



DEPARTMENT OF MATHEMATICS

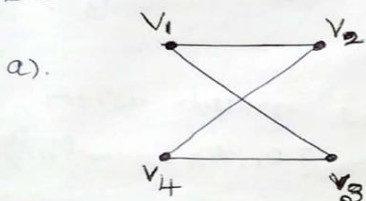
Graph representation:

We can represent a graph in the form of adjacency lists, which are very useful in computer programming.

Adjacency matrix of a simple graph:

Let $G = (V, E)$ be a simple graph with 'n' vertices i.e., v_1, v_2, \dots, v_n , its adjacency matrix is denoted by $A = [a_{ij}] = \begin{cases} 1, & \text{if there exist an edge b/w } v_i \text{ and } v_j \\ 0, & \text{otherwise} \end{cases}$

and degree of each vertex
I. Find the adjacency matrix A of the graph given below.



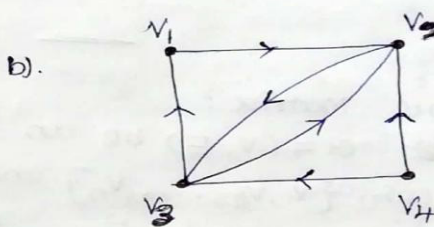
Soln.

Adjacency matrix:

$$A = [a_{ij}] = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

and

$$\begin{aligned} \deg(v_1) &= 2 \\ \deg(v_2) &= 2 \\ \deg(v_3) &= 2 \\ \deg(v_4) &= 2 \end{aligned}$$



$$A = [a_{ij}] = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

and

$$\begin{aligned} \deg(v_1) &= 1 \quad (\text{Sum of the entries in 1st row}) \\ \deg(v_2) &= 1 \\ \deg(v_3) &= 2 \\ \deg(v_4) &= 2 \end{aligned}$$



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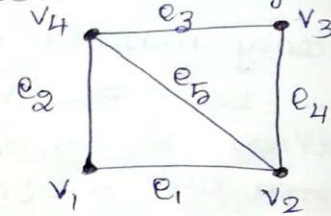
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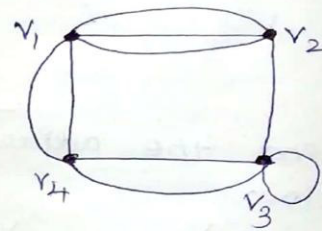


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HW
 1] Find the adjacency matrix of the following graph G. Hence find the degree of each vertex and also find A^2 & A^3 . What is your observations regarding the entries in A^2 and A^3 .



2] Obtain Adjacency matrix to represent the pseudograph



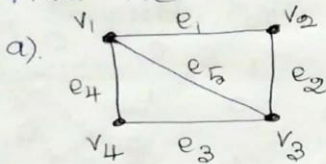
Incidence matrix:

Let $G=(V, E)$ be an undirected graph with n vertices $\{v_1, v_2, \dots, v_n\}$ and m edges $\{e_1, e_2, \dots, e_m\}$.

Then the $(n \times m)$ matrix $B = [b_{ij}]$ where

$$b_{ij} = \begin{cases} 1 & \text{when edge } e_j \text{ incident on } v_i \\ 0 & \text{otherwise} \end{cases}$$

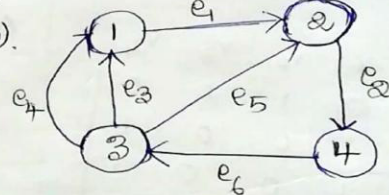
Find the incident matrix for the following graph.



Soln. Incidence matrix

$$B = [b_{ij}] = \begin{matrix} & e_1 & e_2 & e_3 & e_4 & e_5 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

b)



$$B = [b_{ij}] = \begin{matrix} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$



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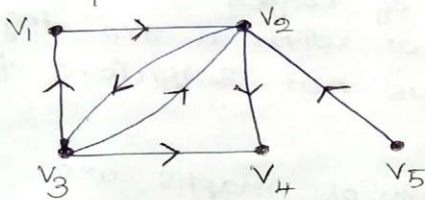
path matrix:

If $G = (V, E)$ be a simple digraph in which $|V| = n$ and the nodes of G are assumed to be ordered. An $n \times n$ matrix P whose elts. are given by,

$$P_{ij} = \begin{cases} 1 & \text{if there exists a path from } v_i \text{ to } v_j \\ 0 & \text{otherwise} \end{cases}$$

is called the path matrix (reachability matrix) of the graph G .

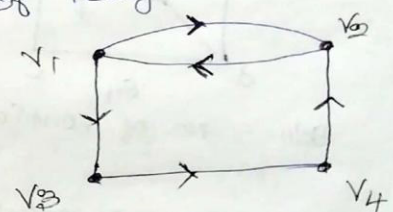
1. Find path matrix of



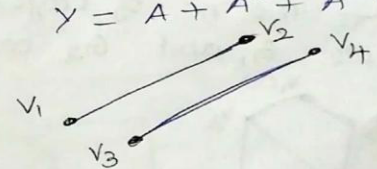
$$P = \begin{matrix} & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

Hw.

2. Consider the following digraph. Find the No. of possible elementary paths of length 3 from vertex $v_1 - v_2$.



3. Find the adjacency matrix of following graph also find $\gamma = A + A^2 + A^3 + A^4$



Problems:
1. Let $\delta(G)$ and $\Delta(G)$ denotes minimum and maximum degrees of all the vertices of G resly. Then show that for a non directed graph G .

$$\delta(G) \leq \frac{2|E|}{|V|} \leq \Delta(G)$$