Conditional Proof:

1. Show that $R \to S$ can be derived from the Premises $P \to (Q \to S)$, $\neg R \vee P$ and Q.

Set		Premises	Rule
	1.	R. (Assume)	P (Assumed premise) Rule CP
	2.	$\neg R \vee P$	P
{1,2}	3.	P	T $\neg P, P \vee Q \Rightarrow Q$
	4.	$P \to (Q \to S)$	P
{3,4}	5.	$Q \to S$	T $P, P \to Q \Rightarrow Q$
	6.	Q	P
{5,6}	7.	S	T $P, P \to Q \Rightarrow Q$
{1,7}	8.	$R \to S$	CP

2. Derive the following using CP:

i) $P \to Q \Rightarrow P \to (P \wedge Q)$

ii) $P, P \to (Q \to (R \wedge S)) \Rightarrow Q \to S$

iii) $P \to Q \Rightarrow P \to (P \wedge Q)$

iv) $\neg P \vee Q, \neg Q \vee R, R \to S \Rightarrow P \to S$

i) $P \to Q \Rightarrow P \to (P \wedge Q)$

Step		Premises	Rule
	1.	P	P (Assumed premise)
	2.	$P \to Q$	P
{1,2}	3.	Q	T $P, P \to Q \Rightarrow Q$
{1,3}	4.	$P \wedge Q$	T $P, Q \Rightarrow P \wedge Q$
	5.	$P \to (P \wedge Q)$	CP

ii). $P,\ P \to (Q \to (R \wedge S)) \Rightarrow Q \to S$

Step	Premises	Rule
1.	Q	P(Assumed premise)
2.	P	P
3.	$P \to (Q \to (R \wedge S))$	P
{2,3} 4.	$Q \to (R \wedge S)$	T — P, $P \to Q \Rightarrow Q$
{1,4} 5.	$R \wedge S$	T — P, $P \to Q \Rightarrow Q$
{5} 6.	S	T — $P \wedge Q \Rightarrow Q$
{1,6} 7.	$Q \to S$	CP

iii). $P \to Q \Rightarrow P \to (P \wedge Q) \neg P \vee Q,\ \neg Q \vee R,\ R \to S \Rightarrow P \to S$

Step	Premises	Rule
1.	P	P(Assumed premise)
2.	$\neg P \vee Q$	P
{2} 3.	$P \to Q$	T — $\neg P \vee Q \Leftrightarrow P \to Q$
{1,3} 4.	Q	T — P, $P \to Q \Rightarrow Q$
5.	$\neg Q \vee R$	P
{5} 6.	$Q \to R$	T
{4,6} 7.	R	T — P, $P \to Q \Rightarrow Q$
8.	$R \to S$	P
{7,8} 9.	S	T
{1,9} 10.	$P \to S$	CP

Inconsistent Proof:

A set of premises $H_1, H_2, \ldots, H_m$ is said to be inconsistent if $H_1 \wedge H_2 \wedge \ldots \wedge H_m \Rightarrow F$ which stands for a contradiction.

i.e., $H_1 \wedge H_2 \wedge \ldots \wedge H_m = A \wedge \neg A$, where A is any variable.

1. Prove that the premises $P \to Q$, $Q \to R$, $R \to S$ $S \to TR$ and $P \wedge S$ are inconsistent.

	Step	Premises	Rule
	1.	$P \to Q$	P
	2.	$Q \to R$	P
{1, 2}	3.	$P \to R$	T $P \to Q, Q \to R \Rightarrow P \to$
	4.	$S \to TR$	P
{4}	5.	$R \to TS$	T $P \to Q \Leftrightarrow TQ \to TP$
{3, 5}	6.	$P \to TS$	T $P \to Q, Q \to R \Rightarrow P \to$
	7.	$R \to S$	P
{6}	8.	$TP \vee TS$	T $P \to Q \Leftrightarrow TP \vee Q$
{8}	9.	$T(P \wedge S)$	T $TP \vee TQ \Leftrightarrow T(P \wedge$
	10.	$P \wedge S$	P
{9, 10}	11.	$(P \wedge S) \wedge T(P \wedge S)$	T $P, Q \Rightarrow P \wedge Q$
	12.	F	T

2. Show that the premises $P \to q$, $P \to r$, $q \to Tr$, P are inconsistent.

	1.	P	P
	2.	$P \to q$	P
{1, 2}	3.	q	T $P, P \to Q \Rightarrow Q$
	4.	$q \to Tr$	P
{3, 4}	5.	Tr	T $P, P \to Q \Rightarrow Q$
	6.	$P \to r$	P
{6}	7.	$Tr \to TP$	T $P \to Q \Leftrightarrow TQ \to TP$

{5,7}	8.	$\neg P$	T	$P, P \to Q \Rightarrow Q$
{1,8}	9.	$P \wedge \neg P$	T	$P, Q \Rightarrow P \wedge Q$
	10.	F	T	$P \wedge \neg P \Leftrightarrow F$

3. Prove that the premises $a \to (b \to c)$, $d \to (b \wedge \neg c)$, $a \wedge d$ are inconsistent.

	1.	$a \wedge d$	P	
{1}	2.	a	T	$P \wedge Q \Rightarrow P, Q$
	3.	$a \to (b \to c)$	P	
{2,3}	4.	$b \to c$	T	$P, P \to Q \Rightarrow P \to Q$
{1}	5.	d	T	
	6.	$d \to (b \wedge \neg c)$	P	
{5,6}	7.	$b \wedge \neg c$	T	$P, P \to Q \Rightarrow Q$
{7}	8.	$\neg (b \to c)$	T	$\neg(P \to Q) \Leftrightarrow P \wedge \neg Q$
{4,8}	9.	$(b \to c) \wedge \neg (b \to c)$	T	$P, Q \Rightarrow P \wedge Q$
	10.	F	T	

4. Show that the following premises are inconsistent.
1. If Jack misses many classes through illness, then he fails high school.
2. If Jack fails high school, then he is uneducated.
3. If Jack reads a lot of books, then he is not uneducated.
4. Jack misses many classes through illness and reads a lot of books.

P : Jack misses many classes through illness
Q : Jack fails high school.
R : Jack reads a lot of books
S : Jack is uneducated.

The premises are

$P \to Q,\ Q \to R,\ S \to \neg R,\ P \wedge S$.

Step	Premises	Rule	
1.	$P \to Q$	P	
2.	$Q \to R$	P	
{1,2} 3.	$P \to R$	T	$P \to Q, Q \to R \Rightarrow P \to R$
4.	$S \to \neg R$	P	
{4} 5.	$R \to \neg S$	T	$P \to Q \Leftrightarrow \neg Q \to \neg P$
{3,5} 6.	$P \to \neg S$	T	$P \to Q, Q \to R \Rightarrow P \to R$
{6} 7.	$\neg P \vee \neg S$	T	$P \to Q \Leftrightarrow \neg P \vee Q$
{7} 8.	$\neg (P \wedge S)$	T	$\neg (P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$
9.	$P \wedge S$	P	
{8,9} 10.	$(P \wedge S) \wedge \neg (P \wedge S)$	T	$P, Q \Rightarrow P \wedge Q$
11.	F	T	$P \wedge \neg P \Leftrightarrow F$.

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5. i). If there is a ball game, then travelling was difficult.
ii). If they arrived on time then travelling was not difficult.
iii). They arrived on time
iv). Therefore there was no ball game.
Show that the above statements are valid statement.

Let P : There was a ball game
Q : Travelling was difficult
R : They arrived on time.

The premises are $P \to Q,\ R \to \neg Q,\ R$,
The conclusion is $\neg P$.

Step	Premises	Rule
1.	R	P
2.	$R \to \neg Q$	P
{1,2} 3.	$\neg Q$	T $P, P \to Q \Rightarrow Q$
4.	$P \to Q$	P
{3,4} 5.	$\neg P$	T, $P \to Q, \neg Q \Rightarrow \neg P$