PARTIAL DIFFERENTIAL EQUATIONS

Defn:

A partial differential equation is an equation involving a function of two or more variables and some of its partial derivatives. Therefore, a partial differential equation contains one dependent and more than one independent variable.

Let $z = f(x,y)$ is a function of x & y then z is the dependent variable and x, y are independent variable. The partial derivatives of z with respect to x and y are $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}$ and we shall use the following notations:

$$\frac{\partial z}{\partial x} = p, \quad \frac{\partial z}{\partial y} = q, \quad \frac{\partial^2 z}{\partial x^2} = r, \quad \frac{\partial^2 z}{\partial y^2} = t, \quad \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = s$$

ORDER OF PDE:

The order of a PDE is the order of a highest partial derivative occurring in the equation.

Eg: $\frac{\partial u}{\partial x} + \left(\frac{\partial u}{\partial y}\right)^2$, order is 1, Degree is 2.

Eg: $\left(\frac{\partial u}{\partial x}\right)^3 + \left(\frac{\partial^2 u}{\partial x^2}\right) + 2\left(\frac{\partial u}{\partial t}\right) = p$, order is 2, Degree is 1.

1) Form the partial diff. Eqn. by eliminating the arbitrary constants in $z = ax+by+a^2+b^2$

$z = ax+by+a^2+b^2$ ——— (1)

p.d.w.r.to x.

$\frac{\partial z}{\partial x} = a$

$p = a$.

p.d.w.r.to y

$\frac{\partial z}{\partial y} = b$

$q = b$.

Sub (a) & (b) in (1)

$z = px+qy+p^2+q^2$.

Arbitary Constant → A.C.
Indep. variable → I.V
A.C ≤ I.V then use p & q
A.C > I.V then use p, q, r, s, t

2) $z = (x^2+a)(y^2+b)$.

p.d.w.r.to x.

$\frac{\partial z}{\partial x} = 2x\,(y^2+b)$.

$p = 2x\,(y^2+b)$

$y^2+b = \frac{p}{2x}$.

p.d.w.r to y.

$$\frac{\partial z}{\partial y} = 2y.(x^2+a)$$

$$q = 2y(x^2+a)$$

$$\Rightarrow x^2+a = \frac{q}{2y}$$

$\therefore$ Eqn. becomes $z = \frac{p}{2x} \cdot \frac{q}{2y}$

$$\Rightarrow 4xyz = pq$$

) $z = (x-a)^2 (y-b)^2$

p.d.w.r. to x.

$$\frac{\partial z}{\partial x} = 2(x-a)(y-b)^2$$

$$p = 2(x-a)(y-b)^2$$

$$\frac{\partial^2 z}{\partial x^2} = 2(y-b)^2.$$

$$r = 2(y-b)^2 \Rightarrow (y-b)^2 = r/2.$$

p.d.w.r. to y.

$$\frac{\partial z}{\partial y} = 2(y-b)(x-a)^2.$$

$$\frac{\partial^2 z}{\partial y^2} = 2(x-a)^2$$

$$t = 2(x-a)^2 \Rightarrow (x-a)^2 = t/2$$

$\therefore$ Eqn becomes $4z = rt$.

) $z = f(x+at) + g(x-at)$

p.d.w.r to x

$$\frac{\partial z}{\partial x} = f'(x+at) + g'(x-at)$$

$$p = f'(x+at) + g'(x-at)$$

p.d.w.r to y

$$q = 0$$

p.d.w.r. to x

$$\frac{\partial^2 z}{\partial x^2} = f''(x+at) + g''(x-at)$$

$$r = f''(x+at) + g''(x-at)$$

p.d.w.r. to 't'

$$\frac{\partial z}{\partial t} = f'(x+at).a + g'(x-at)(-a)$$

$$q = f'(x+at)a - g'(x-at)a.$$

p.d.w.r. to 't'

$$\frac{\partial^2 z}{\partial t^2} = f''(x+at)a^2 + g''(x-at).a^2$$

$$t = f''(x+at)a^2 + g''(x-at)a^2$$

$$t = a^2[f''(x+at) + g''(x-at)]$$

$$\boxed{t = a^2 r}$$