1- (14) Statically Determinate Structure

If a structure can be solved for the forces, displacements, strains and stresses using only the static equations of Equilibrium ($\Sigma F_x = 0, \Sigma F_y = 0$ $\Sigma F_z = 0, \Sigma M_x = 0, \Sigma M_y = 0$ $\Sigma M_z = 0$), then the structure is a statically determinate structure.

$\Sigma F_x = 0$ Forces acting on the x-direction (Summation of)

$\Sigma M_x = 0$ Summation of moments about the x-axis.

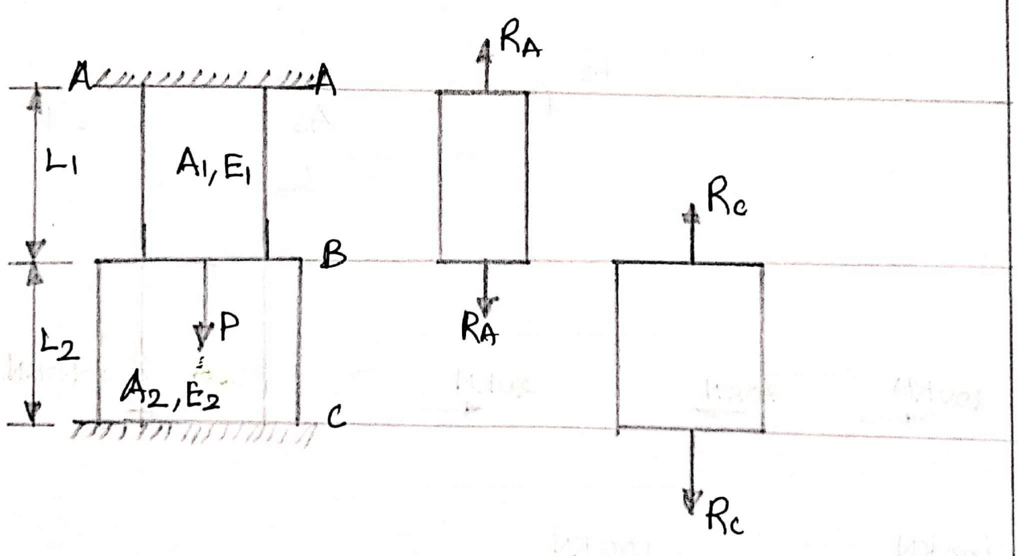
Statically Indeterminate structure.

If a structure can not be solved.....

How do you solve a statically Indeterminate structure.

* using compatibility Equation
  Displacement compatibility Equation
  strain compatibility equation.

*.

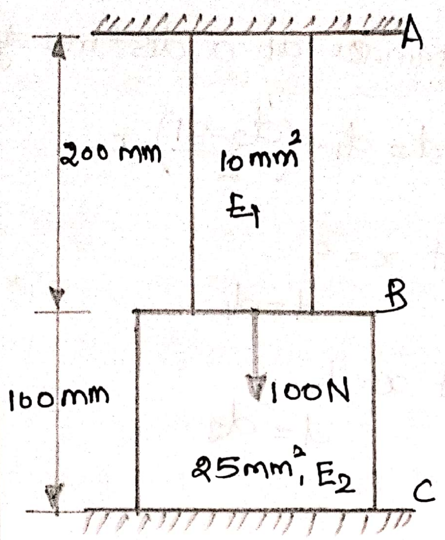


Force Equilibrium Equation ⇒ $R_A + R_C = P$

Moment equilibrium equation does not exist because There are no moments produced in the given bar.

Compatibility Equation ⇒ Elongation of portion AB = Shortening of portion BC

$$\frac{R_A L_1}{A_1 E_1} = \frac{R_C L_2}{A_2 E_2}$$



Take $E_1 = 200\ GPa$

$E_1 = 200 \times 10^3 \frac{N}{mm^2}$

$E_2 = 100\ GPa$

$E_2 = 100 \times 10^3 \frac{N}{mm^2}$.

Force Equilibrium Equation $R_A + R_c = 100\ N.$ — ①

Displacement compatibility equation:

Elongation of Portion AB = Shortening of portion BC

$$\frac{R_A\ L_1}{A_1 E_1} = \frac{R_c\ L_2}{A_2 E_2}$$

$$\frac{R_A\ \cancel{200}}{10 \times \cancel{200}} = \frac{R_c\ \cancel{100}}{25 \times \cancel{100}}$$

$$R_A = R_c \frac{10}{25} = 0.4 R_c \quad —— ②$$

Substituting ② in ①

$R_A + R_C = 100\ N$

$0.4 R_c + R_C = 100\ N$

$1.4\ R_C = 100\ N$

$R_C = 71.428\overline{57}\ N$ 143

$R_A = 100 - 71.42857\ N$

$R_A = 28.571428\ N$ 57

$\delta_{AB} = \frac{R_A L_1}{A_1 E_1}$

$\delta_{AB} = \frac{28.57 \times 200}{10 \times 200}$

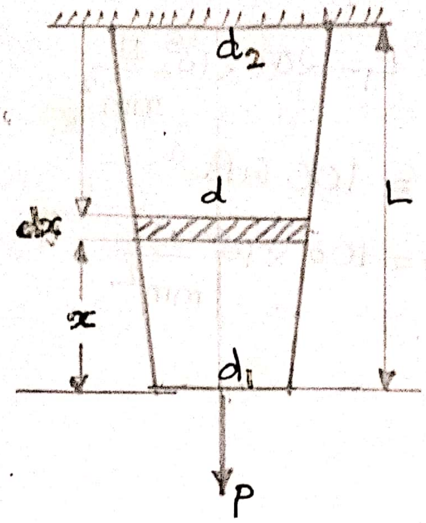
$\delta_{AB} = 2.8571\ mm.$

$\delta_{BC} = \frac{R_c L_2}{A_2 E_2}$

$\delta_{BC} = \frac{71.428 \times 100}{25 \times 100}$

$\delta_{BC} = 2.8571\ mm.$

1-(16) Elongation of a tapered bar.



diameter at a distance x

$$d = d_1 + \left(\frac{d_2 - d_1}{L}\right) x$$

At $x = 0$,
$d = d_1$

At $x = L$,
$d = d_2$

Cross-sectional Area at distance x $= \frac{\pi}{4} d^2$

$$A = \frac{\pi}{4}\left[d_1 + \left(\frac{d_2 - d_1}{L}\right) x\right]^2$$

Let $g = \frac{d_2 - d_1}{L}$

$$A = \frac{\pi}{4}(d_1 + gx)^2$$

Elongation of the shaded region $= \frac{P\,dx}{AE}$

$$= \frac{P\,dx}{\frac{\pi}{4}(d_1 + gx)^2 E.}$$

Elongation of the tapered bar $= \int_0^L \frac{P\,dx}{\frac{\pi}{4}(d_1 + gx)^2 E}$

$$= \frac{4P}{\pi E}\int_0^L (d_1 + gx)^{-2} dx = \frac{4P}{\pi E}\left[\frac{(d_1 + gx)^{-2+1}}{(-2+1)\cdot g.}\right]_0^l$$

$$= \frac{4P}{\pi E}\frac{(-1)}{g}\left[\frac{1}{(d_1 + g x).}\right]_0^l$$

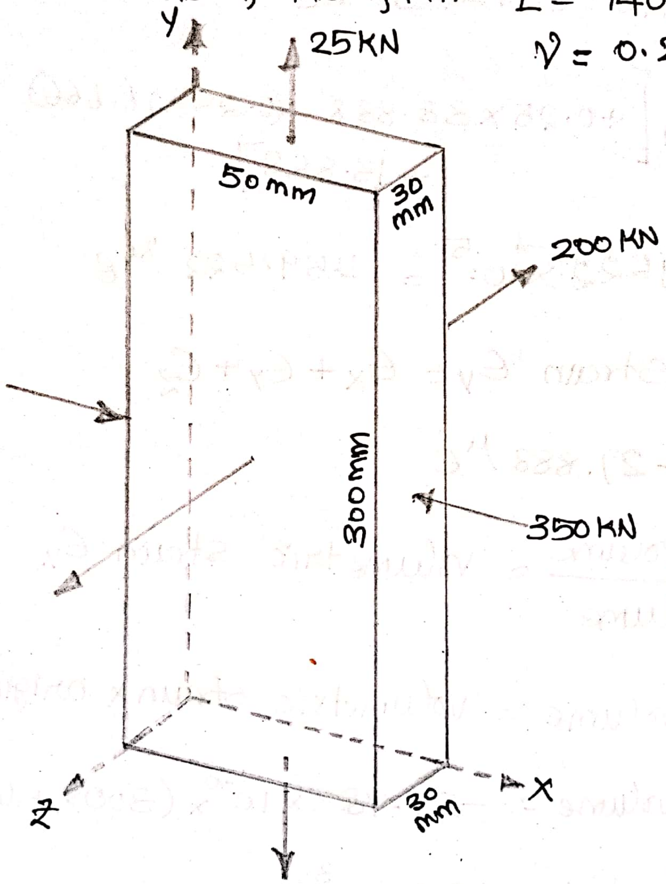
$$= \frac{4P}{\pi E}(-1)\frac{1}{\frac{d_2 - d_1}{L}}\left[\frac{1}{d_1 + \frac{d_2 - d_1}{L}.L} - \frac{1}{d_1 + \frac{d_2 - d_1}{L}0}\right]$$

$$= \frac{4P}{\pi E}(-1)\frac{L}{d_2 - d_1}\left[\frac{1}{d_2} - \frac{1}{d_1}\right] = \frac{4P}{\pi E}(-1)\frac{L}{d_2 - d_1}\left[\frac{d_1 - d_2}{d_1 d_2}\right]$$

$$= \frac{4P}{\pi E}\frac{L}{d_2 - d_1}\cdot\frac{d_2 - d_1}{d_1 d_2} = \frac{4PL}{\pi E d_1 d_2}$$

* A cast iron flat 300mm long and of 30 mm x 50mm uniform section is acted upon by the following forces uniformly distributed over the respective cross-section 25 KN in the direction of length (tensile); 350 KN in the direction of width (compressive); and 200 KN in the direction of thickness (tensile). Determine the change in volume of the flat. $E = 140\,GPa$ (cast iron)

$\nu = 0.25$



$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\epsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\epsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\sigma_x = \frac{-350\,KN}{300 \times 30\,mm^2}$$

$$\sigma_x = -0.038888\,\frac{KN}{mm^2}$$

$\sigma_x = 38.888\,MPa$ Compressive

$$\sigma_y = \frac{25\,KN}{50mm \times 30mm}$$

$\sigma_y = 16.6666\,MPa$ tensile

$$\sigma_z = \frac{+200\,KN}{300 \times 50\,mm^2}$$

$\sigma_z = 13.333\,MPa$ tensile

1-18

$$\epsilon_x = \frac{10^6}{140 \times 10^9}\left[-38.888 - (0.25 \times 16.6666) - (0.25 \times 13.333)\right]$$

$$\epsilon_x = -3.3134 \times 10^{-4} = -33.135 \times 10^{-6}$$

$$\epsilon_x = -331.341 \mu\epsilon$$

$$\epsilon_y = \frac{10^6}{140 \times 10^9}\left[+0.25 \times 38.888 + 16.6666 - (0.25 \times 13.333)\right]$$

$$\epsilon_y = 1.6467 \times 10^{-4} = 164.67 \mu\epsilon$$

$$\epsilon_z = \frac{10^6}{140 \times 10^9}\left[+0.25 \times 38.888 - (0.25 \times 16.666) + 13.888\right]$$

$$\epsilon_z = \cancel{1.38}882 \times 10^{-4} = 138.882 \mu\epsilon$$

volumetric strain $\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$

$$\epsilon_v = -27.788 \mu\epsilon$$

$$\frac{\text{change in volume}}{\text{original volume}} = \text{volumetric strain } \epsilon_v$$

change in volume = volumetric strain × original volume

change in volume = $-27.788 \times 10^{-6} \times (300 \times 50 \times 30)$ mm^3

$$\delta V = 12.5 mm^3$$

increase
(Change) in length = $300 \times \epsilon_y = 0.049401$ mm.

final length = $300 + 0.049401 = 300.049401$ mm.

decrease in width = $50 \times 331.341 \mu\epsilon = 50\,\epsilon_x = -0.016567$ mm

Final width = $50 - 0.016567 = 49.98343$ mm.

Increase in thickness = $30\,\epsilon_z = 0.00416646$ mm.

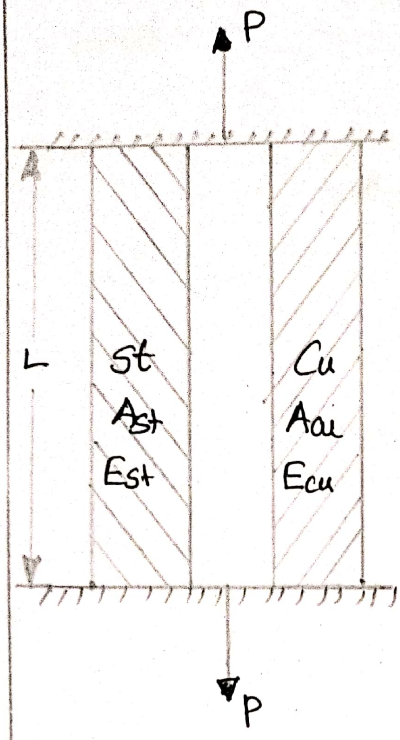
Final thickness = 30.00416646

Final volume = 449,987.4334

original volume = $300 \times 50 \times 30$ mm^3 = 450 000 mm^3

Change in volume = 12.5665 mm^3

Stresses in Compound Bars - Indeterminate structure



$$P = P_{st} + P_{cu} \quad \text{—— ①}$$

Elongation of steel bar = Elongation of copper bar.

$$\frac{P_{st} L_{st}}{A_{st} E_{st}} = \frac{P_{cu} L_{cu}}{A_{cu} E_{cu}}$$

Since both lengths are the same and since $\sigma = \frac{P}{A}$,

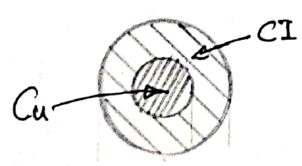
$$\frac{\sigma_{st}}{E_{st}} = \frac{\sigma_{cu}}{E_{cu}} \quad \text{—— ②}$$

Equation ① can be rewritten as

$$P = \sigma_{st} A_{st} + \sigma_{cu} A_{cu} \quad \text{—— ①}$$

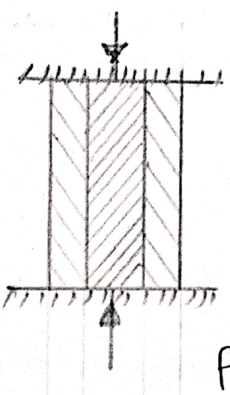
A copper rod of 40 mm diameter is surrounded tightly by a cast-iron tube of 80 mm external diameter, the ends being firmly fastened together. When put to a compressive load of 30 KN, what load will be shared by each? Also determine the amount by which the compound bar shortens if it is 2 m long.

$$E_{ci} = 175 \, G\frac{N}{m^2} \qquad E_{cu} = 75 \, G\frac{N}{m^2}$$



$$A_{cu} = \frac{\pi}{4} 40^2 = 1256.637 \, mm^2$$

$$A_{ci} = \frac{\pi}{4}(80^2 - 40^2) = 3769.911 \, mm^2$$



$$\frac{\sigma_{cu}}{E_{cu}} = \frac{\sigma_{ci}}{E_{ci}} \Rightarrow \sigma_{cu} = \sigma_{ci} \frac{E_{cu}}{E_{ci}}$$

$$\sigma_{cu} = \sigma_{ci} \frac{75}{175} = 0.428571 \, \sigma_{ci} \quad \text{— ②}$$

$$P = 30 \, KN = \sigma_{cu} A_{cu} + \sigma_{ci} A_{ci} \quad \text{— ①}$$

$$\sigma_{ci} = 6.963 \frac{N}{mm^2} \qquad \sigma_{cu} = 2.9841 \frac{N}{mm^2}$$

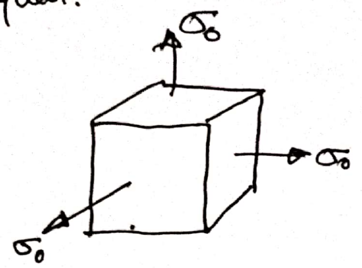
$$P_{ci} = 26.24989 \, KN \qquad P_{cu} = 3.74993 \, KN$$

1-20

$$\delta = \frac{PL}{AE} = \frac{\sigma L}{E} \Rightarrow \delta = \frac{\sigma_{cu} L_{cu}}{E_{cu}} = 0.0795776 \text{ mm.}$$

Relationship between E and K.

A spherical state of stress is a special triaxial stress state in which all three normal stresses are equal.



$$\sigma_x = \sigma_y = \sigma_z = \sigma_0$$

$$\epsilon_0 = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$\epsilon_0 = \frac{1}{E}(\sigma_0 - \nu\sigma_0 - \nu\sigma_0)$$

$$\epsilon_0 = \frac{\sigma_0}{E}(1-2\nu)$$

unit volume change $e = \frac{\Delta V}{V_0} = \epsilon_x + \epsilon_y + \epsilon_z$

$$e = \frac{\Delta V}{V_0} = \left(\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}\right) + \left(-\nu\frac{\sigma_x}{E} + \sigma_y - \nu\frac{\sigma_z}{E}\right)$$
$$+\left(-\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)$$

$$e = \frac{\Delta V}{V_0} = \frac{3\sigma_0}{E}(1-2\nu)$$

$$\sigma_0 = \frac{E}{3(1-2\nu)}e$$

or $\sigma_0 = Ke$

where $K = \frac{E}{3(1-2\nu)}$ is called the volume modulus of Elasticity or Bulk modulus of Elasticity.