



Tautology:

A statement formula which is true always irrespective of the truth values of the individual variables is called a tautology.

Eg:

$P \vee \neg P$ is a tautology.

Contradiction:

A statement formula which is always false is called a contradiction (or) absurdity.

Eg:

$P \wedge \neg P$ is a contradiction.

Contingency:

A statement formula which is neither tautology nor contradiction is called contingency.

Eg:

$P \leftrightarrow Q$ is contingency.

Q. Show that $[P \wedge (P \wedge Q)] \rightarrow Q$ is a tautology.

P	Q	$\neg P$	$P \wedge Q$	$\neg P \wedge (P \wedge Q)$	$[P \wedge (P \wedge Q)] \rightarrow Q$
T	T	F	T	F	T
T	F	F	F	F	T
F	T	T	F	F	T
F	F	T	F	F	T

Since all the entries in the resulting column is true, the given expression is a tautology.

R. Show that $\neg(P \wedge Q) \wedge \neg(P \vee Q)$ is a contradiction.

P	Q	$P \wedge Q$	$P \vee Q$	$\neg(P \vee Q)$	$\neg(P \wedge Q) \wedge \neg(P \vee Q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Since all the entries in the resulting column are F, the given expression is a contradiction.



3. Identify the given expression

$$\neg(P \vee Q) \vee (\neg P \vee \neg Q)$$

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$\neg P \vee \neg Q$	$\neg(P \vee Q) \vee (\neg P \vee \neg Q)$
T	T	F	F	T	F	F	F
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	F	T	T	T

Since the entries in the resulting column are T as well as F, the given expression is a contingency.

Examine whether $[(P \vee Q) \rightarrow R] \leftrightarrow [\neg R \rightarrow \neg(P \vee Q)]$ is a tautology?

Equivalence:

Two statement formulas P and Q are equivalent iff $P \leftrightarrow Q$ is a tautology. It is denoted by $P \Leftrightarrow Q$.

Show that the propositions are logically equivalent for the following.

- (i) $P \rightarrow Q \Leftrightarrow \neg P \vee Q$
- (ii) $P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$

(i) $P \rightarrow Q \Leftrightarrow \neg P \vee Q$

P	Q	$P \rightarrow Q$	$\neg P$	$\neg P \vee Q$	$P \rightarrow Q \Leftrightarrow \neg P \vee Q$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Since $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$ is a tautology.

Hence $(P \rightarrow Q) \Leftrightarrow (\neg P \vee Q)$



(ii) $P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$

P	Q	R	$Q \vee R$	$P \rightarrow (Q \vee R)$	$P \rightarrow Q$	$P \rightarrow R$	$(P \rightarrow Q) \vee (P \rightarrow R)$	$P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
F	T	T	T	T	T	T	T	T
T	F	F	F	F	F	F	F	T
F	F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Here $[P \rightarrow (Q \vee R)] \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$ is a tautology.
 Hence $P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$.