$$RHS = e^{ax+by} + \sin(ax+by),$$

(or)

$$e^{ax+by} + \cos(ax+by)$$

J. Solve $(D^2 - DD' - 20D'^2)z = e^{5x+y} + \sin(4x-y)$

Soln.

AE

$$m^2 - m - 20 = 0 \qquad D \to m$$

$$(m-5)(m+4) = 0 \qquad D' \to 1$$

$$m = 5, -4$$

$$CF = f_1(y-4x) + f_2(y+5x)$$

$$PI = \frac{1}{D^2 - DD' - 20D'^2}\left[e^{5x+y} + \sin(4x-y)\right]$$

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$$= \frac{1}{D^2 - DD' - 20D'^2} e^{5x+y} + \frac{1}{D^2 - DD' - 20D'^2} \sin(4x-y)$$

$PI = PI_1 + PI_2$

$$PI_1 = \frac{1}{D^2 - DD' - 20D'^2} e^{5x+y}$$

$$= \frac{1}{25 - 5(1) - 20(1)^2} e^{5x+y} \qquad D \to a = 5$$

$$D' \to b = 1$$

$$= \frac{1}{0} e^{5x+y}$$

$$= x \frac{1}{2D - D'} e^{5x+y}$$

$$= x \frac{1}{2(5) - 1} e^{5x+y}$$

$$PI_1 = \frac{x}{9} e^{5x+y}$$

$a = 4, \; b = -1$

$$PI_2 = \frac{1}{D^2 - DD' - 20D'^2} \sin(4x-y)$$

$D^2 \to -a^2 = -16$

$DD' \to -ab = -4(-1) = 4$

$D'^2 \to -b^2 = -(-1)^2 = -1$

$$= \frac{1}{-16 - 4 - 20(-1)} \sin(4x-y)$$

$$= x \frac{1}{2D - D'} \sin(4x-y)$$

$$= x \frac{(2D + D')}{(2D - D')(2D + D')} \sin(4x-y)$$

$$= x \frac{(2D + D') \sin(4x-y)}{4D^2 - D'^2}$$

$$= \frac{x}{-64+1} (2D + D') \sin(4x-y)$$

$$= -\frac{x}{63} [2D \sin(4x-y) + D' \sin(4x-y)]$$

$$= -\frac{x}{63}\left[8\cos(4x-y) - \cos(4x-y)\right]$$

$$= \frac{-7x\cos(4x-y)}{63}$$

$$= -\frac{x}{9}\cos(4x-y)$$

The Soln is, $z = CF + PI$

$$z = f_1(y-4x) + f_2(y+5x) + \frac{x}{9}e^{5x+y} - \frac{x}{9}\cos(4x-y)$$

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HW 1]. $(D^2+4DD'-5D'^2)z = e^{2x-y} + \sin(x-2y)$

2]. $(D^2-DD'-30D'^2)z = xy + e^{6x+y}$

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Solve $r+s-6t = y\cos x$

Soln :

Given $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$

$$(D^2 + DD' - 6D'^2)z = y\cos x$$

AE $m^2+m-6=0$

$$(m+3)(m-2)=0$$

$$m = -3, 2$$

$-6$: $3$, $-2$

$$CF = f_1(y-3x) + f_2(y+2x)$$

$$PI = \frac{1}{(D^2+DD'-6D'^2)} y\cos x$$

factor $\to D-2D'$

where $y = c-2x$

$D \to c$

$D' \to x$

$$= \frac{1}{(D+3D')(D-2D')} y\cos x$$

$$= \frac{1}{(D+3D')} \int (c-2x)\cos x \, dx$$

$$= \frac{1}{D+3D'}\left[(c-2x)\sin x - (-2)(-\cos x)\right]$$

$$= \frac{1}{D+3D'}\left[y\sin x - 2\cos x\right]$$

factor $\to D+3D'$

$y \to c+3x$

$$= \int \left[(c+3x)\sin x - 2\cos x\right] dx$$

$$= (c+3x)(-\cos x) - 3(-\sin x) - 2\sin x$$

$$= -y\cos x + 3\sin x - 2\sin x$$

$$= -y\cos x + \sin x$$

Type

RHS $= e^{ax+by}\,\phi(x,y)$

Replace $D$ by $D+a$

$D'$ by $D'+b$

Q. To find the PI of $(D^2-2DD'+D'^2)z = x^2y^2e^{x+y}$

Soln. :

$$PI = \frac{1}{D^2-2DD'+D'^2} x^2y^2e^{x+y}$$

$$= \frac{1}{(D-D')^2} e^{x+y} x^2y^2$$

$D \to D+a = D+1$

$D' \to D'+b = D'+1$

$$= e^{x+y} \frac{1}{(D+1-(D'+1))^2} x^2 y^2$$

$$= e^{x+y} \frac{1}{(D-D')^2} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2 - 2DD' + D'^2} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2\left[1 - \frac{2D'}{D} + \frac{D'^2}{D^2}\right]} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2} \left[1 - \left(\frac{2D'}{D} - \frac{D'^2}{D^2}\right)\right]^{-1} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2} \left[1 + \frac{2D'}{D} - \frac{D'^2}{D^2} + \frac{4D'^2}{D^2}\right] x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2} \left[x^2y^2 + \frac{2D'}{D} x^2y^2 + \frac{3D'^2}{D^2} x^2 y^2\right]$$

$$= e^{x+y} \left[\frac{1}{D^2} x^2 y^2 + \frac{2}{D^3} 2x^2 y + \frac{3}{D^4} (2x^2)\right]$$

$$\cancel{=} \cancel{e^{x+y} [}$$

$\frac{1}{D^2} x^2y^2 \xrightarrow{1st} \frac{x^3}{3} y^2$

$\xrightarrow{2nd} \frac{x^4}{12} y^2$

$\frac{1}{D^3} 4x^2 y \to 4 \frac{x^3}{3} y$

$\xrightarrow{2nd} \frac{4x^4}{12} y \xrightarrow{3rd} \frac{4x^5}{60} y$

$\frac{1}{D^4} 6x^2 \to \frac{6x^3}{3} \to \frac{6x^4}{12}$

$\to \frac{6x^5}{60} \to \frac{6x^7}{420}$

$$PI = e^{x+y}\left[\frac{x^4 y^2}{12} + \frac{x^5 y}{15} + \frac{x^7}{70}\right]$$