# SNS COLLEGE OF TECHNOLOGY

**Coimbatore-35**
**An Autonomous Institution**

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

## DEPARTMENT OF MECHATRONICS ENGINEERING

## 19MCT201 - DESIGN OF DIGITAL CIRCUITS

II YEAR - III SEM

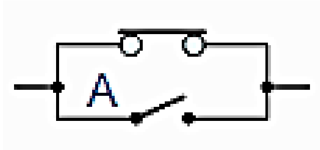
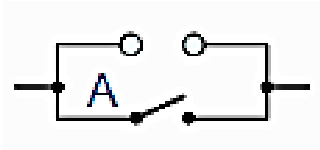
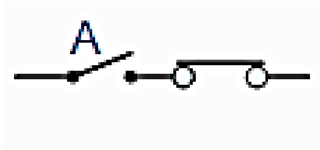
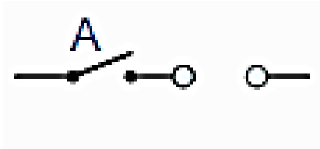
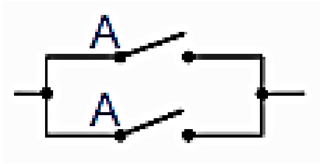
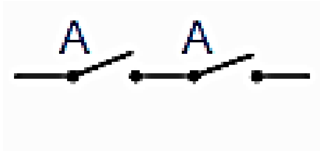
## UNIT 1 – MINIMIZATION TECHNIQUES AND LOGIC GATES

TOPIC 2 –Minimization of Boolean Expression

# Recap

- ✓Symbol represents by A, B, C, etc.,,
- ✓It can either 0 or 1
- ✓Boolean expression Y = A+ BC
- ✓AND (.) Eg. Y=A.B or Y=AB
- ✓OR (+) Eg. Y=A + B

# Rules of Boolean Algebra

| Boolean Expression | Description | Equivalent Switching Circuit | Boolean Algebra Law or Rule |
|---|---|---|---|
| A + 1 = 1 | A in parallel with closed = "CLOSED" |  | Annulment |
| A + 0 = A | A in parallel with open = "A" |  | Identity |
| A . 1 = A | A in series with closed = "A" |  | Identity |
| A . 0 = 0 | A in series with open = "OPEN" |  | Annulment |
| A + A = A | A in parallel with A = "A" |  | Idempotent |
| A . A = A | A in series with A = "A" |  | Idempotent |

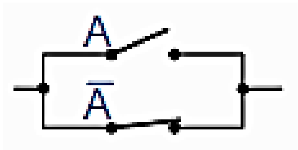
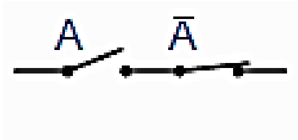
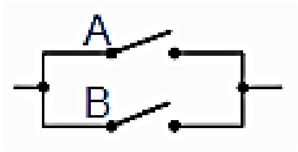
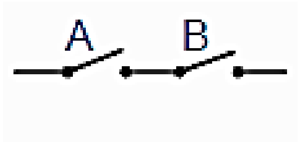
| | | | |
|---|---|---|---|
| NOT $\overline{A}$ = A | NOT NOT A (double negative) = "A" | | Double Negation |
| A + $\overline{A}$ = 1 | A in parallel with NOT A = "CLOSED" |  | Complement |
| A . $\overline{A}$ = 0 | A in series with NOT A = "OPEN" |  | Complement |
| A+B = B+A | A in parallel with B = B in parallel with A |  | Commutative |
| A.B = B.A | A in series with B = B in series with A |  | Commutative |
| $\overline{A+B} = \overline{A}.\overline{B}$ | invert and replace OR with AND | | de Morgan's Theorem |
| $\overline{A.B} = \overline{A}+\overline{B}$ | invert and replace AND with OR | | de Morgan's Theorem |

Image Courtesy: softbankrobotics.com

# Laws of Boolean Algebra

**T1 : Commutative Law**

(a) $A + B = B + A$

(b) $A\,B = B\,A$

**T2 : Associate Law**

(a) $(A + B) + C = A + (B + C)$

(b) $(A\,B)\,C = A\,(B\,C)$

**T3 : Distributive Law**

(a) $A\,(B + C) = A\,B + A\,C$

(b) $A + (B\,C) = (A + B)\,(A + C)$

**T4 : Identity Law**

(a) $A + A = A$

(b) $A\,A = A$

**T5 :**

(a) $AB + AB = A$

(b) $(A+B)(A+\overline{B}) = A$

**T6 : Redundance Law**

(a) $A + A\,B = A$

(b) $A\,(A + B) = A$

**T7 :**

(a) $0 + A = A$

(b) $0\,A = 0$

**T8 :**

(a) $1 + A = 1$

(b) $1\,A = A$

**T9 :**

(a) $\overline{A} + A = 1$

(b) $\overline{A}\,A = 0$

**T10 :**

(a) $A + \overline{A}\,B = A + B$

(b) $A\,(\overline{A} + B) = A\,B$

**T11 : De Morgan's Theorem**

(a) $(\overline{A + B}) = \overline{A}\;\overline{B}$

(b) $(\overline{A\,B}) = \overline{A} + \overline{B}$

# Boolean Algebra

$$A + AB = A$$

$$A + AB = A \cdot 1 + AB$$

$$= A\,(1 + B)$$

$$= A \cdot 1$$

$$= A$$

$$A + \overline{A}B = A + B$$

$$= A + AB + \overline{A}B$$

$$= A + B \cdot (A + \overline{A})$$

$$= A + B \cdot 1$$

$$= A + B$$

# Boolean Algebra

$$\bar{A}B\bar{C}D + \bar{A}BCD + ABD$$

$$= \bar{A}BD(\bar{C} + C) + ABD$$

$$= \bar{A}BD + ABD$$

$$= BD(\bar{A} + A)$$

$$= BD$$

$$\overline{\overline{AB} + \bar{A} + AB}$$

$$= \overline{\bar{A} + \bar{B} + \bar{A} + AB}$$

DeMorgan's Theorem 1 : $[\overline{AB} = \bar{A} + \bar{B}]$

$$= \overline{\bar{A} + \bar{B} + \bar{A} + B}$$

$[A + \bar{A}B = A + B]$

$$= \overline{\bar{A} + 1}$$

$[A + A = A]$

$[A + \bar{A} = 1]$

$$= \bar{1}$$

$$= 0$$

Work out

Problem 1:

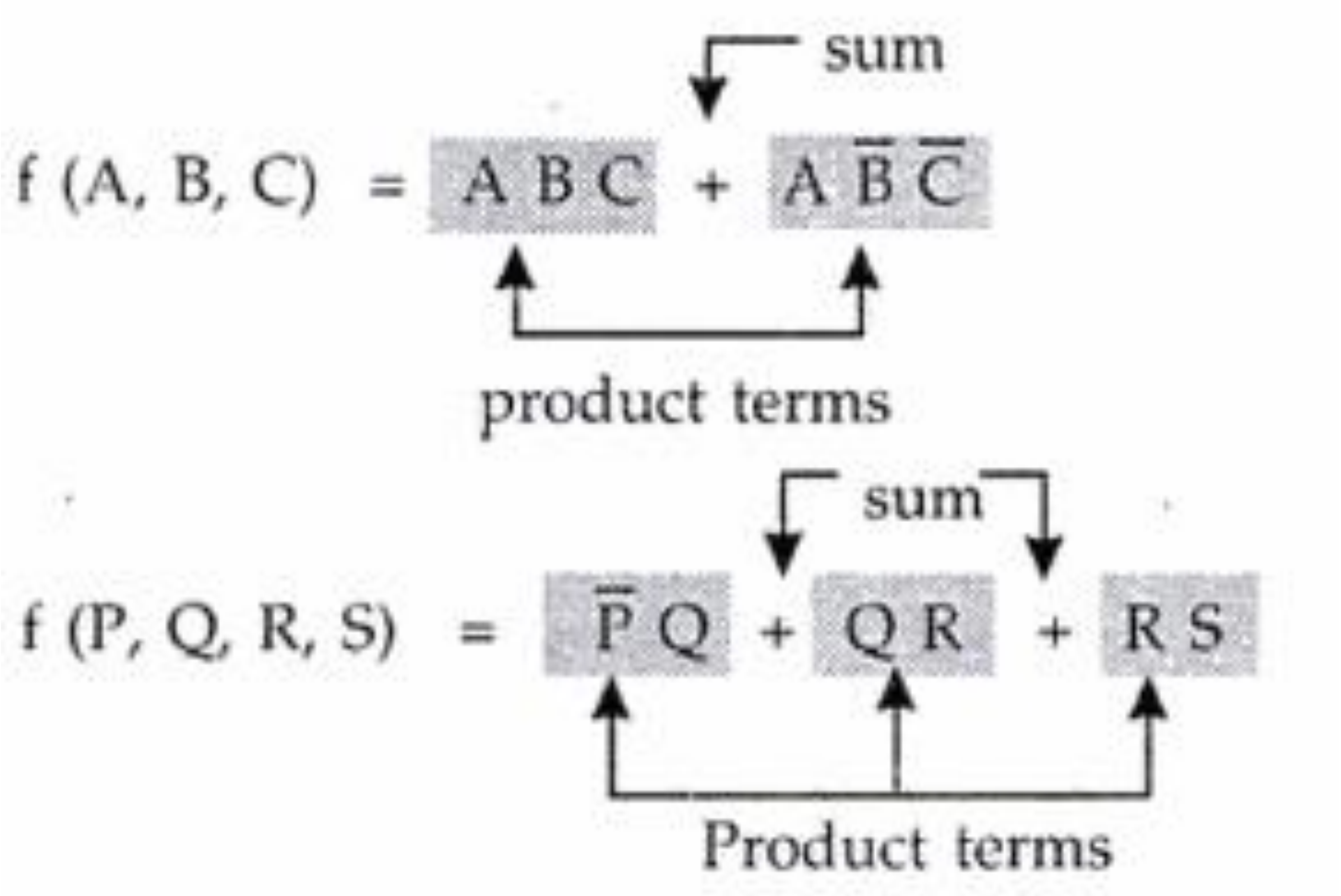
$$AB + \overline{AC} + A\bar{B}C(AB + C)$$

Problem 2:

$$Z = AB + A\bar{B} \cdot (\overline{\bar{A}\bar{C}}).$$

# Sum of Product (SOP) - Minterm

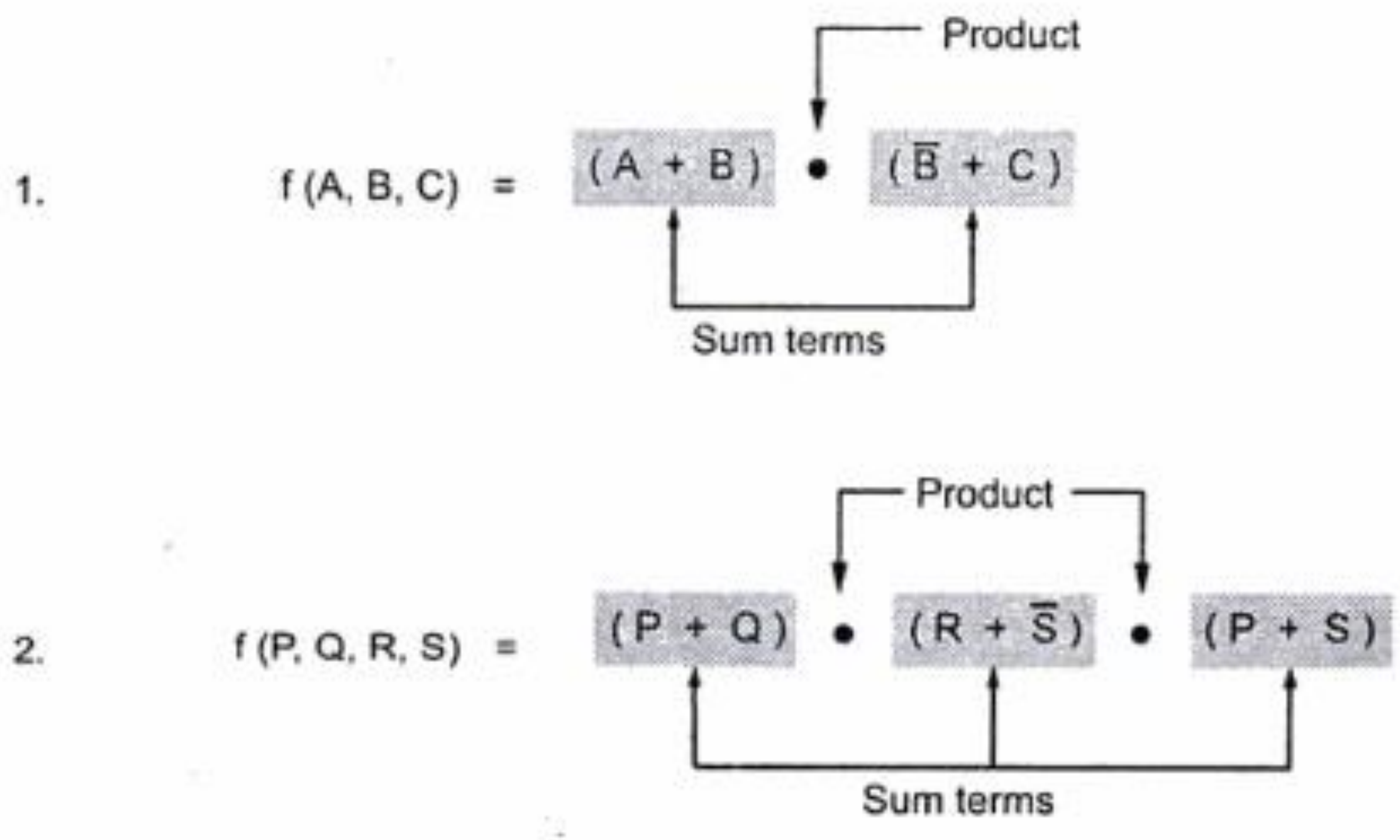
sum of products expressions consist of two or more product terms (AND) that are ORed together.




| Variables | | | Min terms |
|---|---|---|---|
| A | B | C | $m_i$ |
| 0 | 0 | 0 | A' B' C' = m 0 |
| 0 | 0 | 1 | A' B' C = m 1 |
| 0 | 1 | 0 | A' B C' = m 2 |
| 0 | 1 | 1 | A' B C = m 3 |
| 1 | 0 | 0 | A B' C' = m 4 |
| 1 | 0 | 1 | A B' C = m 5 |
| 1 | 1 | 0 | A B C' = m 6 |
| 1 | 1 | 1 | A B C = m 7 |

# Product of Sum (POS) - Maxterm

A product of sums is any groups of sum terms ANDed together. Some examples of this form are :




| Variables | | | Max terms |
|---|---|---|---|
| A | B | C | $M_i$ |
| 0 | 0 | 0 | A + B + C = M 0 |
| 0 | 0 | 1 | A + B + C' = M 1 |
| 0 | 1 | 0 | A + B' + C = M 2 |
| 0 | 1 | 1 | A + B' + C' = M 3 |
| 1 | 0 | 0 | A' + B + C = M 4 |
| 1 | 0 | 1 | A' + B + C' = M 5 |
| 1 | 1 | 0 | A' + B' + C = M 6 |
| 1 | 1 | 1 | A' + B' + C' = M 7 |

# Converting SOP to standard SOP

Steps to convert SOP to standard SOP form

**Step 1 :** Find the missing literal in each product term if any.

**Step 2 :** AND each product term having missing literal/s with term/s form by ORing the literal and its complement.

**Step 3 :** Expand the terms by applying distributive law and reorder the literals in the product terms.

**Step 4 :** Reduce the expression by omitting repeated product terms if any. Because A + A = A.

# Converting POS to standard POS

**Steps to convert POS to standard POS form**

**Step 1 :** Find the missing literals in each sum term if any

**Step 2 :** OR each sum term having missing literal/s with term/s form by ANDing the literal and its complement.

**Step 3 :** Expand the terms by applying distributive law and reorder the literals in the sum terms.

**Step 4 :** Reduce the expression by omitting repeated sum terms if any. Because $A \cdot A = A$.

# Minterms & Maxterms

| Variables | | | Minterms | Maxterms |
|---|---|---|---|---|
| A | B | C | $m_i$ | $M_i$ |
| 0 | 0 | 0 | $\bar{A}\bar{B}\bar{C} = m_0$ | $A + B + C = M_0$ |
| 0 | 0 | 1 | $\bar{A}\bar{B}C = m_1$ | $A + B + \bar{C} = M_1$ |
| 0 | 1 | 0 | $\bar{A}B\bar{C} = m_2$ | $A + \bar{B} + C = M_2$ |
| 0 | 1 | 1 | $\bar{A}BC = m_3$ | $A + \bar{B} + \bar{C} = M_3$ |
| 1 | 0 | 0 | $A\bar{B}\bar{C} = m_4$ | $\bar{A} + B + C = M_4$ |
| 1 | 0 | 1 | $A\bar{B}C = m_5$ | $\bar{A} + B + \bar{C} = M_5$ |
| 1 | 1 | 0 | $AB\bar{C} = m_6$ | $\bar{A} + \bar{B} + C = M_6$ |
| 1 | 1 | 1 | $ABC = m_7$ | $\bar{A} + \bar{B} + \bar{C} = M_7$ |

# ASSESSMENT - 1

## How Laws relates with us….

### Question 1

**What does the expression AD + ABCD + ACD + AB + ACD + AB on minimization result into ?**

- a. A + D
- b. AD + A'
- c. AD
- d. A'+D

### Question 2

**A + BC is equivalent to**

- a. (A + B) (A + C)
- b. A + B
- c. A + C
- d. (A + B') (A + C)'

# References


- https://brilliant.org/wiki/de-morgans-laws/
- https://circuitglobe.com/demorgans-theorem.html
- https://www.electrical4u.com/