Discrete Mathematics

Unit - I

Logics and Proofs

* Propositional logic
* Proportional Equivalences
* predicates and Quantifiers
* Nested Quantifiers
* Rules of inference
* Introduction to proofs
* Proof methods and strategy

Logics and proofs

Proposition:

A proposition (or) Statement is a declarative sentence which is either true or false but not both.

Egs:

1. Newdelhi is the capital of India. [True]
2. Chennai is in England. [False]

Non proposition:

Questions, Exclamations and commands are non proposition.

Egs:

1. What is the height of himalaya [Interrogative sentence]
2. Obey my orders. [Command]
3. $x+5=3$ [Neither true nor false]

Types of proposition:

Simple proposition:

A declarative sentence which cannot be further split up into simple sentences are called primary (or) atomic (or) Simple Statements.

Eg: Nandhini is a lawyer.

Compound proposition:

A Statement which contain one or more primary statements and some connectives are called compound (or) molecular (or) Composite Statements.

Eg: Lotus is a flower and it is the national flower of India.

Connectives:

Connective is an operation which is used to connect two or more than two statements. are known

There are five basic connectives.

S.No.	Logical connectives	Name	Symbols	Type of operator
1.	NOT	Negation (or) Denial	$\neg$ (or) $\sim$	Unary
	Eg: P: Chennai is a city $\neg P$: Chennai is not a city.			
2.	AND	Conjunction	$\wedge$	Binary
	Eg: P: It is raining Q: I am getting cold. $P \wedge Q$: It is raining and I am getting cold.			
3.	OR	Disjunction	$\vee$	Binary
	Eg: P: $3+5=8$ Q: $5<3$ $P \vee Q$: $3+5=8$ or $5<3$			
4.	If ... then	Conditional	$\rightarrow$	Binary
	Eg: P: There is a flood Q: The crop will be destroyed. $P \rightarrow Q$: If there is a flood then the crop will be destroyed.			
5.	If and only if	Biconditional	$\leftrightarrow$ (or) $\rightleftarrows$	Binary
	Eg: P: Students will come to college Q: Friday is a working day $P \leftrightarrow Q$: Students will come to college if and only if Friday is a working day			

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Truth table :

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$P \leftrightarrow Q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Instead of T & F as 0 and 1, we can use

1. Using the Statements

P : x is rich

Q : x is happy

write the following Statements in symbolic form:

(a) x is poor

(b) x is poor but happy

(c) x is rich or unhappy

(d) x is neither rich nor happy

(e) x is poor or he is both rich and unhappy.

Soln.:

(a) $\neg A$

(b) $\neg A \wedge B$

(c) $\neg A \vee \neg B$

(d) $\neg A \wedge \neg B$

(e) $\neg A \vee (A \wedge \neg B)$

2. write the statements for the following symbolic form

P: It is hot day

Q: Temperature is 45°C.

(i) $\neg P$ (ii) $\neg(P \vee Q)$ (iii) $P \wedge Q$ (iv) $\neg(\neg P)$

(v) $\neg P \wedge \neg Q$ (vi) $\neg P \vee \neg Q$ (vii) $\neg(\neg P \vee \neg Q)$

Soln.:

(i) $\neg P \Rightarrow$ It is not hot day

(ii) $\neg(P \vee Q) \Rightarrow$ It is false that it is hot day or the temperature is 45°C.

(iii) $P \wedge Q \Rightarrow$ It is hot day and the temperature is 45°C

(iv) $\neg(\neg P) \Rightarrow$ It is hot day.

(v) $\neg P \wedge \neg Q \Rightarrow$ It is not hot day and the temperature is not 45°C. (or)
Neither it is hot day nor the temperature is 45°C.

(vi) $\neg P \vee \neg Q \Rightarrow$ It is not hot day or the temperature is not 45°C. (or)
Either it is not hot day or the temp. is not 45°C

(vii) $\neg(\neg P \vee \neg Q) \Rightarrow$ It is false that it is not hot day or the temperature is not 45°C (or)
It is hot day ~~or~~ the temp. is 45°C.

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3]. let P: Triangle ABC is an isoceles
Q: Triangle ABC is an equilateral
R: Triangle ABC is an equiangular.

Translate each of the following notation into a statement.

(i) $Q \to P$ (ii) $\neg P \to \neg Q$ (iii) $Q \leftrightarrow R$ (iv) $P \to \neg Q$
(v) $R \to P$ (vi) $(P \vee Q) \to R$ (vii) $(\neg P \wedge Q) \to \neg R$

Soln.:

(i) $Q \to P \Rightarrow$ If ΔABC is an equilateral then ΔABC is an isoceles.

(ii) $\neg P \to \neg Q$
$\Rightarrow$ If ΔABC is not an isoceles then ΔABC is not an equilateral.

(iii) $Q \leftrightarrow R \Rightarrow \Delta ABC$ is an equilateral iff ΔABC is an equiangular.

(iv) $P \to \neg Q \Rightarrow$ If ΔABC is an isoceles then ΔABC is not an equilateral.

(v) $R \to P \Rightarrow$ If ΔABC is an equiangular then ΔABC is an isoceles

(vi) $(P \vee Q) \to R \Rightarrow$ If either ΔABC is an isoceles or ΔABC is an equilateral then ΔABC is an equiangular.

(vii) $(\neg P \wedge Q) \to \neg R$
$\Rightarrow$ If ΔABC is not an isoceles and equilateral then ΔABC is not an equiangular.

4]. Construct the truth table $\neg(P \vee Q) \vee (\neg P \vee \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg P \vee \neg Q$	$\neg(P \vee Q)$	$\neg(P \vee Q) \vee (\neg P \vee \neg Q)$
T	T	F	F	T	F	F	F
T	F	F	T	T	T	F	T
F	T	T	F	T	T	F	T
F	F	T	T	F	T	T	T

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5]. Construct the truth table for the following:

(i). $(P \to Q) \wedge (Q \to P)$

(ii). $\neg(P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)$

(i) $(P \to Q) \wedge (Q \to P)$

P	Q	$P \to Q$	$Q \to P$	$(P \to Q) \wedge (Q \to P)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(ii) $\neg(P \wedge Q) \longleftrightarrow (\neg P \vee \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$	$\neg(P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

6]. How many rows are needed in the truth table of the given statement formula.

$$(P \to Q) \wedge (\neg R \vee S) \rightleftarrows T$$

Since the given statement formula consisting of P, Q, R, S, T.

Hence the truth table have 2^5 rows $= 32$ rows

7. Negate the statement "for all real numbers x, if $x > 4$ then $x^2 > 16$"

For some x, if $x^2 \le 16$, then $x \le 4$.

Construct the truth table i) $(P \wedge Q) \to \sim r$ ii). $q \wedge (\sim r)$