The Theory of Inference:

Argument:

An argument is a sequence of statements. All statements except the final one are called premises (or assumption or hypothesis). The final statement is called conclusion.

i.e., Let $P_1, P_2, \dots, P_n$ be a sequence of statements that yield conclusion $Q$. It is denoted by $(P_1 \wedge P_2 \wedge \dots, P_n) \rightarrow Q$ is a tautology.

Valid Argument:

An argument is called valid if the conclusion is true when all premises are true.

Invalid Argument:

An argument is called invalid if it is not valid argument.

Rules of Inference:

Rule P: A premise may be introduced at any point in the derivation

Rule T: A formula S may be introduced at any point in a derivation if S is tautologically implied by any one or more of the preceding formula's

Rule CP: If we can derive S from R and a set of premises, then we can derive $R \rightarrow S$ from the set of premises alone.

Types of Proof:

i). Direct Proof
ii). Indirect Proof (or) Proof by contradiction
iii). conditional proof
iv). In consistent proof

Direct Proof:

When a conclusion is derived from a set of premises by using accepted rules of reasoning then such a process of derivation is called direct proof.

Implication Rules:

1. Modus Phones: $P, P \to Q \Rightarrow Q$
2. Modus Tollens: $P \to Q, \neg Q \Rightarrow \neg P$
3. Disjunctive Syllogism: $\neg P, P \vee Q \Rightarrow Q$
4. Hypothetical Syllogism: $P \to Q, Q \to R \Rightarrow P \to Q$ (or) chain Rule
5. Simplification Rule: $P, Q \Rightarrow P \wedge Q$
   $P \wedge Q \Rightarrow P, Q$
6. Addition Rule: $P, Q \Rightarrow P \vee Q$
7. Equivalence Rule: $P \wedge \neg Q \Rightarrow \neg(P \to Q)$

1. Show that R is valid from the premises $P \to Q$, $Q \to R$ and $P$.

| | Step | Premises | Rule |
|---|---|---|---|
| | S1 | $P \to Q$ | P |
| | S2. | $Q \to R$ | P |
| {1,2} | 3. | $P \to R$ | T $[P \to Q, Q \to R \Rightarrow P \to R]$ |
| | 4. | $P$ | P |
| {4,5} | 5. | $R$ | T $[P, P \to Q \Rightarrow Q]$ |

2. Show that $R \vee S$ follows logically from the premises $C \vee D$, $(C \vee D) \to \neg H$, $\neg H \to (A \wedge \neg B)$, and $(A \wedge \neg B) \to R \vee S$

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| Step | Premises | Rule |
|---|---|---|
| 1. | $C \vee D$ | P |
| 2. | $(C \vee D) \rightarrow \neg H$ | P |
| {1,2} 3. | $\neg H$ | T |
| 4. | $\neg H \rightarrow (A \wedge \neg B)$ | P |
| {3,4} 5. | $A \wedge \neg B$ | T |
| 6. | $A \wedge \neg B \rightarrow R \vee S$ | P |
| {5,6} 7. | $R \vee S$ | T |

3]. Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$.

| Step | Premises | Rule |
|---|---|---|
| 1. | $\neg M$ | P |
| 2. | $P \rightarrow M$ | P |
| {1,2} 3. | $\neg P$ | T |
| 4. | $P \vee Q$ | P |
| {3,4} 5. | $Q$ | T |
| 6. | $Q \rightarrow R$ | P |
| {5,6} 7. | $R$ | T |
| {4,7} 8. | $R \wedge (P \vee Q)$ | T |

4]. Show that $A \rightarrow B$, $C \rightarrow B$, $D \rightarrow (A \vee C)$, $D$ is $B$.

| Step | Premises | Rule |
|---|---|---|
| 1. | $D \to (A \vee C)$ | P |
| 2. | $D$ | P |
| {1,2} 3. | $A \vee C$ | T |
| {3} 4. | $\neg A \to C$ | T $[P \to Q \Leftrightarrow \neg P \vee Q]$ |
| 5. | $C \to B$ | P |
| {4,5} 6. | $\neg A \to B$ | T $[P \to Q, Q \to R \Rightarrow P \to R]$ |
| {6} 7. | $\neg B \to A$ | T $[P \to Q \Leftrightarrow \neg Q \to \neg P]$ |
| 8. | $A \to B$ | P |
| {7,8} 9. | $\neg B \to B$ | T $[P \to Q, Q \to R \Rightarrow P \to R]$ |
| {9} 10. | $B \vee B$ | T $P \to Q \Leftrightarrow \neg P \vee Q$ |
| 11. | $B$ | T $P \vee P \Leftrightarrow P$ |

Indirect Proof:

If $H_1, H_2, \ldots, H_m$ are the given premises and $c$ is the conclusion then by indirect proof we get $\neg c \wedge (H_1 \wedge H_2 \wedge \ldots \wedge H_m) \Rightarrow F$, where $F$ is the contradiction.

1]. Prove by indirect method $\neg Q, P \to Q, P \vee R \Rightarrow R$

| Step | Premises | Rule |
|---|---|---|
| 1. | $\neg R$ | negation of conclusion |
| 2. | $P \vee R$ | P |
| {1,2} 3. | $P$ | T, $\neg P, P \vee Q \Rightarrow Q$ |
| 4. | $P \to Q$ | P |
| {3,4} 5. | $Q$ | T, $P, P \to Q \Rightarrow Q$ |
| 6. | $\neg Q$ | P |
| {5,6} 7. | $Q \wedge \neg Q$ | T, $P, Q \Rightarrow P \wedge Q$ |
| {7} 8. | $F$ | T $Q \wedge \neg Q \Leftrightarrow F$ |

1] Show that $\neg P \wedge \neg Q \Rightarrow \neg(P \wedge Q)$ by indirect proof.

| | Step | Premises | Rule | |
|---|---|---|---|---|
| | 1. | $P \wedge Q$ | Negation of conclusion | |
| {1} | 2. | $P$ | T | $P \wedge Q \Rightarrow P$ |
| | 3. | $\neg P \wedge \neg Q$ | P | |
| {3} | 4. | $\neg P$ | T | $\neg P \wedge \neg Q \Rightarrow \neg P$ |
| {2,4} | 5. | $P \wedge \neg P$ | T | $P, Q \Rightarrow P \wedge Q$ |
| {5} | 6. | F | T | |

3] Show that $R \to \neg Q,\ R \vee S,\ S \to \neg Q,\ P \to Q \Rightarrow \neg P$ by indirect method.

| | Step | Premises | Rule | |
|---|---|---|---|---|
| | 1. | $P$ | Negation of conclusion | |
| | 2. | $P \to Q$ | P | |
| {1,2} | 3. | $Q$ | T | $P, P \to Q \Rightarrow Q$ |
| | 4. | $R \to \neg Q$ | P | |
| {3,4} | 5. | $\neg R$ | T | $P \to Q, \neg Q \Rightarrow \neg P$ |
| | 6. | $R \vee S$ | P | |
| {5,6} | 7. | $S$ | T | $\neg R, R \vee S \Rightarrow S$ |
| | 8. | $S \to \neg Q$ | P | |
| {7,8} | 9. | $\neg Q$ | T | $P, P \to Q \Rightarrow Q$ |
| {3,9} | 10. | $Q \wedge \neg Q$ | T | $P, Q \Rightarrow P \wedge Q$ |
| | 11. | F | T | $P \wedge \neg P \Leftrightarrow F$ |

HW

1] $\neg Q,\ P \to Q \Rightarrow \neg P$

2] $\{(P \to Q) \wedge (R \to S),\ (Q \to T) \wedge (S \to U),\ \neg(T \wedge U),\ P \to R\} \Rightarrow \neg P$

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Conditional Proof:

1. Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\neg R \vee P$ and $Q$.

| Set | Premises | Rule | |
|---|---|---|---|
| 1. | $R$ | P (Assumed premise) ~~Rule CP~~ | |
| 2. | $\neg R \vee P$ | P | |
| {1,2} 3. | $P$ | T | $\neg P, P \vee Q \Rightarrow Q$ |
| 4. | $P \to (Q \to S)$ | P | |
| {3,4} 5. | $Q \to S$ | T | $P, P \to Q \Rightarrow Q$ |
| 6. | $Q$ | P | |
| {5,6} 7. | $S$ | T | $P, P \to Q \Rightarrow Q$ |
| {1,7} 8. | $R \to S$ | CP | |

2. Derive the following using CP:

i) $P \to Q \Rightarrow P \to (P \wedge Q)$

ii) $P, P \to (Q \to (R \wedge S)) \Rightarrow Q \to S$

~~iii) $P \to Q \Rightarrow P \to (P \wedge Q)$~~

iiv) $\neg P \vee Q, \neg Q \vee R, R \to S \Rightarrow P \to S$

i) $P \to Q \Rightarrow P \to (P \wedge Q)$

| Step | Premises | Rule | |
|---|---|---|---|
| 1. | $P$ | P (Assumed premise) | |
| 2. | $P \to Q$ | P | |
| {1,2} 3. | $Q$ | T | $P, P \to Q \Rightarrow Q$ |
| {1,3} 4. | $P \wedge Q$ | T | $P, Q \Rightarrow P \wedge Q$ |
| 5. | $P \to (P \wedge Q)$ | CP | |

ii). $P,\ P \to (Q \to (R \wedge S)) \Rightarrow Q \to S$

| Step | Premises | Rule | |
|---|---|---|---|
| 1. | $Q$ | P(Assumed premise) | |
| 2. | $P$ | P | |
| 3. | $P \to (Q \to (R \wedge S))$ | P | |
| {2,3} 4. | $Q \to (R \wedge S)$ | T | $P, P\to Q \Rightarrow Q$ |
| {1,4} 5. | $R \wedge S$ | T | $P, P\to Q \Rightarrow Q$ |
| {5} 6. | $S$ | T | $P \wedge Q \Rightarrow Q$ |
| {1,6} 7. | $Q \to S$ | CP | |

iii). $P \to Q \Rightarrow P \to (P \wedge Q)\ \neg P \vee Q,\ \neg Q \vee R,\ R \to S \Rightarrow P \to S$

| Step | Premises | Rule | |
|---|---|---|---|
| 1. | $P$ | P(Assumed premise) | |
| 2. | $\neg P \vee Q$ | P | |
| {2} 3. | $P \to Q$ | T | $\neg P \vee Q \Leftrightarrow P \to Q$ |
| {1,3} 4. | $Q$ | T | $P, P\to Q \Rightarrow Q$ |
| 5. | $\neg Q \vee R$ | P | |
| {5} 6. | $Q \to R$ | T | |
| {4,6} 7. | $R$ | T | $P, P\to Q \Rightarrow Q$ |
| 8. | $R \to S$ | P | |
| {7,8} 9. | $S$ | T | |
| {1,9}. 10. | $P \to S$ | CP | |

Inconsistent proof:

A set of premises $H_1, H_2, \ldots, H_m$ is said to be inconsistent if $H_1 \wedge H_2 \wedge \ldots \wedge H_m \Rightarrow F$ which stands for a contradiction.

i.e., $H_1 \wedge H_2 \wedge \ldots \wedge H_m = A \wedge \neg A$, where $A$ is any variable.

1. Prove that the premises $P \to Q$, $Q \to R$, $R \to S$, $S \to \neg R$ and $P \wedge S$ are inconsistent.

| | Step | Premises | Rule |
|---|---|---|---|
| | 1. | $P \to Q$ | P |
| | 2. | $Q \to R$ | P |
| {1, 2} | 3. | $P \to R$ | T $P \to Q, Q \to R \Rightarrow P \to$ |
| | 4. | $S \to \neg R$ | P |
| {4} | 5. | $R \to \neg S$ | T $P \to Q \Leftrightarrow \neg Q \to \neg P$ |
| {3, 5} | 6. | $P \to \neg S$ | T $P \to Q, Q \to R \Rightarrow P \to$ |
| | 7. | $R \to S$ | P |
| {6} | 8. | $\neg P \vee \neg S$ | T $P \to Q \Leftrightarrow \neg P \vee Q$ |
| {8} | 9. | $\neg(P \wedge S)$ | T $\neg P \vee \neg Q \Leftrightarrow \neg(P \wedge$ |
| | 10. | $P \wedge S$ | P |
| {9, 10} | 11. | $(P \wedge S) \wedge \neg(P \wedge S)$ | T $P, Q \Rightarrow P \wedge Q$ |
| | 12. | F | T |

2. Show that the premises $p \to q$, $p \to r$, $q \to \neg r$, $p$ are inconsistent.

| | Step | Premises | Rule |
|---|---|---|---|
| | 1. | $p$ | P |
| | 2. | $p \to q$ | P |
| {1, 2} | 3. | $q$ | T $P, P \to Q \Rightarrow Q$ |
| | 4. | $q \to \neg r$ | P |
| {3, 4} | 5. | $\neg r$ | T $P, P \to Q \Rightarrow Q$ |
| | 6. | $p \to r$ | P |
| {6} | 7. | $\neg r \to \neg p$ | T $P \to Q \Leftrightarrow \neg Q \to \neg P$ |

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|---|---|---|---|---|
| {5,7} | 8. | $\neg P$ | T | $P, P \to Q \Rightarrow Q$ |
| {1,8} | 9. | $P \wedge \neg P$ | T | $P, Q \Rightarrow P \wedge Q$ |
| | 10. | F | T | $P \wedge \neg P \Leftrightarrow F$ |

3). Prove that the premises $a \to (b \to c)$, $d \to (b \wedge \neg c)$, and $a \wedge d$ are inconsistent.

| | | | | |
|---|---|---|---|---|
| | 1. | $a \wedge d$ | P | |
| {1} | 2. | $a$ | T | $P \wedge Q \Rightarrow P, Q$ |
| | 3. | $a \to (b \to c)$ | P | |
| {2,3} | 4. | $b \to c$ | T | $P, P \to Q \Rightarrow P \to Q$ |
| {1} | 5. | $d$ | T | |
| | 6. | $d \to (b \wedge \neg c)$ | P | |
| {5,6} | 7. | $b \wedge \neg c$ | T | $P, P \to Q \Rightarrow Q$ |
| {7} | 8. | $\neg (b \to c)$ | T | $\neg(P \to Q) \Leftrightarrow P \wedge \neg Q$ |
| {4,8} | 9. | $(b \to c) \wedge \neg (b \to c)$ | T | $P, Q \Rightarrow P \wedge Q$ |
| | 10. | F | T | |

4). Show that the following premises are inconsistent.

1. If Jack misses many classes through illness, then he fails high school.
2. If Jack fails high school, then he is uneducated.
3. If Jack reads a lot of books, then he is not uneducated.
4. Jack misses many classes through illness and reads a lot of books.

P : Jack misses many classes through illness
Q : Jack fails high school.
R : Jack reads a lot of books
S : Jack is uneducated.

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The premises are

$P \to Q,\ Q \to R,\ S \to \neg R,\ P \wedge S.$

| Step | | Premises | Rule | |
|---|---|---|---|---|
| | 1. | $P \to Q$ | P | |
| | 2. | $Q \to R$ | P | |
| {1,2} | 3. | $P \to R$ | T | $P \to Q, Q \to R \Rightarrow P \to R$ |
| | 4. | $S \to \neg R$ | P | |
| {4} | 5. | $R \to \neg S$ | T | $P \to Q \Leftrightarrow \neg Q \to \neg P$ |
| {3,5} | 6. | $P \to \neg S$ | T | $P \to Q, Q \to R \Rightarrow P \to R$ |
| {6} | 7. | $\neg P \vee \neg S$ | T | $P \to Q \Leftrightarrow \neg P \vee Q$ |
| {7} | 8. | $\neg(P \wedge S)$ | T | $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ |
| | 9. | $P \wedge S$ | P | |
| {8,9} | 10. | $(P \wedge S) \wedge \neg(P \wedge S)$ | T | $P, Q \Rightarrow P \wedge Q$ |
| | 11. | F | T | $P \wedge \neg P \Leftrightarrow F$. |

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5. i). If there is a ball game, then travelling was difficult.

ii). If they arrived on time then travelling was not difficult.

iii). They arrived on time

iv). Therefore there was no ball game.

Show that the above statements are valid statement.

Let P : There was a ball game

Q : Travelling was difficult

R : They arrived on time.

The premises are $P \to Q,\ R \to \neg Q,\ R,$

The conclusion is $\neg P$.

| | Step | Premises | Rule |
|---|---|---|---|
| | 1. | $R$ | P |
| | 2. | $R \to \neg Q$ | P |
| {1,2} | 3. | $\neg Q$ | T $P, P \to Q \Rightarrow Q$ |
| | 4. | $P \to Q$ | P |
| {3,4} | 5. | $\neg P$ | T , $P \to Q, \neg Q \Rightarrow \neg P$ |