



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

Solve the recurrence relation for the Fibonacci sequence $0, 1, 1, 2, 3, 5, 8, 13, \dots$

Soln.:

Fibonacci sequence $0, 1, 1, 2, 3, 5, 8, 13, \dots$
Satisfies the recurrence relation $f_n = f_{n-1} + f_{n-2}$
with $f_0 = 0, f_1 = 1$

$$\text{ie., } f_n - f_{n-1} - f_{n-2} = 0$$

Characteristic eqn.: $m^2 - m - 1 = 0$

$$m = \frac{1 \pm \sqrt{5}}{2}$$

$$\begin{aligned} \text{HS} &= A \left(\frac{1+\sqrt{5}}{2} \right)^n + B \left(\frac{1-\sqrt{5}}{2} \right)^n \\ \text{PS} &= 0 \end{aligned}$$

Since RHS = 0 \Rightarrow PS = 0

$$\therefore \text{The soln. is } f_n = A \left(\frac{1+\sqrt{5}}{2} \right)^n + B \left(\frac{1-\sqrt{5}}{2} \right)^n$$

$$\text{Given } f_0 = 0 \Rightarrow f_0 = A \left(\frac{1+\sqrt{5}}{2} \right)^0 + B \left(\frac{1-\sqrt{5}}{2} \right)^0$$

$$0 = A + B$$

$$A + B = 0 \rightarrow (1)$$

$$\text{and } f_1 = 1 \Rightarrow f_1 = A \left(\frac{1+\sqrt{5}}{2} \right)^1 + B \left(\frac{1-\sqrt{5}}{2} \right)^1 \rightarrow$$

$$A \left(\frac{1+\sqrt{5}}{2} \right) + B \left(\frac{1-\sqrt{5}}{2} \right) = 1 \rightarrow (2)$$

Solving (1) & (2), we get

$$A = \frac{1}{\sqrt{5}} ; B = -\frac{1}{\sqrt{5}}$$

$$\therefore f_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right]$$

Q. Find the solution to the recurrence relation
 $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with $a_0 = 2, a_1 = 5$
and $a_2 = 15$.

$$\text{Given } a_n - 6a_{n-1} + 11a_{n-2} - 6a_{n-3} = 0$$



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DEPARTMENT OF MATHEMATICS

characteristic eqn:

$$m^3 - 6m^2 + 11m - 6 = 0$$

$$m = 1, 2, 3$$

Since RHS = 0 \Rightarrow P.C = 0

$$\therefore a_n = A(1)^n + B(2)^n + C(3)^n$$

Given $a_0 = 2$

$$A + B + C = 2 \rightarrow (1)$$

$$a_1 = 5$$

$$A + 2B + 3C = 5 \rightarrow (2)$$

and $a_2 = 15$

$$A + B(2^2) + C(3^2) = 15$$

$$A + 4B + 9C = 15 \rightarrow (3)$$

Solving (1), (2) and (3),

$$(1) \Rightarrow C = 2 - A - B \rightarrow (4)$$

Sub (4) in (2),

$$A + 2B + 3(2 - A - B) = 5$$

$$A + 2B + 6 - 3A - 3B = 5$$

$$-2A - B = 5 - 6 = -1$$

$$2A + B = 1 \rightarrow (5)$$

Sub. (4) in (3),

$$A + 4B + 9(2 - A - B) = 15$$

$$A + 4B + 18 - 9A - 9B = 15$$

$$-8A - 5B = -3$$

$$8A + 5B = 3 \rightarrow (6)$$

Solving (5) & (6),

$$(5) \times 4 \Rightarrow 8A + 4B = 4$$

$$(6) \Rightarrow 8A + 5B = 3$$

$$(5) \times 5 \Rightarrow 10A + 5B = 5$$

$$(6) \Rightarrow 8A + 5B = 3$$

$$2A = 2 \Rightarrow A = 1$$

$$\begin{array}{c} | \\ 1 \quad | \quad \begin{array}{ccc} 1 & -6 & 11 \\ 0 & 1 & -5 \\ 1 & -5 & 6 \end{array} \\ | \end{array}$$

$$m=1, \quad m^2 - 5m + 6 = (m-2)(m-3)$$



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Sub A=1 in (5),

$$2 + B = 1 \Rightarrow B = -1$$

$$(1) \Rightarrow A + B + C = 2$$

$$1 - 1 + C = 2$$

$$C = 2$$

\therefore solution is $a_n = 1(1)^n - 1(2)^n + 2(3)^n$

$$a_n = 1^n - 2^n + 2(3)^n$$

6]. solve the recurrence relation
 $a_n = 2a_{n-1} - 2a_{n-2}$, $n \geq 2$ and $a_0 = 1, a_1 = 2$

Given.

$$a_n - 2a_{n-1} + 2a_{n-2} = 0$$

characteristic eqn.

$$m^2 - 2m + 2 = 0$$

$$m = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{4 - 8}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$$= \frac{2[1 \pm i]}{2}$$

$$m = 1 \pm i \quad (\alpha \pm i\beta)$$

\therefore solution is $a_n = r^n (A \cos n\theta + B \sin n\theta)$

where $r = \sqrt{\alpha^2 + \beta^2}$ and $\theta = \tan^{-1}(\beta/\alpha)$

$$r = \sqrt{2}$$

$$= \tan^{-1}(1/1) = \tan^{-1}(1)$$

$$\theta = \pi/4$$

$$\therefore a_n = (\sqrt{2})^n \left[A \cos \frac{n\pi}{4} + B \sin \frac{n\pi}{4} \right] \rightarrow (A)$$

Given $a_0 = 1 \Rightarrow a_0 = A = 1$

and $a_1 = 2 \Rightarrow a_1 = (\sqrt{2}) \left[A \cos \frac{\pi}{4} + B \sin \frac{\pi}{4} \right] = 2$

$$A \sqrt{2} \times \frac{1}{\sqrt{2}} + B \sqrt{2} \times \frac{1}{\sqrt{2}} = 2$$



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$$A + B = 2$$

$$B = 2 - A$$

$$B = 1$$

∴ solution is $a_n = (\sqrt{2})^n \left(\cos \frac{n\pi}{4} + i \sin \frac{n\pi}{4} \right)$

HW J. Solve the recurrence relation

$$S(n) - 10S(n-1) + 9S(n-2) = 0 \text{ with } S(0) = 2, S(1) = 1$$

Linear Non-Homogeneous Recurrence Equations:

A recurrence relation of the form

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k} + F(n)$$

where c_1, c_2, \dots, c_k are real nos. & $F(n)$ is a fcn. not identically zero depending only on n , is called a non homogeneous recurrence relation with constant coefficients.

J. Solve the recurrence relation

$$S(k) - 7S(k-1) + 10S(k-2) = 8k + 6 \text{ with } S(0) = 1, S(1) = 2 \rightarrow (1)$$

$$S(1) = 2$$

Given. $S(k) - 7S(k-1) + 10S(k-2) = 0$

CE: $m^2 - 7m + 10 = 0$

$$(m-2)(m-5) = 0$$

$$m = 2, 5$$

$$HS = A(2)^k + B(5)^k \rightarrow (2)$$

PS

$$RHS = 8k + 6$$

Take $S(k) = ck + d$

$$S(k-1) = c(k-1) + d$$

$$S(k-2) = c(k-2) + d$$

} → (A)



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Subs. (A) In (1),

$$ck+d - 7 [c(k-1)+d] + 10 [c(k-2)+d] = 8k+6$$

$$ck+d - 7ck+7c-7d + 10ck-20c + 10d = 8k+6$$

$$4ck - 13c + 4d = 8k+6$$

Equating the coefficient of n and constant,

$$4c = 8$$

$$c = 2$$

$$-13c + 4d = 6$$

$$-13(2) + 4d = 6$$

$$4d = 6 + 26$$

$$4d = 32$$

$$d = 8$$

$$\therefore ps = ck + d$$

$$= 2k + 8 \rightarrow (3)$$

General soln.

$$f(k) = A(2)^k + B(5)^k + 2k + 8 \rightarrow (4)$$

Given $S(0) = 1$

$$S(0) = A + B + 8 = 1$$

$$A + B = -7 \rightarrow (5)$$

and $S(1) = 2$

$$S(1) = A(2) + B(5) + 2 + 8 = 2$$

$$2A + 5B = -8 \rightarrow (6)$$

Solving (5) and (6),

$$A + B = -7 \rightarrow (5)$$

$$2A + 5B = -8 \rightarrow (6)$$

$$(5) \times 2 \Rightarrow \begin{array}{r} 2A + 2B = -14 \\ \underline{-(2A + 5B = -8)} \\ 3B = -6 \end{array}$$

$$3B = -6 \Rightarrow B = -2$$



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Subs. $B = 2$ in (5),

$$A + B = -7$$

$$A = -2 - 7 = -9$$

$$A = -9$$

Subs. A & B in (4),

$$S(k) = -9(2)^k + 2(5)^k + 2k + 8$$

3. Solve the Recurrence Relation

$$a_n - a_{n-1} - 6a_{n-2} = -30, \quad a_0 = 0, \quad a_1 = -5, \quad n \geq 2$$

Given. $a_n - a_{n-1} - 6a_{n-2} = -30 \rightarrow (1)$

CE: $m^2 - m - 6 = 0$

$$(m-3)(m+2) = 0$$

$$m = 3, -2$$

$$HS = A(3)^n + B(-2)^n \rightarrow (2)$$

PS

RHS = a constant

Take $a_n = a_{n-1} = a_{n-2} = d$

$$(1) \Rightarrow d - d - 6d = -30$$

$$-6d = -30$$

$$d = 5$$

$$PS = 5 \rightarrow (3)$$

General soln.

$$a_n = A(3)^n + B(-2)^n + 5 \rightarrow (4)$$

Given. $a_0 = 0$

$$A + B + 5 = 0$$

$$A + B = -5 \rightarrow (5)$$

and $a_1 = -5$

$$3A - 2B + 5 = -5$$



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$$3A - 2B = -10 \rightarrow (6)$$

Solving (5) and (6),

$$(5) \times 2 \Rightarrow 2A + 2B = -10$$

$$3A - 2B = -10$$

$$\hline 5A = -20$$

$$A = \frac{-20}{5} = -4$$

$$(5) \Rightarrow -4 + B = -5$$

$$B = -5 + 4$$

$$B = -1$$

$$(4) \Rightarrow a_n = -4(3)^{n-1}(-2)^{n+5}$$

$$1. \text{ Solve } a_n - 2a_{n-1} - 3a_{n-2} = 4^n + 6$$

$$\text{Givn. } a_n - 2a_{n-1} - 3a_{n-2} = 4^n + 6 \rightarrow (1)$$

$$E: m^2 - 2m - 3 = 0$$

$$(m-3)(m+1) = 0$$

$$m = 3, -1$$

$$\text{HS} = A(3)^n + B(-1)^n \rightarrow (2)$$

$$\text{PS: RHS} = 4^n + 6$$

$$\text{PS} = \text{PS}_1 + \text{PS}_2$$

$$\text{PS}_1: \text{ Take } \left. \begin{aligned} a_n &= d \cdot 4^n \\ a_{n-1} &= d \cdot 4^{n-1} \\ a_{n-2} &= d \cdot 4^{n-2} \end{aligned} \right\} \rightarrow (3)$$

$$\text{Subs. (3) in (1), } d \cdot 4^n - 2d \cdot 4^{n-1} - 3d \cdot 4^{n-2} = 4^n$$

$$d \cdot 4^n - 2d \cdot \frac{4^n}{4} - 3d \cdot \frac{4^n}{16} = 4^n$$



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$$4^n \left[d - \frac{d}{2} - \frac{3d}{16} \right] = 4^n$$

$$\frac{16d - 8d - 3d}{16} = 1$$

$$\frac{5d}{16} = 1$$

$$d = \frac{16}{5}$$

$$PS_1 = \frac{16}{5} (4)^n$$

PS₂:

RHS = a constant

$$\text{Take } a_n = a_{n-1} = a_{n-2} = d$$

$$d - 2d - 3d = 6$$

$$-4d = 6$$

$$d = \frac{6}{-4}$$

$$d = \frac{-3}{2}$$

$$PS_2 = \frac{-3}{2}$$

$$PS = \frac{16}{5} (4)^n - \frac{3}{2}$$

General soln.

$$a_n = A(3)^n + B(-1)^n + \frac{16}{5} (4)^n - \frac{3}{2}$$

4]. solve $a_n - 4a_{n-1} + 4a_{n-2} = 2^n + 3n, n \geq 2$

Given. $a_n - 4a_{n-1} + 4a_{n-2} = 2^n + 3n$
↳ (1)

CE: $m^2 - 4m + 4 = 0$

$$(m-2)^2 = 0$$

$$m = 2, 2$$

$$HS = (A + nB) 2^n$$



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PS:

$$RHS = 2^n + 3n$$

$$PS = PS_1 + PS_2$$

$$PS_1 = 2^n$$

Take $a_n = dn^2 2^n$ since base of RHS is 2, which is a double root of CE

$$a_{n-1} = d(n-1)^2 2^{n-1}$$

$$a_{n-2} = d(n-2)^2 2^{n-2}$$

$$\therefore (1) \Rightarrow dn^2 2^n - 4[d(n-1)^2 2^{n-1}] + 4[d(n-2)^2 2^{n-2}] = 2^n$$

$$\div 2^n \quad dn^2 - 4d(n-1)^2 2^{-1} + 4d(n-2)^2 2^{-2} = 1$$

$$dn^2 - \frac{4d}{2}(n^2 + 1 - 2n) + \frac{4d}{4}(n^2 + 4 - 4n) = 1$$

$$dn^2 - 2d(n^2 + 1 - 2n) + d(n^2 + 4 - 4n) = 1$$

$$dn^2 - 2dn^2 - 2d + 4dn + dn^2 + 4d - 4dn = 1$$

$$2d = 1$$

$$d = \frac{1}{2}$$

$$PS_1 = \frac{1}{2} n^2 (2)^n$$

$$PS_2 = 3n$$

Take $a_n = d_0 + d_1 n$

$$a_{n-1} = d_0 + d_1(n-1)$$

$$a_{n-2} = d_0 + d_1(n-2)$$

$$(1) \Rightarrow d_0 + d_1 n - 4[d_0 + d_1(n-1)] + 4[d_0 + d_1(n-2)] = 3n$$

$$d_0 + d_1 n - 4d_0 - 4d_1 n + 4d_1 + 4d_0 + 4d_1 n - 8d_1 = 3n$$

$$d_0 - 4d_1 + d_1 n = 3n$$



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Equating the coeff. of n and constant,

$$d_1 = 3; \quad d_0 - 4d_1 = 0$$

$$d_0 = 4d_1 = 12$$

$$d_0 = 12$$

$$P_{S_2} = 12 + 3n$$

$$P_B = \frac{1}{2}(n)^2(2)^n + 12 + 3n$$

General soln.

$$a_n = (A + nB)(2)^n + \frac{1}{2}(n)^2(2)^n + 12 + 3n$$

Hw J. Solve $S(n) - 5S(n-1) + 6S(n-2) = 2$ with

$$S(0) = 1, \quad S(1) = -1.$$