



# **SNS COLLEGE OF TECHNOLOGY**

## **(AN AUTONOMOUS INSTITUTION)**

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## **Department of Biomedical Engineering**

**Course Name: Control Systems**

**III Year : V Semester**

**Unit II -Time Response Analysis**

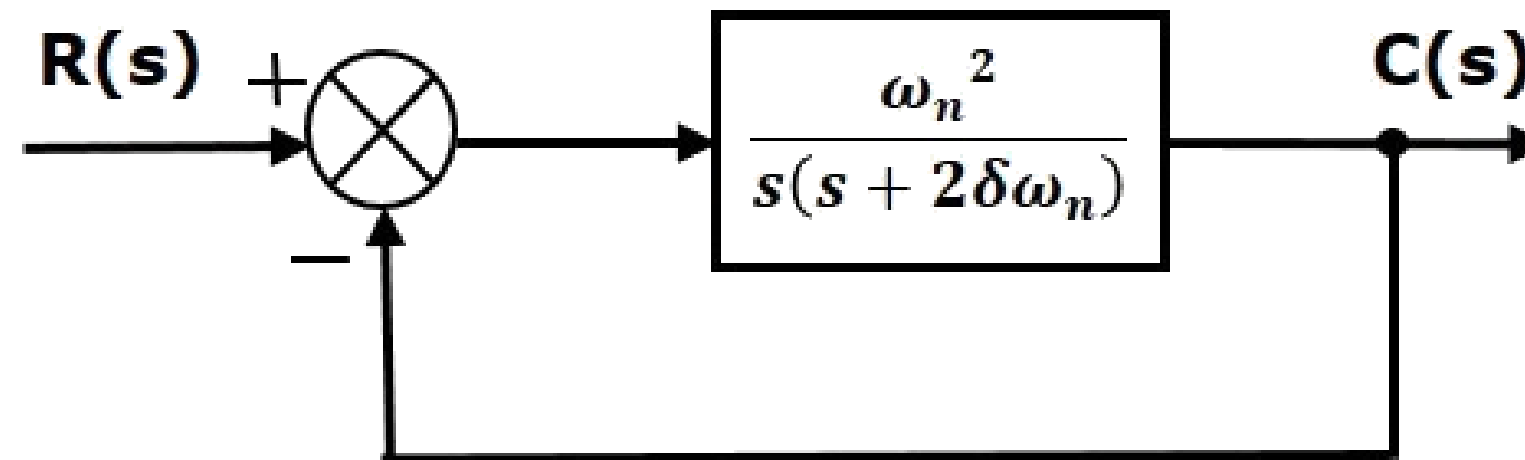
**Topic : Second Order System**



## Introduction

Consider the following block diagram of the closed loop control system.

- Here, an open loop transfer function,  $\frac{\omega_n^2}{s(s+2\zeta\omega_n)}$  is connected with a unity negative feedback. The system is called as second order system



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$



## Second Order Response



- Depending on the value of damping ratio, second order system can be classified into:

1. Undamped system,  $\zeta = 0$
2. Underdamped system,  $0 < \zeta < 1$
3. Critically damped system,  $\zeta = 1$
4. Overdamped system,  $\zeta > 1$

- The characteristic equation is given by,

$$s^2 + 2\delta\omega_n s + \omega_n^2 = 0$$



## Second Order Response

- The roots of characteristic equation is given by,

$$s = -\delta\omega_n \pm \omega_n \sqrt{\delta^2 - 1}$$

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Vision Title 3

- The roots are imaginary when  $\zeta = 0$
- The roots are real and equal when  $\zeta = 1$
- The roots are real and unequal when  $\zeta > 1$
- The roots are complex conjugate when  $0 < \zeta < 1$



# Undamped System

- Step Response of undamped second order system:

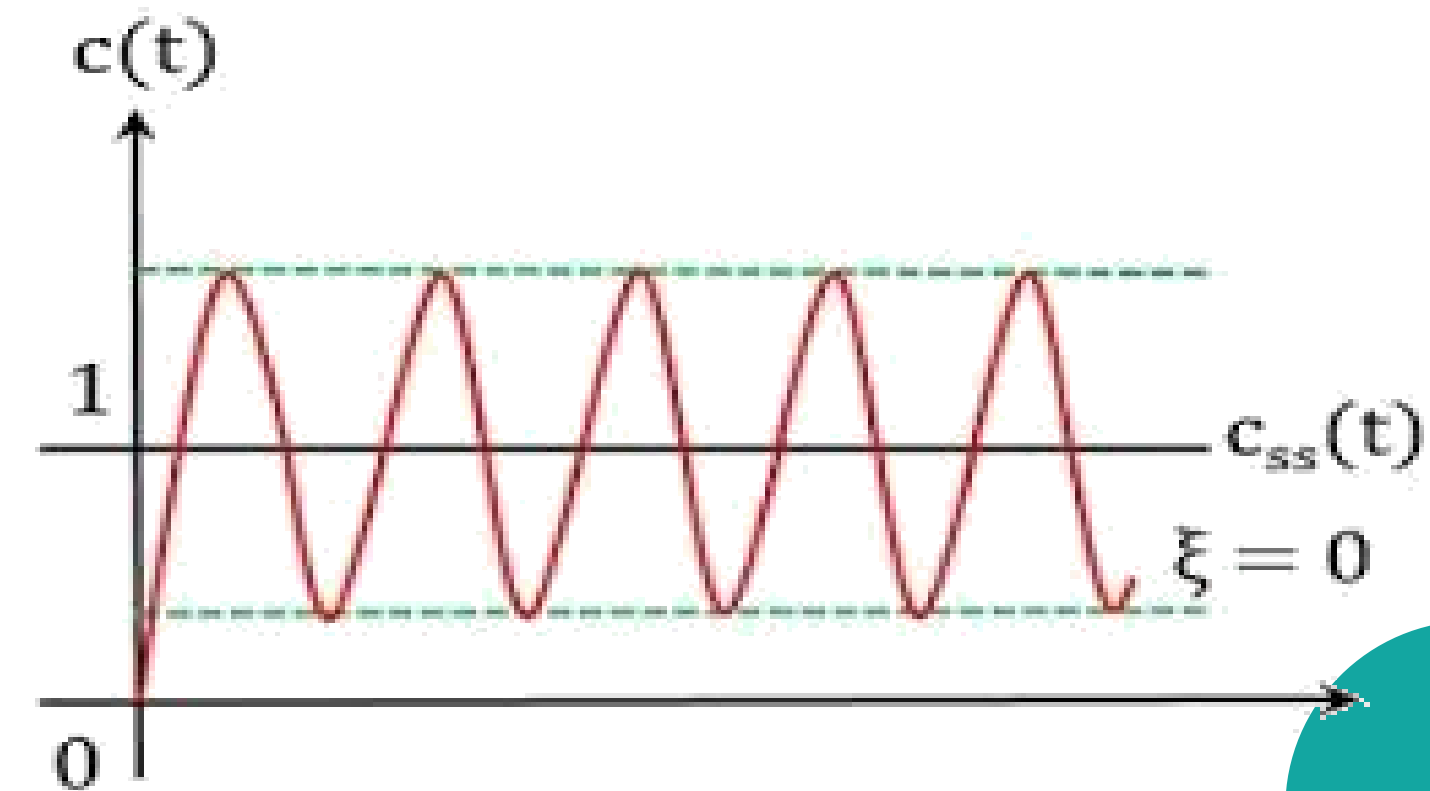
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- When  $\zeta = 0$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

- The response of the system for unit step input is

$$c(t) = (1 - \cos(\omega_n t))$$





## Under damped System

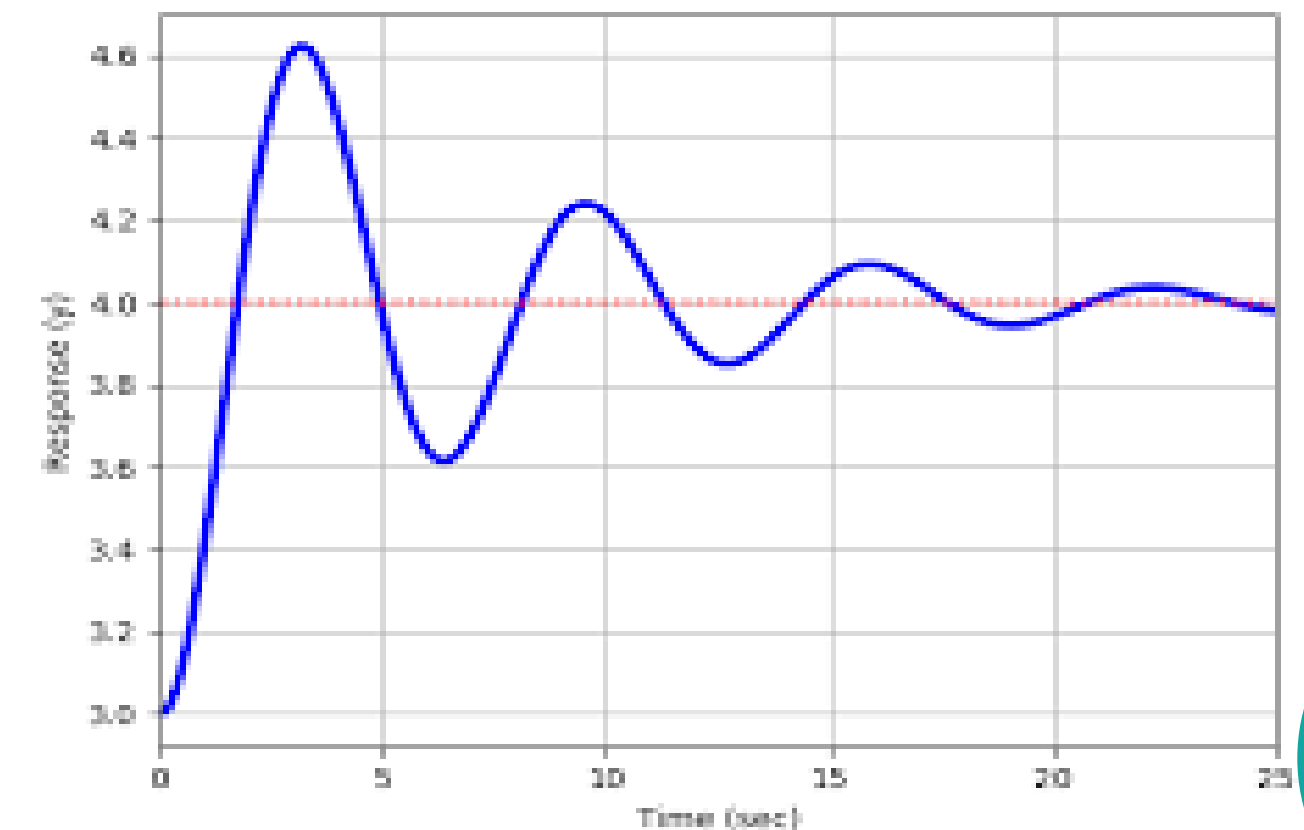
- Step Response of underdamped second order system:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- When  $0 < \zeta < 1$
- The response of the system for unit step input is

$$\frac{C(s)}{R(s)} = \frac{A}{s} + \frac{(Bs + C)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$c(t) = \left( 1 - \left( \frac{e^{-\delta\omega_n t}}{\sqrt{1 - \delta^2}} \right) \sin(\omega_d t + \theta) \right)$$





## Critically damped System

- Step Response of underdamped second order system:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- When  $\zeta = 1$ :
- The response of the system for unit step input is

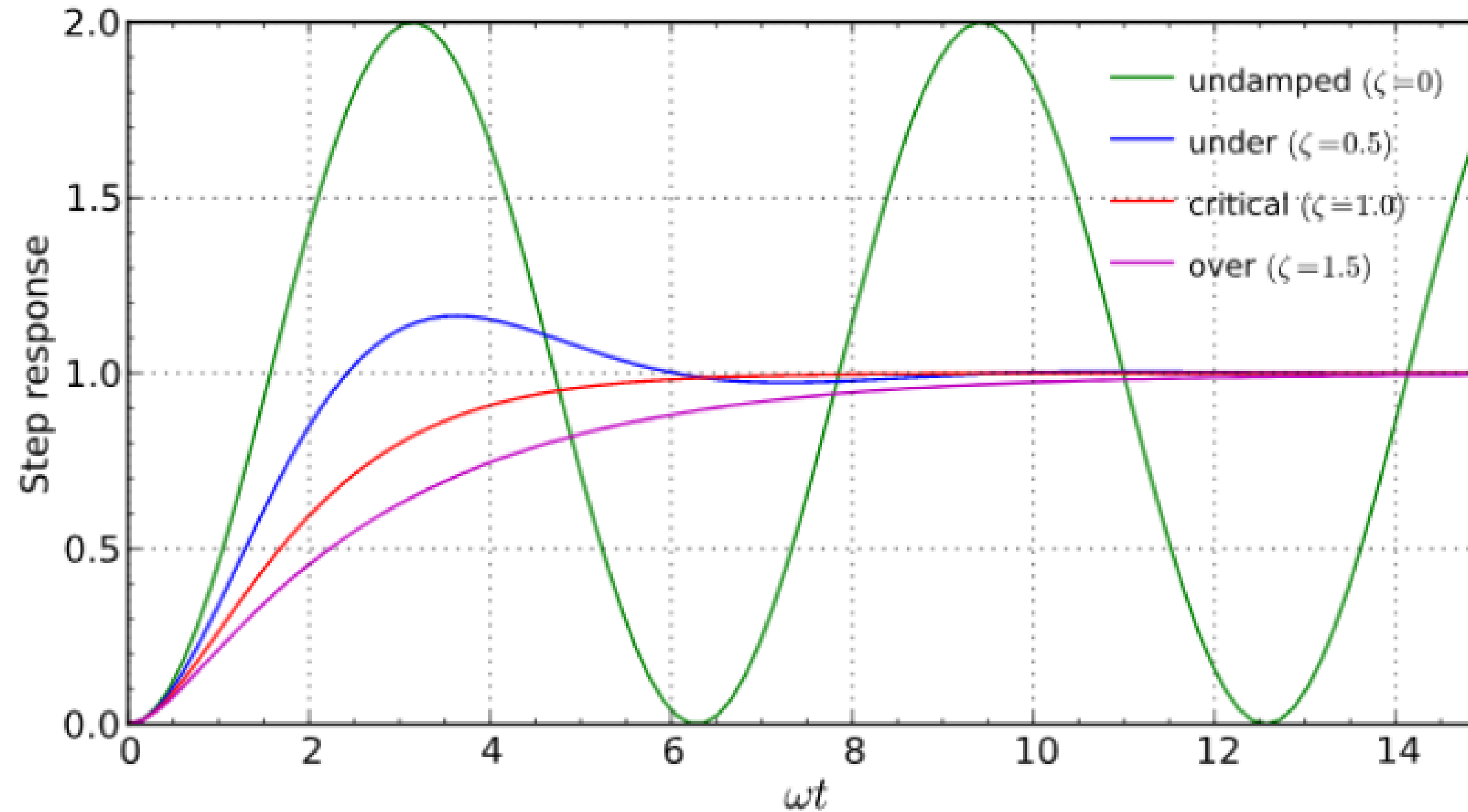
$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s(s + \omega_n)^2} = \frac{A}{s} + \frac{B}{s + \omega_n} + \frac{C}{(s + \omega_n)^2}$$

$$c(t) = (1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t})$$



# Second Order Response







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*Thank You*