EULER GRAPH & HAMILTON GRAPH:

EULERIAN PATH:

A path of a graph G is called an Eulerian path, if it contains each edge of the graph exactly once.

EULERIAN CIRCUIT OR EULERIAN CYCLE:

A circuit or cycle of a graph G is called an Eulerian circuit or cycle, if it includes each edge of G exactly once and starting & ending points are same.

EULERIAN GRAPH:

Any graph containing an Eulerian circuit or cycle is called an Eulerian graph.

NOTE: A connected graph is Euler graph iff each of its vertices is of even degree.

HAMILTONIAN GRAPH:

HAMILTONIAN PATH:

A path of a graph G is called a Hamiltonian path, if it includes each vertex of G exactly once

HAMILTONIAN CIRCUIT OR CYCLE :

A circuit (cycle) of a graph G is called a Hamiltonian circuit (cycle) if it includes each vertex of G exactly once, except the starting & ending vertices.

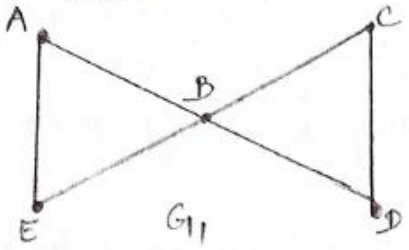
HAMILTONIAN GRAPH :

Any graph containing a Hamiltonian circuit or cycle is called a Hamiltonian graph.

(1) Give an example of a graph which is

i) Eulerian but not Hamiltonian
ii) Hamiltonian but not Eulerian
iii) Both Eulerian and Hamiltonian.
iv) Non Eulerian and non Hamiltonian.

i) Eulerian graph but not a Hamiltonian graph.



G_1 contains the Eulerian cycle is

$A \to B \to C \to D \to B \to E \to A$ (all edges are exactly once.)

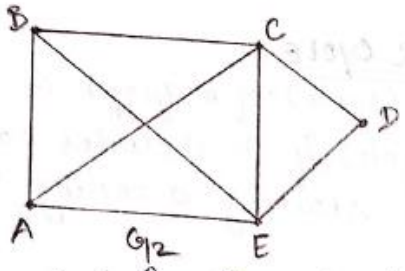
The degree of all vertices of G_1 is even.

$\therefore$ G_1 is Euler graph.

As the vertex B is repeated twice, G_1 is not a Hamiltonian graph.

$\therefore$ G_1 is Eulerian but not Hamiltonian

ii) Hamiltonian but not Eulerian :



G_2

G_2 contains the Hamiltonian cycle.

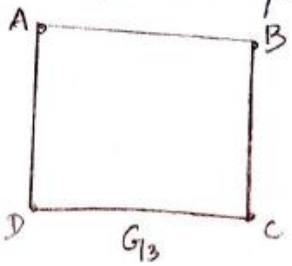
$A \to B \to C \to D \to E \to A$ (all vertices are exactly once)

$\therefore$ G_2 is Hamiltonian graph.

Since the degree of A and B is not an even, G_2 is not Eulerian graph.

$\therefore$ G_2 is Hamiltonian but not Eulerian.

iii) Eulerian and Hamiltonian graph :



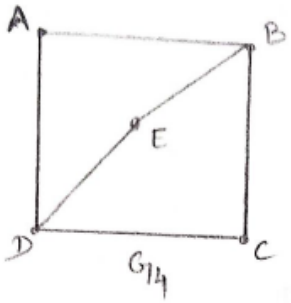
G_3

In G_{13}, contains the cycle, $A \to B \to C \to D \to A$

Since the cycle contains all the edges exactly once & all the vertices exactly once. And also all the vertices are of even degree.

$\therefore$ G_{13} is an example of both Hamiltonian & Eulerian.

Non-Eulerian and Non-Hamiltonian :



G_{14}

In G_{14}, deg B & deg D are not an even numbers. therefore, it is not an Eulerian graph.

In G_{14}, the cycle is $A \to B \to C \to D \to E \to B \to A$, as the vertex B is repeated twice, the graph is not a Hamiltonian graph.