Minterms: [product consisting of all variables]

(i) $P \wedge Q$, $\neg P \wedge Q$, $P \wedge \neg Q$, $\neg P \wedge \neg Q$ are minterms in P, Q.

(ii) $P \wedge Q \wedge R$, $P \wedge \neg Q \wedge R$, $\neg P \wedge Q \wedge R$, $P \wedge Q \wedge \neg R$, $\neg P \wedge \neg Q \wedge R$, $\neg P \wedge Q \wedge \neg R$, $P \wedge \neg Q \wedge \neg R$, $\neg P \wedge \neg Q \wedge \neg R$ are minterms in P, Q and R.

Maxterms: [Sum consisting of all variables]

(i) $P \vee Q$, $\neg P \vee Q$, $P \vee \neg Q$, $\neg P \vee \neg Q$ are maxterms in P, Q.

(ii) $P \vee Q \vee R$, $P \vee \neg Q \vee R$, $\neg P \vee Q \vee R$, $P \vee Q \vee \neg R$, $P \vee \neg Q \vee \neg R$, $\neg P \vee Q \vee \neg R$, $\neg P \vee \neg Q \vee R$, $\neg P \vee \neg Q \vee \neg R$ are maxterms in P, Q, R.

PDNF principal disjunctive normal form

The PDNF of a given formula is an equivalent formula which consisting of disjunction of minterms only. i.e., PDNF = (minterms) $\vee$ (minterms) $\vee \dots \vee$ (minterms)

Principal conjunctive normal form (PCNF)

The PCNF of a given formula is an equivalent formula which consisting of conjunction of maxterms only. i.e., PCNF = (maxterms) $\wedge$ (maxterms) $\wedge \dots \wedge$ (maxterms)

Obtain the PDNF of

i) $\neg P \vee Q$

ii) $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

i) $\neg P \vee Q \Leftrightarrow (\neg P \wedge T) \vee (Q \wedge T)$

$\Leftrightarrow (\neg P \wedge (Q \vee \neg Q)) \vee (Q \wedge (P \vee \neg P))$

$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (Q \wedge P) \vee (Q \wedge \neg P)$ Distributive laws

$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q) \vee (Q \wedge \neg P)$ Commutative law

ii) $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

$\Leftrightarrow (P \wedge Q \wedge T) \vee (\neg P \wedge R \wedge T) \vee (Q \wedge R \wedge T)$

$\Leftrightarrow (P \wedge Q \wedge (R \vee \neg R)) \vee (\neg P \wedge R \wedge (Q \vee \neg Q)) \vee (Q \wedge R \wedge (P \vee \neg P))$

$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge R \wedge Q)$
$\vee (\neg P \wedge R \wedge \neg Q) \vee (Q \wedge R \wedge P) \vee (Q \wedge R \wedge \neg P)$

$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R)$
$\vee (\neg P \wedge \neg Q \wedge R) \vee (\neg$

iii). obtain the PDNF of $P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$

$P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$
$\Leftrightarrow \neg P \vee ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$
$\Leftrightarrow \neg P \vee ((\neg P \vee Q) \wedge \neg(\neg Q \vee \neg P))$
$\Leftrightarrow \neg P \vee ((\neg P \vee Q) \wedge (Q \wedge P))$
$\Leftrightarrow \neg P \vee [(\neg P \wedge (Q \wedge P)) \vee (Q \wedge (Q \wedge P))]$
$\Leftrightarrow \neg P \vee [(\neg P \wedge (P \wedge Q)) \vee [(Q \wedge Q) \wedge P)]]$
$\Leftrightarrow \neg P \vee [((\neg P \wedge P) \wedge Q) \vee ((Q \wedge Q) \wedge P)]$
$\Leftrightarrow \neg P \vee [(F \wedge Q) \vee (Q \wedge P)]$
$\Leftrightarrow \neg P \vee [F \vee (P \wedge Q)]$
$\Leftrightarrow \neg P \vee (P \wedge Q)$
$\Leftrightarrow (\neg P \wedge T) \vee (P \wedge Q)$
$\Leftrightarrow (\neg P \wedge (Q \vee \neg Q)) \vee (P \wedge Q)$
$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q)$

Obtain the PCNF & PDNF of i). $(\neg P \rightarrow R) \wedge (Q \leftrightarrows P)$ and
PCNP of ii). $P \rightarrow (\neg P \wedge (Q \rightarrow P))$

i). $(\neg P \rightarrow R) \wedge (Q \leftrightarrows P)$
PCNF $\Leftrightarrow (P \vee R) \wedge [(Q \rightarrow P) \wedge (P \rightarrow Q)]$
$\Leftrightarrow (P \vee R) \wedge [(\neg Q \vee P) \wedge (\neg P \vee Q)] \Leftrightarrow (P \vee R \vee F) \wedge (\neg Q \vee P \vee F) \wedge (\neg P \vee Q \vee F)$
$\Leftrightarrow (P \vee R \vee (Q \wedge \neg Q)) \wedge (\neg Q \vee P \vee (R \wedge \neg R)) \wedge (\neg P \vee Q \vee (R \wedge \neg R))$
$\Leftrightarrow (P \vee R \vee Q) \wedge (P \vee R \vee \neg Q) \wedge (\neg Q \vee P \vee R) \wedge (\neg Q \vee P \vee \neg R)$
$\wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$

$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge$
$(\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$

PDNF :

TS: Conjunction of the remaining maxterms.

TS: $(P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)$

$\neg(TS) : \neg(P \vee Q \vee \neg R) \vee \neg(\neg P \vee \neg Q \vee R) \vee \neg(\neg P \vee \neg Q \vee \neg R)$

$\Leftrightarrow (\neg P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R)$ which is the required PDNF.

ii). $P \rightarrow (\neg P \wedge (Q \rightarrow P))$

PCNF :

$\Leftrightarrow \neg P \vee [\neg P \wedge (Q \rightarrow P)]$ material implication Rule

$\Leftrightarrow \neg P \vee [\neg P \wedge (\neg Q \vee P)]$ material implication Rule

$\Leftrightarrow (\neg P \vee \neg P) \wedge (\neg P \vee (\neg Q \vee P))$ Distributive law

$\Leftrightarrow \neg P \wedge (\neg P \vee (P \vee \neg Q))$ Idempotent law, Commutative law

$\Leftrightarrow \neg P \wedge [(\neg P \vee P) \vee \neg Q]$ Associative law

$\Leftrightarrow \neg P \wedge (T \vee \neg Q)$

$\Leftrightarrow \neg P \wedge T$

$\Leftrightarrow \neg P$

$\Leftrightarrow \neg P \vee F$

$\Leftrightarrow [\neg P \vee (Q \wedge \neg Q)]$

$\Leftrightarrow (\neg P \vee Q) \wedge (\neg P \vee \neg Q)$ which is the required PCNF.

iii). $(Q \rightarrow P) \wedge (\neg P \wedge Q)$

$\Leftrightarrow (\neg Q \vee P) \wedge (\neg P \wedge Q)$

$\Leftrightarrow (\neg Q \vee P) \wedge [(\neg P \wedge Q) \vee F]$

$\Leftrightarrow (\neg Q \vee P) \wedge [(\neg P \wedge Q) \vee (P \wedge \neg P)]$

$\Leftrightarrow (\neg Q \vee P) \wedge [(\neg P \wedge Q) \vee P] \wedge [(\neg P \wedge Q) \vee \neg P]$

$\Leftrightarrow (\neg Q \vee P) \wedge (\neg P \vee P) \wedge (Q \vee P) \wedge (\neg P \vee \neg P) \wedge (Q \vee \neg P)$

$\Leftrightarrow (\neg Q \vee P) \wedge T \wedge (Q \vee P) \wedge (\neg P \vee (Q \vee \neg P))$

$\wedge F$

$\Leftrightarrow (P \vee \neg Q) \wedge (P \vee Q) \wedge (\neg P \vee (\neg P \vee Q))$

$\Leftrightarrow (P \vee \neg Q) \wedge (P \vee Q) \wedge ((\neg P \vee \neg P) \vee Q)$

$\Leftrightarrow (P \vee \neg Q) \wedge (P \vee Q) \wedge (\neg P \vee Q)$

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Truth Table (PDNF & PCNF)

~~PDNF:~~

JJ. Obtain PDNF & PCNF for the following. using truth table

i). $P \wedge (Q \rightarrow R)$

ii). $P \rightarrow [(P \rightarrow Q) \wedge (\neg Q \vee \neg P)]$

i). $P \wedge (Q \rightarrow R)$

| P | Q | R | $Q \rightarrow R$ | $P \wedge (Q \rightarrow R)$ | minterms | maxterms |
|---|---|---|---|---|---|---|
| T | T | T | T | T | $P \wedge Q \wedge R$ | |
| T | T | F | F | F * | | $\neg P \vee \neg Q \vee R$ |
| T | F | T | T | T | $P \wedge \neg Q \wedge R$ | |
| T | F | F | T | T | $P \wedge \neg Q \wedge \neg R$ | |
| F | T | T | T | F * | | $P \vee \neg Q \vee \neg R$ |
| F | T | F | F | F * | | $P \vee \neg Q \vee R$ |
| F | F | T | T | F * | | $P \vee Q \vee \neg R$ |
| F | F | F | T | F * | | $P \vee Q \vee R$ |

PDNF : Disjunction of minterms

$= (P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$

PCNF : conjunction of maxterms

$= (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee \neg R) \wedge (P \vee Q \vee R)$

S ⇔

ii). $P \to [(P \to Q) \wedge (\neg Q \vee \neg P)]$

| P | Q | $P \to Q$ | $\neg Q$ | $\neg P$ | $\neg Q \vee \neg P$ | $[(P \to Q) \wedge (\neg Q \vee \neg P)]$ | S | min terms | max terms |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F | F | | $\neg P \vee \neg Q$ |
| T | F | F | T | F | F | F | F | | $\neg P \vee Q$ |
| F | T | T | F | T | F | F | T | $\neg P \wedge Q$ | |
| F | F | T | T | T | T | T | T | $\neg P \wedge \neg Q$ | |

PDNF : Disjunction of minterms

$= (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$

PCNF : Conjunction of maxterms.

$= (\neg P \vee \neg Q) \wedge (\neg P \vee Q)$

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Hw ij. $(\neg P \to R) \wedge (Q \leftrightarrow P)$ using truthtable.