



DEPARTMENT OF MATHEMATICS

Linear Non-Homogeneous Recurrence Equations:

A recurrence relation of the form

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k} + F(n)$$

where c_1, c_2, \dots, c_k are real nos. & $F(n)$ is a fcn. not identically zero depending only on n , is called a non homogeneous recurrence relation with constant coefficients.

Q. Solve the recurrence relation

$$S(k) - 7S(k-1) + 10S(k-2) = 8k + 6 \quad \text{with } S(0) = 1, \\ \rightarrow (1)$$

$$S(1) = 2$$

$$\text{Givn. } S(k) - 7S(k-1) + 10S(k-2) = 0$$

$$\text{CE: } m^2 - 7m + 10 = 0$$

$$(m-2)(m-5) = 0$$

$$m = 2, 5$$

$$\text{H.S.} = A(2)^k + B(5)^k \rightarrow (2)$$

PS RHS = $8k + 6$

$$\left. \begin{aligned} \text{Take } S(k) &= ck + d \\ S(k-1) &= c(k-1) + d \\ S(k-2) &= c(k-2) + d \end{aligned} \right\} \rightarrow (A)$$



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Subs. (A) In (1),

$$ck+d - 7 [c(k-1)+d] + 10 [c(k-2)+d] = 8k+6$$

$$ck+d - 7ck+7c-7d + 10ck-20c = 8k+6$$

$$4ck - 13c + 4d \qquad \qquad \qquad + 10d$$

$$= 8k+6$$

Equating the coefficient of n and constant,

$$4c = 8$$

$$c = 2$$

$$-13c + 4d = 6$$

$$-13(2) + 4d = 6$$

$$4d = 6 + 26$$

$$4d = 32$$

$$d = 8$$

$$\therefore ps = ck+d$$

$$= 2k+8 \rightarrow (3)$$

General soln.

$$s(k) = A(2)^k + B(5)^k + 2k + 8 \rightarrow (4)$$

Given $s(0) = 1$

$$s(0) = A + B + 8 = 1$$

$$A + B = -7 \rightarrow (5)$$

and $s(1) = 2$

$$s(1) = A(2) + B(5) + 2 + 8 = 2$$

$$2A + 5B = -8 \rightarrow (6)$$

Solving (5) and (6),

$$A + B = -7 \rightarrow (5)$$

$$2A + 5B = -8 \rightarrow (6)$$

$$(5) \times 2 \Rightarrow \begin{array}{r} 2A + 2B = -14 \\ (-) \quad (-) \quad (+) \\ \hline \end{array}$$

$$3B = 6 \Rightarrow B = 2$$



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Subs. $B = 2$ in (5),

$$A + B = -7$$

$$A = -2 - 7 = -9$$

$$A = -9$$

Subs. A & B in (4),

$$S(k) = -9(2)^k + 2(5)^k + 2k + 8$$

3. Solve the Recurrence Relation

$$a_n - a_{n-1} - 6a_{n-2} = -30, \quad a_0 = 0, \quad a_1 = -5, \quad n \geq 2$$

Given: $a_n - a_{n-1} - 6a_{n-2} = -30 \rightarrow (1)$

CE: $m^2 - m - 6 = 0$

$$(m-3)(m+2) = 0$$

$$m = 3, -2$$

$$HS = A(3)^n + B(-2)^n \rightarrow (2)$$

PS

RHS = a constant

Take $a_n = a_{n-1} = a_{n-2} = d$

$$(1) \Rightarrow d - d - 6d = -30$$

$$-6d = -30$$

$$d = 5$$

$$PS = 5 \rightarrow (3)$$

General soln.

$$a_n = A(3)^n + B(-2)^n + 5 \rightarrow (4)$$

Given: $a_0 = 0$

$$A + B + 5 = 0$$

$$A + B = -5 \rightarrow (5)$$

and $a_1 = -5$

$$3A - 2B + 5 = -5$$

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$$3A - 2B = -10 \rightarrow (6)$$

Solving (5) and (6),

$$(5) \times 2 \Rightarrow 2A + 2B = -10$$

$$3A - 2B = -10$$

$$\hline 5A = -20$$

$$A = \frac{-20}{5} = -4$$

$$(5) \Rightarrow -4 + B = -5$$

$$B = -5 + 4$$

$$B = -1$$

$$(4) \Rightarrow a_n = -4(3)^{n-1}(-2)^{n+5}$$

$$3]. \text{ Solve } a_n - 2a_{n-1} - 3a_{n-2} = 4^n + 6$$

$$\text{Givn. } a_n - 2a_{n-1} - 3a_{n-2} = 4^n + 6 \rightarrow (1)$$

$$\text{CE: } m^2 - 2m - 3 = 0$$

$$(m-3)(m+1) = 0$$

$$m = 3, -1$$

$$\text{HS} = A(3)^n + B(-1)^n \rightarrow (2)$$

$$\text{RS: } \text{RHS} = 4^n + 6$$

$$\text{PS} = \text{PS}_1 + \text{PS}_2$$

$$\text{PS}_1: \text{ Take } \left. \begin{array}{l} a_n = d \cdot 4^n \\ a_{n-1} = d \cdot 4^{n-1} \\ a_{n-2} = d \cdot 4^{n-2} \end{array} \right\} \rightarrow (3)$$

Subs. (3) in (1),

$$d \cdot 4^n - 2d \cdot 4^{n-1} - 3d \cdot 4^{n-2} = 4^n$$

$$d \cdot 4^n - 2d \cdot \frac{4^n}{4} - 3d \cdot \frac{4^n}{16} = 4^n$$



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$$4^n \left[d - \frac{d}{2} - \frac{3d}{16} \right] = 4^n$$

$$\frac{16d - 8d - 3d}{16} = 1$$

$$\frac{5d}{16} = 1$$

$$d = \frac{16}{5}$$

$$PS_1 = \frac{16}{5} (4)^n$$

PS₂:

RHS = a constant

$$\text{Take } a_n = a_{n-1} = a_{n-2} = d$$

$$d - 2d - 3d = 6$$

$$-4d = 6$$

$$d = \frac{6}{-4}$$

$$d = \frac{-3}{2}$$

$$PS_2 = \frac{-3}{2}$$

$$PS = \frac{16}{5} (4)^n - \frac{3}{2}$$

General soln.

$$a_n = A(3)^n + B(-1)^n + \frac{16}{5} (4)^n - \frac{3}{2}$$

4]. solve $a_n - 4a_{n-1} + 4a_{n-2} = 2^n + 3n, n \geq 2$

Given. $a_n - 4a_{n-1} + 4a_{n-2} = 2^n + 3n$
↳ (1)

CE: $m^2 - 4m + 4 = 0$

$$(m-2)^2 = 0$$

$$m = 2, 2$$

$$HS = (A + nB) 2^n$$



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PS:

$$RHS = 2^n + 3n$$

$$PS = PS_1 + PS_2$$

$$PS_1 = 2^n$$

Take $a_n = dn^2 2^n$ since base of RHS is 2, which is a double root of CE

$$a_{n-1} = d(n-1)^2 2^{n-1}$$

$$a_{n-2} = d(n-2)^2 2^{n-2}$$

$$\therefore (1) \Rightarrow dn^2 2^n - 4[d(n-1)^2 2^{n-1}] + 4[d(n-2)^2 2^{n-2}] = 2^n$$

$$\div 2^n \quad dn^2 - 4d(n-1)^2 2^{-1} + 4d(n-2)^2 2^{-2} = 1$$

$$dn^2 - \frac{4d}{2}(n^2 + 1 - 2n) + \frac{4d}{4}(n^2 + 4 - 4n) = 1$$

$$dn^2 - 2d(n^2 + 1 - 2n) + d(n^2 + 4 - 4n) = 1$$

$$dn^2 - 2dn^2 - 2d + 4dn + dn^2 + 4d - 4dn = 1$$

$$2d = 1$$

$$d = \frac{1}{2}$$

$$PS_1 = \frac{1}{2} n^2 (2)^n$$

$$PS_2 = 3n$$

Take $a_n = d_0 + d_1 n$

$$a_{n-1} = d_0 + d_1(n-1)$$

$$a_{n-2} = d_0 + d_1(n-2)$$

$$(1) \Rightarrow d_0 + d_1 n - 4[d_0 + d_1(n-1)] + 4[d_0 + d_1(n-2)] = 3n$$

$$d_0 + d_1 n - 4d_0 - 4d_1 n + 4d_1 + 4d_0 + 4d_1 n - 8d_1 = 3n$$

$$d_0 - 4d_1 + d_1 n = 3n$$



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Equating the coeff. of n and constant,

$$d_1 = 3; \quad d_0 - 4d_1 = 0$$

$$d_0 = 4d_1 = 12$$

$$d_0 = 12$$

$$P_{S_2} = 12 + 3n$$

$$P_B = \frac{1}{2} (n)^2 (2)^n + 12 + 3n$$

General soln.

$$a_n = (A + nB)(2)^n + \frac{1}{2} (n)^2 (2)^n + 12 + 3n$$

Hw J. Solve $S(k) - 5S(k-1) + 6S(k-2) = 2$ with

$$S(0) = 1, \quad S(1) = -1.$$