Equivalence Rules:

1]. Idempotent laws	$P \wedge P \Leftrightarrow P$, $P \vee P \Leftrightarrow P$
2]. Associative laws	$P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$ $P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$
3]. Commutative Laws	$P \wedge Q \Leftrightarrow Q \wedge P$, $P \vee Q \Leftrightarrow Q \vee P$
4]. Demorgan's Laws	$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$
5]. Distributive laws	$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$ $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$
6]. Complement laws Negation law	$P \wedge \neg P \Leftrightarrow F$, $P \vee \neg P \Leftrightarrow T$
7]. Dominance Laws tion	$P \vee T \Leftrightarrow T$, $P \wedge F \Leftrightarrow F$
8]. Identity Laws	$P \wedge T \Leftrightarrow P$, $P \vee F \Leftrightarrow P$
9]. Absorption Laws	$P \vee (P \wedge Q) \Leftrightarrow P$ $P \wedge (P \vee Q) \Leftrightarrow P$
10]. Double negation law	$\neg(\neg P) \Leftrightarrow P$
11]. Transportation law	$P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$
12]. material implication law	$P \rightarrow Q \Leftrightarrow \neg P \vee Q$
13]. material Equivalence law	$P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$ $P \leftrightarrow Q \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$
14]. Exportation law	$P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$

1). Show that $(P \vee Q) \wedge \neg(\neg P \wedge Q) \Leftrightarrow P$

$(P \vee Q) \wedge \neg(\neg P \wedge Q)$

$\Leftrightarrow (P \vee Q) \wedge [\neg(\neg P) \vee \neg Q]$ Demorgan's law

$\Leftrightarrow (P \vee Q) \wedge [P \vee \neg Q]$ Double negation law

$\Leftrightarrow P \vee (Q \wedge \neg Q)$ Distributive law

$\Leftrightarrow P \vee F$ Complement law

$\Leftrightarrow P$ Identity law

$(P \vee Q) \wedge \neg(\neg P \wedge Q) \Leftrightarrow P$

2). Without using truth tables, Show that

$P \rightarrow (Q \rightarrow R) \Leftrightarrow P \rightarrow (\neg Q \vee R) \Leftrightarrow (P \wedge Q) \rightarrow R$

Now $P \rightarrow (Q \rightarrow R)$ - material Implication law

$\Leftrightarrow P \rightarrow (\neg Q \vee R)$ material Implication law

$\Leftrightarrow \neg P \vee (\neg Q \vee R)$ Associative law

$\Leftrightarrow (\neg P \vee \neg Q) \vee R$ Demorgan's law

$\Leftrightarrow \neg(P \wedge Q) \vee R$ material Implication law.

$\Leftrightarrow (P \wedge Q) \rightarrow R$

3). Show that $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow \neg P \vee Q$

Now $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q))$ material Implication law

$\Leftrightarrow \neg(\neg(P \wedge Q)) \vee (\neg P \vee (\neg P \vee Q))$ Involution law (or) Double negation law

$\Leftrightarrow (P \wedge Q) \vee (\neg P \vee (\neg P \vee Q))$ Associative law

$\Leftrightarrow (P \wedge Q) \vee ((\neg P \vee \neg P) \vee Q)$ Idempotent law

$\Leftrightarrow (P \wedge Q) \vee (\neg P \vee Q)$ Distributive law

$\Leftrightarrow (P \vee (\neg P \vee Q)) \wedge (Q \vee (\neg P \vee Q))$ Associative law

$\Leftrightarrow ((P \vee \neg P) \vee Q) \wedge (Q \vee (Q \vee \neg P))$ Commutative law

$\Leftrightarrow (T \vee Q) \wedge ((Q \vee Q) \vee \neg P)$ complement law, Associative law

$\Leftrightarrow (Q \vee T) \wedge (Q \vee \neg P)$ Commutative law, Idempotent law

$\Leftrightarrow T \wedge (\neg P \vee Q)$ Commutative law, Dominance law

$\Leftrightarrow (\neg P \vee Q) \wedge T$ Commutative law

$\Leftrightarrow \neg P \vee Q$ Identity law.

4]. Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$

Now,

$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$

$\Leftrightarrow [(\neg P \wedge \neg Q) \wedge R] \vee [(Q \vee P) \wedge R]$ Associative law, Distributive law

$\Leftrightarrow [(\neg P \wedge \neg Q) \vee (Q \vee P)] \wedge R$ Distributive law

$\Leftrightarrow [\neg (P \vee Q) \vee (P \vee Q)] \wedge R$ Demorgan's law, commutative law

$\Leftrightarrow T \wedge R$ compliment law

$\Leftrightarrow R$ Identity law

5]. Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.

Now,

$(P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R)) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$

$\Leftrightarrow (P \vee Q) \wedge \neg(\neg P \wedge \neg(Q \wedge R)) \vee (\neg P \wedge (\neg Q \vee \neg R))$
Demorgan's law
Distributive law

$\Leftrightarrow (P \vee Q) \wedge \neg(\neg(P \vee (Q \wedge R))) \vee (\neg P \wedge \neg(Q \wedge R))$
Demorgan's law,

$\Leftrightarrow (P \vee Q) \wedge (P \vee (Q \wedge R)) \vee \neg(P \vee (Q \wedge R))$ Double Negation law, Demorgan's law

$\Leftrightarrow (P \vee Q) \wedge ((P \vee Q) \wedge (P \vee R)) \vee \neg(P \vee (Q \wedge R))$ Distributive law

$\Leftrightarrow ((P \vee Q) \wedge (P \vee Q)) \wedge (P \vee R)) \vee \neg(P \vee (Q \wedge R))$ Distributive law

$\Leftrightarrow ((P \vee Q) \wedge (P \vee R)) \vee \neg(P \vee (Q \wedge R))$ Idempotent law

$\Leftrightarrow (P \vee (Q \wedge R)) \vee \neg(P \vee (Q \wedge R))$ Distributive law, compliment law.

$\Leftrightarrow T$

$\therefore ((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.

6] Show that $(P \wedge Q) \rightarrow (P \vee Q)$ is a tautology.

Now,

$(P \wedge Q) \rightarrow (P \vee Q)$

$\Leftrightarrow \neg (P \wedge Q) \vee (P \vee Q)$ Material Implication

$\Leftrightarrow (\neg P \vee \neg Q) \vee (P \vee Q)$ Demorgan's law

$\Leftrightarrow (\neg P \vee \neg Q) \vee (Q \vee P)$ Commutative law

$\Leftrightarrow \neg P \vee (\neg Q \vee Q) \vee P$ Associative law

$\Leftrightarrow (\neg P \vee T) \vee P$ Negation law

$\Leftrightarrow T \vee P$ Dominance law

$\Leftrightarrow T$ Dominance law

$\therefore (P \wedge Q) \rightarrow (P \vee Q)$ is a tautology.

Hw 1. Show that $Q \rightarrow (P \rightarrow Q)$ is a tautology.

2. $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology.

3]. Show that $(P \rightarrow (Q \rightarrow P)) \Leftrightarrow \neg P \rightarrow (P \rightarrow Q)$

4. $(P \wedge Q) \rightarrow P$ is tauto

5. $Q \rightarrow (P \rightarrow Q)$ is a tauto

6. $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tau.

7. $(P \rightarrow Q) \wedge (R \rightarrow Q) \Leftrightarrow (P \vee R) \rightarrow Q$

8. $\neg (P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow \neg P \vee Q$

$(\neg P \vee Q) \wedge (\neg R \vee Q)$

$(\neg P \wedge \neg R) \vee Q$

$\neg (P \vee R) \vee Q$

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