UNIT I

STRESS AND STRAIN

Poisson's Ratio:

Linear Strain or Primary Strain:

The ratio of change in length to the original length is known as linear or primary strain → Tensile

Lateral Strain or Secondary Strain or transverse strains:

Due to the load, the dimensions of the body changes; in all directions at right angles to its line of application the strains are called lateral or secondary or transverse strain. → Compressive

Poisson's ratio:

The ratio of lateral strain to linear strain is known as Poisson's ratio.

$$\mu = \frac{\text{Lateral strain or transverse strain}}{\text{Linear or primary strain}} = \frac{1}{m}$$

Relations between the Elastic Moduli:

(a) Relation between Young's Modulus (E) and Bulk Modulus (K):

Let a cube of side 'L' be subjected to three mutually $\perp^r$ like stresses of equal intensity p. By definition of Bulk Modulus

$$K = \frac{p}{e_v} \qquad e_v = \frac{\delta v}{V} = \frac{p}{K} \quad \text{--- (i)}$$

Total linear strain of each side $= e = \frac{p}{E} - \frac{p}{mE} - \frac{p}{mE}$

$$\boxed{e_v = \frac{p}{E}\left(1 - 2/m\right)}$$

$$\boxed{\begin{aligned} e_1 &= \frac{p_1}{E} - \frac{p_2}{mE} - \frac{p_3}{mE} \\ e_2 &= \frac{p_2}{E} - \frac{p_3}{mE} - \frac{p_1}{mE} \\ e_3 &= \frac{p_3}{E} - \frac{p_1}{mE} - \frac{p_2}{mE} \end{aligned}}$$

$$e_1 + e_2 + e_3 = (1 - 2/m)\left(\frac{p_1 + p_2 + p_3}{E}\right)$$

$$\boxed{Ee = p(1 - 2/m)}$$

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$$\frac{\delta L}{L} = e = \frac{p}{E}\left(1 - 2/m\right) \quad \text{—— (ii)}$$

$$V = L^3$$

$$\delta V = 3L^2 \cdot \delta L$$

$$\frac{\delta V}{V} = e_V = \frac{3\,\delta L}{E} = 3e = \frac{3p}{E}\left(1 - 2/m\right) \quad \text{—— (iii)}$$

Equating (i) & (iii)

$$\frac{p}{K} = \frac{3p}{E}\left(1 - 2/m\right)$$

Relation b/w E, K & m

$$\boxed{E = 3K\left(1 - 2/m\right)} \quad \text{— (I)}$$

(b) Relation between young's modulus (E) and Modulus of Rigidity (N):

We have seen in equ. that the linear strain e of the diagonal AC in fig. = $e = \frac{1}{2}\phi = \frac{q}{2N}$ —— (i)

We have also seen in equ. a state of simple shear produces tensile and compressive stresses along diagonal planes and the value of such direct stresses is numerically equal to q. Thus, on diagonal AC, in fig., there is a tensile stress p, and on diagonal BD there is a compressive stress p.

Hence from equ., the linear strain e of the diagonal AC, due to these two mutually perpendicular direct stresses, given by

$$e = \frac{p}{E} - \left(\frac{-p}{mE}\right) = \frac{p}{E}\left(1 + 1/m\right) \quad \text{—— (ii)}$$

But $p = q$

$$\therefore e = \frac{q}{E}\left(1 + 1/m\right) \quad \text{—— (iii)}$$

Equating (i) & (iii), we get,

$$\frac{1}{2}\frac{q}{N} = \frac{q}{E}\left(1 + 1/m\right)$$

Reln. b/w E, N & m

$$\boxed{E = 2N\left(1 + 1/m\right)} \quad \text{— (II)}$$

already $E = 3k\left(1 - 2/m\right)$

$$\therefore \quad E = 2N\left(1 + 1/m\right) = 3k\left(1 - 2/m\right)$$

$$\frac{1}{m} = \frac{3k - 2N}{6k + 2N}$$

$$\boxed{E = \frac{9kN}{N + 3k}}$$