



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

Minterms: [product consisting of all variables]

(i) $PAQ, TPAQ, PATQ, TPAQT$ are minterms in P, Q .

(ii) $PAQAR, PATQAR, TPAQAR, PAQATR, TPAQATR$

$PATQATR, TPAQATR, TPAQATR$ are minterms in P, Q and R .

maxterms: [sum consisting of all variables]

(i) $PVQ, TPVQ, PVQ, TPVQ$ are maxterms in P, Q .

(ii) $PVQVR, PVQVR, TPVQVR, PVQVR, TPVQVR$

$PVQVR, TPVQVR, TPVQVR$ are maxterms in P, Q, R .

PDNF principal disjunctive normal form
The PDNF of a given formula is an equivalent formula which consisting of disjunction of minterms only. i.e., $PDNF = (minterms) \vee (minterms) \vee \dots \vee (minterms)$

Principal conjunctive normal form (PCNF)
The PCNF of a given formula is an equivalent formula which consisting of conjunction of maxterms only. e.g., $PCNF = (maxterms) \wedge (maxterms) \wedge \dots \wedge (maxterms)$

Obtain the PDNF of

i) $TPVQ$

ii) $(PAQ) \vee (TPAR) \vee (QAR)$

i) $TPVQ \Leftrightarrow (TP \wedge T) \vee (Q \wedge T)$

$\Leftrightarrow (TP \wedge (Q \vee TQ)) \vee (Q \wedge (PVTP))$

$\Leftrightarrow (TP \wedge Q) \vee (TP \wedge TQ) \vee (Q \wedge P) \vee (Q \wedge TP)$

$\Leftrightarrow (TP \wedge Q) \vee (TP \wedge TQ) \vee (P \wedge Q) \vee (Q \wedge TP)$

Distributive laws
Commutative law

ii) $(PAQ) \vee (TPAR) \vee (QAR)$

$\Leftrightarrow (PAQ \wedge T) \vee (TPAR \wedge T) \vee (QAR \wedge T)$

$\Leftrightarrow (PAQ \wedge TR \vee TR) \vee (TPAR \wedge (Q \vee TQ)) \vee (Q \wedge R \wedge (PVTP))$



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$$\begin{aligned} &\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge R \wedge Q) \\ &\quad \vee (\neg P \wedge R \wedge \neg Q) \vee (Q \wedge R \wedge P) \vee (Q \wedge R \wedge \neg P) \\ &\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \\ &\quad \vee (\neg P \wedge Q \wedge \neg R) \vee (P \wedge R \wedge Q) \vee (P \wedge R \wedge \neg Q) \end{aligned}$$

iii) obtain the PDNF of $P \rightarrow ((P \rightarrow Q) \wedge \neg (T \wedge Q \vee TP))$

$$\begin{aligned} &P \rightarrow ((P \rightarrow Q) \wedge \neg (T \wedge Q \vee TP)) \\ &\Leftrightarrow \neg P \vee ((P \rightarrow Q) \wedge \neg (T \wedge Q \vee TP)) \\ &\Leftrightarrow \neg P \vee ((\neg P \vee Q) \wedge \neg (T \wedge Q \vee TP)) \\ &\Leftrightarrow \neg P \vee ((\neg P \vee Q) \wedge (Q \wedge \neg P)) \\ &\Leftrightarrow \neg P \vee [(T \wedge P \wedge (Q \wedge \neg P)) \vee (Q \wedge (Q \wedge \neg P))] \\ &\Leftrightarrow \neg P \vee [(T \wedge P \wedge (P \wedge Q)) \vee ((Q \wedge Q) \wedge \neg P)] \\ &\Leftrightarrow \neg P \vee [(T \wedge P \wedge P) \wedge Q \vee (Q \wedge Q) \wedge \neg P] \\ &\Leftrightarrow \neg P \vee [(T \wedge P) \wedge Q \vee (Q \wedge \neg P)] \\ &\Leftrightarrow \neg P \vee [F \vee (P \wedge Q)] \\ &\Leftrightarrow \neg P \vee (P \wedge Q) \\ &\Leftrightarrow (T \wedge \neg P) \vee (P \wedge Q) \\ &\Leftrightarrow (T \wedge (Q \vee \neg Q)) \vee (P \wedge Q) \\ &\Leftrightarrow (T \wedge Q) \vee (T \wedge \neg Q) \vee (P \wedge Q) \end{aligned}$$

obtain the PCNF of i). $(TP \rightarrow R) \wedge (Q \leftrightarrow P)$ and
PCNF of ii). $P \rightarrow (TP \wedge (Q \rightarrow P))$

$$\begin{aligned} &i). (TP \rightarrow R) \wedge (Q \leftrightarrow P) \\ \text{PCNF} &\Leftrightarrow (P \vee R) \wedge [(Q \rightarrow P) \wedge (P \rightarrow Q)] \\ &\Leftrightarrow (P \vee R) \wedge [(T \wedge Q \vee P) \wedge (TP \vee Q)] \Leftrightarrow (P \vee R \vee F) \wedge (T \wedge Q \vee P \vee F) \wedge (TP \vee Q \vee F) \\ &\Leftrightarrow (P \vee R \vee (Q \wedge \neg Q)) \wedge (T \wedge Q \vee P \vee (R \wedge \neg R)) \wedge (TP \vee Q \vee (R \wedge \neg R)) \\ &\Leftrightarrow (P \vee R \vee Q) \wedge (P \vee R \vee \neg Q) \wedge (T \wedge Q \vee P \vee R) \wedge (T \wedge Q \vee P \vee \neg R) \\ &\quad \wedge (TP \vee Q \vee R) \wedge (TP \vee Q \vee \neg R) \end{aligned}$$



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$$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee T \vee Q \vee R) \wedge (P \vee T \vee Q \vee T \vee R) \wedge \\ (T \vee P \vee Q \vee R) \wedge (T \vee P \vee Q \vee T \vee R)$$

PDNF :

TS: conjunction of the remaining maxterms.

$$TS: (P \vee Q \vee T \vee R) \wedge (T \vee P \vee T \vee Q \vee R) \wedge (T \vee P \vee T \vee Q \vee T \vee R)$$

$$T(TS) : \neg(P \vee Q \vee T \vee R) \vee \neg(T \vee P \vee T \vee Q \vee R) \vee \neg(T \vee P \vee T \vee Q \vee T \vee R)$$

$$\Leftrightarrow (\neg P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge \neg T \vee R) \vee (\neg P \wedge \neg Q \wedge \neg R) \text{ which is the required PDNF.}$$

ii). $P \rightarrow (T \vee P \wedge (Q \rightarrow P))$

PCNF :

$$\Leftrightarrow T \vee P \vee [T \vee P \wedge (Q \rightarrow P)] \text{ material implication Rule}$$

$$\Leftrightarrow T \vee P \vee [T \vee P \wedge (\neg Q \vee P)] \text{ material implication Rule}$$

$$\Leftrightarrow (T \vee P \vee T) \wedge (T \vee P \vee (\neg Q \vee P)) \text{ Distributive law}$$

$$\Leftrightarrow T \vee P \wedge (T \vee P \vee (\neg Q \vee P)) \text{ Idempotent law}$$

Commutative law

$$\Leftrightarrow T \vee P \wedge [(T \vee P) \vee (\neg Q \vee P)] \text{ Associative law}$$

$$\Leftrightarrow T \vee P \wedge (T \vee \neg Q \vee P)$$

$$\Leftrightarrow T \vee P \wedge T$$

$$\Leftrightarrow T \vee P$$

$$\Leftrightarrow T \vee P \vee F$$

$$\Leftrightarrow [T \vee P \vee (Q \wedge \neg Q)]$$

$$\Leftrightarrow (T \vee P \vee Q) \wedge (T \vee P \vee \neg Q) \text{ which is the required PCNF.}$$

iii). $(Q \rightarrow P) \wedge (T \vee P \wedge Q)$

$$\Leftrightarrow (\neg Q \vee P) \wedge (T \vee P \wedge Q)$$

$$\Leftrightarrow (\neg Q \vee P) \wedge [(T \vee P \wedge Q) \vee F]$$

$$\Leftrightarrow (\neg Q \vee P) \wedge [(T \vee P \wedge Q) \vee (P \wedge \neg P)]$$

$$\Leftrightarrow (\neg Q \vee P) \wedge [(T \vee P \wedge Q) \vee P] \wedge [(T \vee P \wedge Q) \vee \neg P]$$



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$$\Leftrightarrow (TQVP) \wedge (TPVP) \wedge (QVP) \wedge (TPVTP) \wedge (QVTP)$$

$$\Leftrightarrow (TQVP) \wedge T \wedge (QVP) \wedge (TPV(QVTP))$$

~~A & P~~

$$\Leftrightarrow (PVTQ) \wedge (PVQ) \wedge (TPV(TPVQ))$$

$$\Leftrightarrow (PVTQ) \wedge (PVQ) \wedge ((TPVTP) \vee Q)$$

$$\Leftrightarrow (PVTQ) \wedge (PVQ) \wedge (TPVQ)$$

main \rightarrow true
main \rightarrow false

Truth Table (PDNF & PCNF)

~~PDNF:~~

J. Obtain PDNF & PCNF for the following using truth table

i). $P \wedge (Q \rightarrow R)$

ii). $P \rightarrow [(P \rightarrow Q) \wedge (TQVTP)]$

i). $P \wedge (Q \rightarrow R)$

P	Q	R	$Q \rightarrow R$	$P \wedge (Q \rightarrow R)$	Minterms	Maxterms
T	T	T	T	<u>T</u>	$P \wedge Q \wedge R$	$\overline{P} \vee \overline{Q} \vee \overline{R}$
T	T	F	F	F*		
T	F	T	T	<u>T</u>	$P \wedge \overline{Q} \wedge R$	
T	F	F	T	<u>T</u>	$P \wedge \overline{Q} \wedge \overline{R}$	
F	T	T	T	F*		$\overline{P} \vee T \vee R$
F	T	F	F	F*		$\overline{P} \vee T \vee \overline{R}$
F	F	T	T	F*		$\overline{P} \vee \overline{Q} \vee R$
F	F	F	T	F*		$\overline{P} \vee \overline{Q} \vee \overline{R}$

PDNF: Disjunction of minterms
 $= (P \wedge Q \wedge R) \vee (P \wedge \overline{Q} \wedge R) \vee (P \wedge \overline{Q} \wedge \overline{R})$

PCNF: Conjunction of maxterms
 $= (\overline{P} \vee T \vee R) \wedge (\overline{P} \vee T \vee \overline{R}) \wedge (\overline{P} \vee \overline{Q} \vee R) \wedge (\overline{P} \vee \overline{Q} \vee \overline{R})$



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$S \leftrightarrow$

ii). $P \rightarrow [(P \rightarrow Q) \wedge (\neg Q \wedge TP)]$

P	Q	$P \rightarrow Q$	$\neg Q$	TP	$\neg Q \wedge TP$	$[(P \rightarrow Q) \wedge (\neg Q \wedge TP)]$	S	min terms	max terms
T	T	T	F	F	F	F	F		$\neg P \vee \neg Q$
T	F	F	T	F	F	F	F		$\neg P \vee Q$
F	T	T	F	T	F	F	T	$TP \wedge Q$	
F	F	T	T	T	T	T	T	$TP \wedge \neg Q$	

Th
A
A
P
S

PDNF : Disjunction of minterms

$$= (TP \wedge Q) \vee (TP \wedge \neg Q)$$

PCNF : Conjunction of maxterms

$$= (TP \vee \neg Q) \wedge (TP \vee Q)$$

HW J. $(TP \rightarrow R) \wedge (Q \leftrightarrow P)$ using fourth table