2}{r_1}\right]} = \\ Q = \cancel{\ln [r_2 - r_1]} \end{array}}$$

$$Q \left[\ln r_2 - \ln r_1\right] = -K \times 2\pi L \left[T_2 - T_1\right]$$

$$Q \ln\left[\frac{r_2}{r_1}\right] = K\, 2\pi L \left[T_1 - T_2\right]$$

$$Q = \frac{2\pi L K \left[T_1 - T_2\right]}{\ln\left[\frac{r_2}{r_1}\right]}$$

$$Q = \frac{T_1 - T_2}{\frac{1}{2\pi L K}\ln\left(\frac{r_2}{r_1}\right)}$$

$$\Delta T = T_1 - T_2$$

$$R = \frac{1}{2\pi L K}\ln\left(\frac{r_2}{r_1}\right)$$ → Thermal ~~radiat~~ resistance of hollow cylinder.

$$\boxed{Q = \frac{\Delta T}{R}}$$

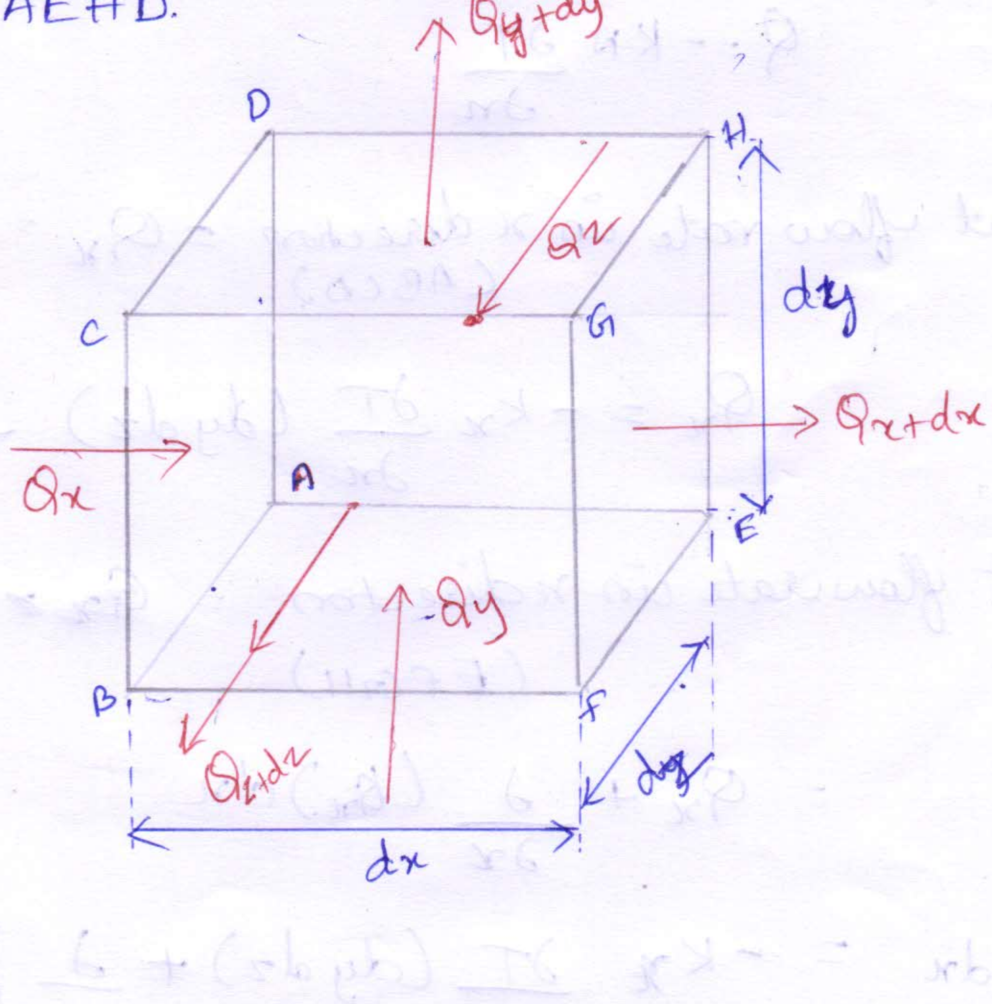
General Heat Conduction equation in Cartesian Coordinates

Let us consider the cube of side ABCDEFGH

Let Q_x be the heat passed through the face ABCD

Let Q_y be the heat passed through the face ABEF

Let Q_z be the heat passed through the face AEHD.



The energy balance of this rectangular element is obtained from first law of thermodynamics

Net heat conducted into element from all the coordinate direction + Heat generated with in the system = Heat stored in the element $\rightarrow$ (1)

We know that from fourier law of conduction

$$Q = -KA \frac{\partial T}{\partial x}$$

Heat flow rate in x direction (ABCD) $= Q_x = -K_x (dy\, dz) \frac{\partial T}{\partial x}$

$$Q_x = -K_x \frac{\partial T}{\partial x} (dy\, dz) \rightarrow (2)$$

Heat flow rate in x direction (EFGH) $= Q_{x+dx}$

$$= Q_x + \frac{\partial}{\partial x} (Q_x)\, dx$$

$$Q_{x+dx} = -K_x \frac{\partial T}{\partial x} (dy\, dz) - \frac{\partial}{\partial x} \left[K_x \frac{\partial T}{\partial x} dy\, dz\right] dx$$

$\rightarrow$ (3)

Subtracting (2) & (3) we get

$$Q_x - Q_{x+dx} = -K_x \frac{\partial T}{\partial x}(dy\,dz) - \left[-K_x \frac{\partial T}{\partial x} dy\,dz - \frac{\partial}{\partial x}\left[K_x \frac{\partial T}{\partial x} dy\,dz\right]dx\right]$$

$$= \cancel{-K_x \frac{\partial T}{\partial x} dy}\,dz + \cancel{K_x \frac{\partial T}{\partial x} dy}\,dz + \frac{\partial}{\partial x}\left[K_x \frac{\partial T}{\partial x} dx\,dy\,dz\right]$$

$$Q_x - Q_{x+dx} = \frac{\partial}{\partial x}\left[K_x \frac{\partial T}{\partial x} dx\,dy\,dz\right]$$

$$\boxed{Q_x - Q_{x+dx} = \frac{\partial^2 T}{\partial x^2} K_x\, dx\,dy\,dz} \rightarrow (4)$$

Similarly for Y & Z direction

$$Q_y - Q_{y+dy} = \frac{\partial^2 T}{\partial y^2} K_y\, dx\,dy\,dz \rightarrow (5)$$

$$Q_z - Q_{z+dz} = \frac{\partial^2 T}{\partial z^2} K_z\, dx\,dy\,dz \rightarrow (6)$$

Net heat conducted into the element from all the coordinate direction $= (Q_x - Q_{x+dx}) + (Q_y - Q_{y+dy}) + (Q_z - Q_{z+dz})$

Sub (4) & (5) & (6) in above equation

$$= \frac{\partial^2 T}{\partial x^2} K_x \, dx\, dy\, dz + \frac{\partial^2 T}{\partial y^2} K_y \, dx\, dy\, dz + \frac{\partial^2 T}{\partial z^2} K_z \, dx\, dy\, dx$$

Net heat Conducted into the element $= \left[\frac{\partial^2 T}{\partial x^2} K_x + \frac{\partial^2 T}{\partial y^2} K_y + \frac{\partial^2 T}{\partial y^2} K_z\right] dx\, dy\, dz \rightarrow (7)$

$\boxed{\text{Heat generated within the element} = Q = \dot{q}\, dx\, dy\, dz \rightarrow (8)}$

Heat Stored in the element $= m\, C_p \frac{\partial T}{\partial t}$

$= \rho \times V \times C_p \frac{\partial T}{\partial t}$

$\boxed{\rho = \frac{m}{V} \quad m = \rho V}$

$\boxed{\text{Heat stored in the element} = \rho \times dx\, dy\, dz \times C_p \frac{\partial T}{\partial t}} \rightarrow (9)$

Sub (9) & (8) & (7) in (1)

$$\left(\frac{\partial^2 T}{\partial x^2} k_x + \frac{\partial^2 T}{\partial y^2} k_y + \frac{\partial^2 T}{\partial z^2} k_z\right) dx\,dy\,dz + \dot{q}\,dx\,dy\,dz$$

$$= \rho \times dx\,dy\,dz \cdot C_p \frac{\partial T}{\partial t}$$

$$\left[\left(\frac{\partial^2 T}{\partial x^2} k_x + \frac{\partial^2 T}{\partial y^2} k_y + \frac{\partial^2 T}{\partial z^2}\right) + \dot{q}\right] \cancel{dx}\,\cancel{dy}\,\cancel{dz} =$$

$$\rho\,\cancel{dx}\,\cancel{dy}\,\cancel{dz} \cdot C_p \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} k_x + \frac{\partial^2 T}{\partial y^2} k_y + \frac{\partial^2 T}{\partial z^2} + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

System is Isotropic & Homogeneous So, $k_x = k_y = k_z = k$.

$$\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] k + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

Divide both sides by k.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{\rho C_p}{k} \frac{\partial T}{\partial t}$$

$$\alpha = \frac{k}{\rho C_p}$$

Sub α in above equation.

$$\boxed{\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\dot{q}}{k} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial t}\right)} \rightarrow (10)$$

The above equation is the General Three Dimensional equation for Heat conduction in Cartesian Coordinates

Case(i) : Absence of Internal heat generation ($\dot{q} = 0$)

Sub $\dot{q} = 0$ in (10)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial t}\right)$$

Case(ii) : Steady State condition

Sub $\frac{\partial T}{\partial t} = 0$ in (10)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = 0$$

(poisson equation)

Considering no heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

(Laplace equation)

Case (iii): One dimensional Steady State heat Conduction

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{K} = 0 \qquad \left(\frac{\partial T}{\partial t} = 0\right)$$

If there is no Internal heat generation

$$\frac{\partial^2 T}{\partial x^2} = 0$$

Case (iv): Two dimensional Steady State heat conduction

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\dot{q}}{K} = 0 \qquad \left(\frac{\partial T}{\partial t} = 0\right)$$

If there is no Internal heat generation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Case (v): Un Steady State one dimensional without heat generation.

$$\left(\frac{\partial T}{\partial t} \neq 0\right)$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Fins:

It is possible to increase the heat transfer rate by increasing the surface of heat transfer. The surface used for increasing heat transfer are called extended surface (or) fins.

Types of fins

* Long fin
* Short fin (End is Insulated)
* Short fin (End is not Insulated)

Fin Efficiency:

It is defined as the ratio of actual heat transferred fin to the maximum possible heat transferred by the fin.

$$\eta_{fin} = \frac{Q_{fin}}{Q_{max}}$$

$$\eta_{fin} = \frac{\tanh(mL)}{mL}$$

Fin Effectiveness

It is defined as the ratio of heat transfer with fin to heat transfer with out fin

$$E = \frac{Q_{\text{with fin}}}{Q_{\text{without fin}}}$$

$$E = \frac{\tan(mL)}{\sqrt{\frac{hA}{kP}}}$$

For long fine

a) Temperature distribution

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx}$$

b) Heat transferred

$$Q = (T_b - T_\infty)\sqrt{hPKA}$$

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2) For Short pin

a) Temperature Distribution

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh(mL)}$$

b) Heat transferred

$$Q = (hPKA)^{1/2}(T_b - T_\infty)\tanh(mL)$$

Number of fins $=$ $\dfrac{\text{Heat generated}}{\text{Heat transfer per fin}}$

$T \rightarrow$ Intermediate Temperature

$T_\infty \rightarrow$ Surrounding Temperature

$T_b \rightarrow$ Base Temperature

$x \rightarrow$ Distance

$h \rightarrow$ Heat transfer coefficient

$P \rightarrow$ Perimeter

$K \rightarrow$ Thermal conductivity

$A \rightarrow$ Area.

Problems related to Steady State Heat conduction at Plane wall

1) A furnace wall is of three layers, first layer of insulation brick of 12 cm thickness of conductivity 0.6 W/mK. The face is exposed to gases at 870°C with a convection coefficient of 110 W/m²K. This layer is backed by a 10 cm layer of fire brick of conductivity 0.8 W/mK. There is a contact resistance between the layers of 2.6×10^{-4} m² °C/W. The third layer is the plate backing of 10 mm thickness of conductivity 49 W/mK. The contact resistance between the second and third layer is 1.5×10^{-4} m² °C/W. The plate is exposed to air at 30°C with a conventional coefficient of 15 W/m²K. Determine the heat flow, the surface temperature and the overall heat transfer coefficient.

Given data:

$L_1 = 12\ cm = 0.12\ m$

$k_1 = 0.6\ W/mk = 0.6\ W/m^\circ C$

$T_a = 870^\circ C$

$h_a = 110\ W/m^2 k = 110\ W/m^2\ {}^\circ C$

$L_2 = 10\ cm = 0.1\ m$

$k_2 = 0.8\ W/mk = 0.8\ W/m^\circ C$

$= 0.8\ W/m^\circ C$

$R_{s_1} = 2.6 \times 10^{-4}\ m^2\ {}^\circ C/W$

$L_3 = 10\ mm = 0.01\ m$

$k_3 = 49\ W/mk = 49\ W/m^\circ C$

$= 49\ W/m^\circ C$

$R_{s_2} = 1.5 \times 10^{-4}\ m^2\ {}^\circ C/W$

$T_b = 30^\circ C$

$h_b = 15\ W/m^2 k = 15\ W/m^2\ {}^\circ C$

To find:

(i) Q

(ii) $T_1, T_{21}, T_{22}, T_{31}, T_{32}, T_4$

(iii) U

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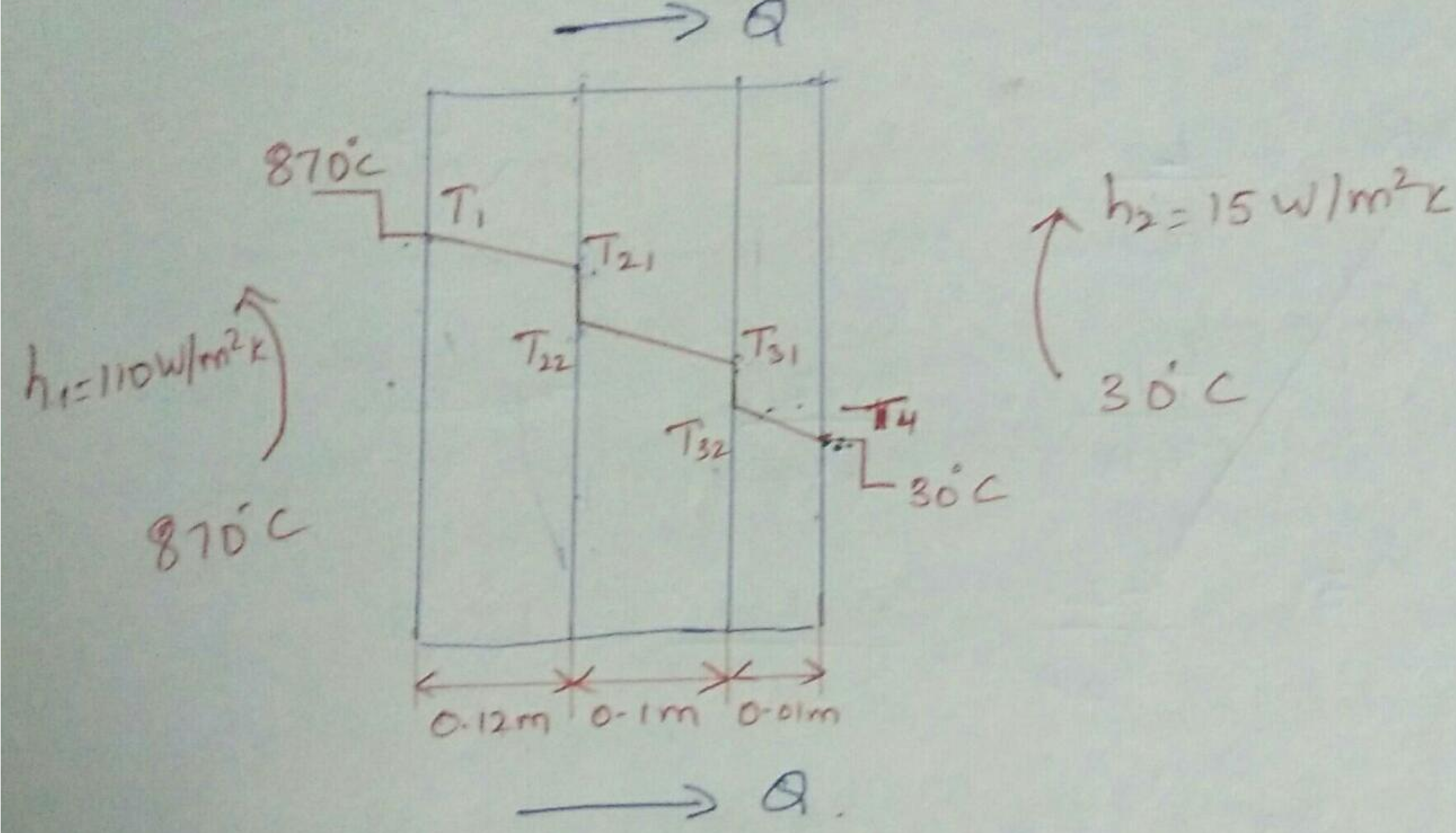
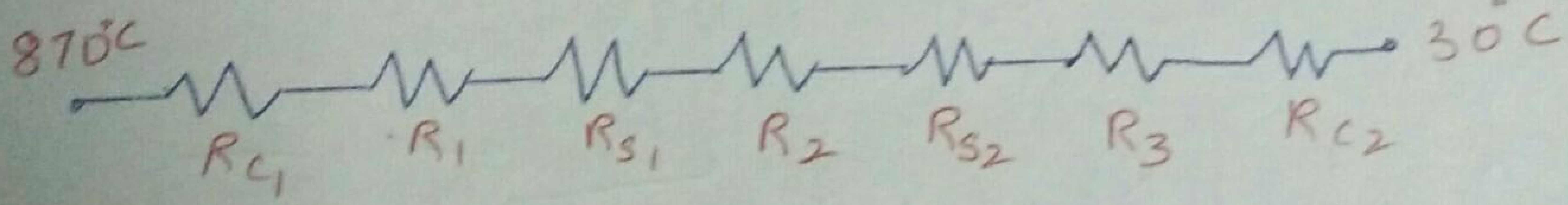
Solution:





we know that $Q = \dfrac{\Delta T}{R}$.

$$\Delta T = T_a - T_b$$

$$= 870 - 30$$

$$\boxed{\Delta T = 840^\circ C}$$

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$$R = R_{ca} + R_1 + R_{s_1} + R_2 + R_{s_2} + R_3 + R_{cb}$$

$$R_{ca} = \frac{1}{h_a A} = \frac{1}{110 \times 1} = 9.09 \times 10^{-3}$$

$$R_1 = \frac{L_1}{k_1 A} = \frac{0.12}{0.6 \times 1} = 0.2$$

$$R_2 = \frac{L_2}{k_2 A} = \frac{0.1}{0.8 \times 1} = 0.125$$

$$R_3 = \frac{L_3}{k_3 A} = \frac{0.01}{49 \times 1} = 2.041 \times 10^{-4}$$

$$R_{cb} = \frac{1}{h_b A} = \frac{1}{15 \times 1} = 0.067$$

$$R = R_{ca} + R_1 + R_{s_1} + R_2 + R_{s_2} + R_3 + R_{cb}$$

$$= 9.09 \times 10^{-3} + 0.2 + 2.6 \times 10^{-4} + 0.125 + 1.5 \times 10^{-4} + 2.041 \times 10^{-4} + 0.067$$

$$\boxed{R = 0.4017 \ ^\circ C/W}$$

$$Q = \frac{\Delta T}{R} = \frac{840}{0.4017}$$

$$\boxed{Q = 2091.1 \ W/m^2}$$

$$Q = UA\,\Delta T$$

$$U = \frac{Q}{A\,\Delta T} = \frac{2091.1}{1 \times 840}$$

$$\boxed{U = 2.49\ W/m^2\,{}^\circ C}$$

For finding Surface temperature.

$$Q = \frac{\Delta T}{R} = \frac{T_a - T_1}{1/h_a}$$

$$Q = \frac{T_a - T_1}{1/h_1} \Rightarrow 2091.1 = \frac{870 - T_1}{1/110}$$

$$\boxed{T_1 = 850.99^\circ C}$$

$$Q = \frac{\Delta T}{R} \Rightarrow \frac{T_1 - T_{21}}{R_1}$$

$$Q = \frac{T_1 - T_{21}}{R_1} \Rightarrow 2091.1 = \frac{850.99 - T_{21}}{0.2}$$

$$\boxed{T_{21} = 432.77^\circ C}$$

$$Q = \frac{\Delta T}{R_2} \Rightarrow \frac{T_{22} - T_{31}}{R_2} \cancel{\Rightarrow 209}$$

$$Q = \frac{T_{22} - T_{31}}{R_2} \Rightarrow 2091.1 =$$

$$Q = \frac{\Delta T}{R} = \frac{T_{21} - T_{22}}{R_{S1}}$$

$$Q = \frac{T_{21} - T_{22}}{R_{S1}} \Rightarrow 2091.1 = \frac{432.77 - T_{22}}{2.6 \times 10^{-4}}$$

$$\boxed{T_{22} = 432.2^{\circ}C}$$

$$Q = \frac{\Delta T}{R} = \frac{T_{22} - T_{31}}{R_2}$$

$$Q = \frac{T_{22} - T_{31}}{R_2} \Rightarrow 2091.1 = \frac{432.2 - T_{31}}{0.125}$$

$$\boxed{T_{31} = 170.81^{\circ}C}$$

$$Q = \frac{\Delta T}{R} = \frac{T_{31} - T_{32}}{R_{S2}}$$

$$Q = \frac{T_{31} - T_{32}}{R_{S2}} \Rightarrow 2091.1 = \frac{170.81 - T_{32}}{1.5 \times 10^{-4}}$$

$$\boxed{T_{32} = 170.49^{\circ}C}$$

$$Q = \frac{\Delta T}{R_3} = \frac{T_{32} - T_4}{R_3}$$

$$Q = \frac{T_{32} - T_4}{R_3} \Rightarrow 2091.1 = \frac{170.49 - T_4}{2.041 \times 10^{-4}}$$

$$\boxed{T_4 = 170.06°C}$$

Result:

(i) $q = 2091.1 \, W/m^2$

(ii) $T_1 = 850.99°C$

$T_{21} = 432.77°C$

$T_{22} = 432.2°C$

$T_{31} = 170.81°C$

$T_{32} = 170.49°C$

$T_4 = 170.06°C$

(iii) $U = 2.49 \, W/m^2 °C$