



UNIT 2 FOURIER SERIES
ODD AND EVEN FUNCTIONS

Even function:-

A real function $f(x)$ is said to be even

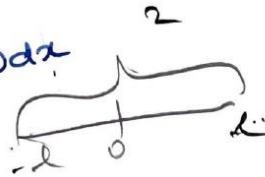
if

$$f(x) = f(-x)$$

$$f(x) = x^2, \quad f(-x) = (-x)^2 = x^2$$
$$f(x) = f(-x)$$

If $f(x)$ is an even function then

$$\int_{-l}^l f(x) dx = 2 \int_0^l f(x) dx$$



Odd function:-

A real function $f(x)$ is said to be odd

if

$$f(x) = -f(-x)$$

$$f(x) = x, \quad f(-x) = -x = (-1)x = -f(x)$$

If $f(x)$ is an odd function then

$$\int_{-l}^l f(x) dx = 0$$



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Note: If $f(x)$ does not satisfy even and odd functions then it is called neither even nor odd function.

Example:-

1. $f(x) = x^2$

$$f(-x) = (-x)^2 = x^2 = f(x)$$

It is Even function

$$\int_{-\pi}^{\pi} x^2 dx = \left[\frac{x^3}{3} \right]_{-\pi}^{\pi} = \frac{\pi^3}{3} - \frac{(-\pi)^3}{3} = \frac{\pi^3}{3} + \frac{\pi^3}{3} = \frac{2\pi^3}{3}$$

$$\int_{-\pi}^{\pi} x^2 dx = \left[\frac{x^3}{3} \right]_{-\pi}^{\pi} = \frac{\pi^3}{3} - \frac{(-\pi)^3}{3} = \frac{\pi^3}{3} + \frac{\pi^3}{3} = \frac{2\pi^3}{3}$$

2. $f(x) = x \cos x$

$$f(-x) = (-x) \cos(-x) = -x \cos x = -f(x)$$

It is odd function

$$\cos x = \cos x$$

$$\cos(-x) = \cos x$$

$$\sin(-x) = -\sin x$$

3. $f(x) = x \sin x \Rightarrow$ Even function

4. $f(x) = |x| \Rightarrow$ Even function

5. $f(x) = x + x^2 \Rightarrow$ Neither Even nor odd.

Note:

1. Even function \times Even fn = Even fn

2. odd fn \times odd fn = Even fn

3. Even fn \times odd fn = odd fn

4. odd fn \times Even fn = odd fn

* For even function, $b_n = 0$

* For odd function $a_0 = 0$ and $a_n = 0$



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① Find the Fourier series for the function $f(x) = |x|$

$$-\pi \leq x \leq \pi$$

$$f(x) = |x| = x$$

$$f(-x) = |-x| = x$$

$$f(x) = f(-x)$$

$\therefore f(x)$ is even function

$$\therefore b_n = 0.$$

The Fourier series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

To find a_0 :

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_0^{\pi} f(x) dx$$

$$\text{Since for even fn, } \int_{-l}^l f(x) dx = 2 \int_0^l f(x) dx$$

$$= \frac{2}{\pi} \int_0^{\pi} x dx$$

$$= \frac{2}{\pi} \left[\frac{x^2}{2} \right]_0^{\pi} = \frac{1}{\pi} [\pi^2]$$

$$a_0 = \pi$$



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TO find a_n :

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \cos nx \, dx = \frac{2}{\pi} \int_0^{\pi} x \cos nx \, dx$$

$$u = x$$

$$v = \cos nx$$

$$u' = 1$$

$$v_1 = \frac{\sin nx}{n}$$

$$u'' = 0$$

$$v_2 = -\frac{\cos nx}{n^2}$$

$$= \frac{2}{\pi} \left[x \frac{\sin nx}{n} - (1) \left(-\frac{\cos nx}{n^2} \right) \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[0 + \frac{\cos n\pi}{n^2} - \frac{\cos 0}{n^2} \right] = \frac{2}{\pi} \left[\frac{(-1)^n - 1}{n^2} \right]$$

$$= \frac{2}{\pi n^2} [(-1)^n - 1]$$

\therefore The Fourier series is

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi} [(-1)^n - 1] \cos nx.$$

Q. Find the Fourier series $f(x) = x$ on $(-\pi, \pi)$

$$f(x) = x$$

$$f(-x) = -x = -f(x)$$

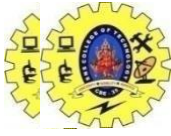
$$\therefore f(-x) = -f(x)$$

$\therefore f(x)$ is odd function

$$\therefore a_0 = 0 \text{ and } a_n = 0.$$

The Fourier series is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx.$$



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$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin nx \, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} x \sin nx \, dx \quad [\because x \sin x \text{ is even fn}]$$

$$u = x$$

$$v = \sin nx$$

$$u' = 1$$

$$v_1 = -\frac{\cos nx}{n}$$

$$u'' = 0$$

$$v_2 = -\frac{\sin nx}{n^2}$$

$$b_n = \frac{2}{\pi} \left[-x \frac{\cos nx}{n} - (1) \left(-\frac{\sin nx}{n^2} \right) \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[-\pi \frac{\cos n\pi}{n} + \frac{\sin n\pi}{n^2} - 0 + 0 \right]$$

$$= \frac{2}{\pi} \left[\frac{-\pi}{n} (-1)^n \right]$$

$$= \frac{-2}{n} (-1)^n$$

$$\therefore f(x) = \sum_{n=1}^{\infty} \frac{(-2)(-1)^n}{n} \sin nx.$$