(1) How many edges are there in a graph with '10' vertices each of degree '6'.

Soln:

Let the no. of edges be 'e'.

Given: No. of vertices = 10

degree of each vertex = 6

By handshaking theorem, WKT

$$\sum deg(v) = 2e$$

$$6 \times 10 = 2e$$

$$60 = 2e$$

$$\boxed{30 = e}$$

(2) How many vertices does a regular graph with degree '4' with '10' edges have?

Soln:

Let the no. of vertices be 'v'

Given: No. of edges = 10

degree of each vertex = 4

By Handshaking theorem, WKT

$$\sum deg(v) = 2e$$

$$4 \times v = 2(10)$$

$$\boxed{v = 5}$$

(3) Can a simple graph exist with '15' vertices each of degree '3'.

Soln:

Let the no. of edges be 'e'.

Given: No. of vertices $= 15$

Degree of each vertex $= 3$

By handshaking Theorem, WKT

$$\sum \deg(v) = 2e$$

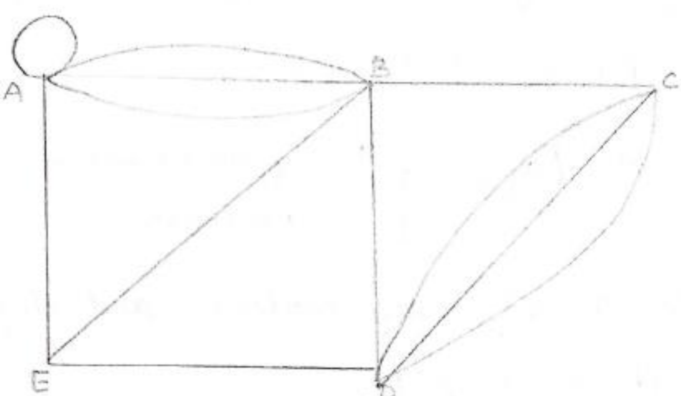
$$3 \times 15 = 2e$$

$$\frac{45}{2} = e$$, which is not an integer.

Therefore, a simple graph cannot exist.

(4) Find the no. of vertices, no. of edges and degree of each vertex in the following undirected graph and also verify handshaking theorem.

Soln:



No. of vertices $= 5$

No. of edges $= 12$

$\deg(A) = 6$

$\deg(B) = 6$

$\deg(C) = 4$

$\deg(D) = 5$

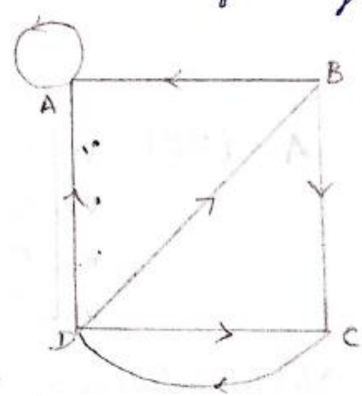
$\deg(E) = 3$

$\therefore$ Total degree $= \Sigma d(v)$
$= 6+6+4+5+3$
$= 24$

$\therefore$ By Handshaking Theorem, WKT
$$\Sigma d(v) = 2e$$
$$24 = 2(12)$$
$$= 24$$, hence proved.

5) Find the indegree of the directed graph & also outdegree of the directed graph. S.T. no. of edges = total no. of indegree

Soln:



Indegree:

$deg^-(A) = 3$
$deg^-(B) = 1$
$deg^-(C) = 2$
$deg^-(D) = 1$

outdegree:

$deg^+(A) = 0$
$deg^+(B) = 2$
$deg^+(C) = 1$
$deg^+(D) = 3$

$\therefore$ Total No. of indegree $= 7$

$\therefore$ No. of edges $= 7$

$\therefore$ No. of edges = total no. of indegree, hence proved