UNIT 1 PARTIAL DIFFERENTIAL EQUATIONS
Solutions of standard types of first order partial differential equations

Solution of Standard types of first order PDE

A partial differential equation in which the partial derivative occurring are of the first degree is said to be linear, otherwise it is said to be non-linear.

Standard types:

Type 1 : $F(p, q) = 0$

Type 2 : $z = px + qy + f(p, q)$ [Clairaut's form]

Type 3 : $f(z, p, q) = 0$

Type 4 : $f_1(x, p) = f_2(y, q)$

Type 1 : Working Rule:

1. Let $z = ax + by + c$ be the Complete integral. $p = \frac{\partial z}{\partial x} = a, q = \frac{\partial z}{\partial y} = b$
2. put $b = \phi(a)$ for general solution
3. There is no singular integral

1. Solve $p + q = pq$

Soln: $p + q = pq \rightarrow (1)$

Let $z = ax + by + c \rightarrow (2)$

Complete Integral:

Diff partially wrt 'x' and y

$\frac{\partial z}{\partial x} = a \qquad \frac{\partial z}{\partial y} = b$

$p = a \qquad q = b$

Sub the above values in (1) we get

$a + b = ab$

$a = ab - b \qquad a = b(a-1) = b = \frac{a}{a-1}$

The complete Integral is,

$z = ax + \left(\frac{a}{a-1}\right) y + c \rightarrow (3)$

Singular Integral:

diff (3) p wrt 'a' and 'c' and equal to zero

$\frac{\partial z}{\partial a} = x + \left[\frac{(a-1)(1) - a(1)}{(a-1)^2}\right] y = 0 \quad , \quad \frac{\partial z}{\partial c} = 1 \neq 0.$

There is no singular Integral