UNIT 2 FOURIER SERIES
HALFRANGE SINE SERIES

Half Range Sine Series [Change of Interval]

The Half range sine series is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

where $b_n = \frac{2}{l}\int_0^l f(x) \sin \frac{n\pi x}{l}\, dx$

① $f(x) = x^2$ in the interval $(0, l)$

Halfrange sine series is :-

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \quad , \quad b_n = \frac{2}{l}\int_0^l f(x) \sin \frac{n\pi x}{l}\, dx$$

$$b_n = \frac{2}{l}\int_0^l x \sin\left(\frac{n\pi x}{l}\right)dx = \frac{2}{l}\left[x\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{(n\pi/l)}\right) - \left(\frac{-\sin\frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_0^l$$

$$= \frac{2}{l}\left[l\left[\frac{-\cos\frac{\pi n l}{l}}{n\pi/l}\right] - (0)\right] = \frac{2}{l}\left[\frac{-l^2}{n\pi}\cos n\pi\right]$$

$$= \frac{-2l}{n\pi}\cos n\pi = \frac{-2l}{n\pi}(-1)^n$$

∴ HR sine series is

$$f(x) = \frac{-2l}{\pi}\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{l}\right)x .$$

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② Express $f(x) = \frac{x(\pi - x)}{2}$, $0 < x < \pi$ as a Fourier series of periodicity 2π containing
1) Sine terms only 2) Cosine terms only.

Half Range Sine Series:

$$f(x) = x(\pi - x), \quad 0 < x < \pi$$

$$b_n = \frac{2}{\pi}\int_0^\pi x(\pi - x)\sin nx\, dx$$

$$= \frac{2}{\pi}\int_0^\pi \pi x \sin nx\, dx - \frac{2}{\pi}\int_0^\pi x^2 \sin nx\, dx$$

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$u = x \quad v = \sin nx$

$u' = 1 \quad v_1 = -\frac{\cos nx}{n}$

$u'' = 0 \quad v_2 = -\frac{\sin nx}{n^2}$

$v_3 = \frac{\cos nx}{n^3}$

$u = x^2 \quad v = \sin nx$

$u' = 2x \quad v_1 = -\frac{\cos nx}{n}$

$u'' = 2 \quad v_2 = -\frac{\sin nx}{n^2}$

$u''' = 0. \quad v_3 = \frac{\cos nx}{n^3}$

$$= 2\left[x\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^\pi$$

$$- \frac{2}{\pi}\left[x^2\left(-\frac{\cos nx}{n}\right) - 2x\left(-\frac{\sin nx}{n^2}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_0^\pi$$

$$= 2\left[-\frac{\pi\cos n\pi}{n}\right] - \frac{2}{\pi}\left[-\pi^2\frac{\cos n\pi}{n} + \frac{2\cos n\pi}{n^3} - \frac{2\cos 0}{n^3}\right]$$

$$= -2\pi\frac{(-1)^n}{n} - \frac{2}{\pi}\left[-\pi^2\frac{(-1)^n}{n} + 2\frac{(-1)^n}{n^3} - \frac{2}{n^3}\right]$$

$$= \cancel{-\frac{2\pi(-1)^n}{n}} + \cancel{\frac{2\pi(-1)^n}{n}} - \frac{4(-1)^n}{\pi n^3} + \frac{4}{\pi n^3}$$

$$= \frac{4}{n^3\pi}\left[1 - (-1)^n\right]$$

$$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{+8}{\pi n^3} & \text{if } n \text{ is odd.} \end{cases}$$

$$\therefore f(x) = \frac{+8}{\pi}\sum_{n=1}^{\infty}\frac{\sin nx}{n^3}.$$