THE PRINCIPLE OF INCLUSION - EXCLUSION :

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

(or)

$$|A \cup B| = |A| + |B| - |A \cap B|$$

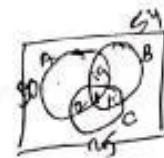$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |B \cap C \cap D| + |A \cap C \cap D| - |A \cap B \cap C \cap D|$$

1) In a survey of 100 students, it was found that 30 studied mathematics, 54 studied statistics, 25 studied Operation Research, 1 studied all the three subjects. 20 studied mathematics and statistics, 3 studied mathematics and operations research and 15 studied statistics and Operations Research.

(i) How many students studied none of these subjects?

(ii) How many students studied only mathematics?

soln: Let A denote the students who studied mathematics

Let B denote the students who studied statistics

Let C denote the students who studied OR.

Then $|A| = 30$; $|B| = 54$; $|C| = 25$

$|A \cap B| = 20$; $|A \cap C| = 3$; $|B \cap C| = 15$

$|A \cap B \cap C| = 1$

(i) No. of students studied none of these subjects $= 100 - |A \cup B \cup C|$

∴ By principle of Inclusion - Exclusion,

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$= 30 + 54 + 25 - 20 - 3 - 15 + 1$

$= 72$

∴ No. of students studied none of these $\left.\begin{array}{c} \\ \text{subjects} \end{array}\right\} = 100 - 72$

$= 28$

(ii) No. of students studied only $\left.\begin{array}{c} \\ \text{Maths & statistics} \end{array}\right\} = n(A \cap B) - n(A \cap B \cap C)$

$= 20 - 1 = 19$

No. of students studied only $\left.\begin{array}{c} \\ \text{Maths & OR} \end{array}\right\} = n(A \cap C) - n(A \cap B \cap C)$

$= 3 - 1 = 2$

Then, No. of students studied $\left.\begin{array}{c} \\ \text{only mathematics} \end{array}\right\} = 30 - 19 - 2$

$= 9$

② How many positive integers not exceeding 1000 are divisible by 7 or 11?

Let A denote the set of +ve integers not exceeding 1000 that are divisible by 7.

Let B denote the set of +ve integers not exceeding 1000 that are divisible by 11.

Then $|A| = \left[\dfrac{1000}{7}\right] = \lceil 142.8 \rceil = 142$.

$|B| = \left[\dfrac{1000}{11}\right] = [90.9] = 90$

$$|A \cap B| = \left\lceil \frac{1000}{7 \times 11} \right\rceil = [12.9] = 12.$$

∴ No. q +ve integer not exceeding 1000 that are divisible by either 7 or 11 is $|A \cup B|$

By principle of Inclusion - Exclusion,

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= 142 + 90 - 12 = 220.$$

③ Find the no. q integers between 1 to 250 that are not divisible by any q the integers 2, 3, 5 & 7.

Sltn: Let A denote the integer from 1 to 250 that are divisible by 2

Let B " " " " 3

Let C " " " " 5

Let D " " " " 7

$$|A| = \left\lceil \frac{250}{2} \right\rceil = 125$$

$$|B| = \left\lceil \frac{250}{3} \right\rceil = 83$$

$$|C| = \left\lceil \frac{250}{5} \right\rceil = 50$$

$$|D| = \left\lceil \frac{250}{7} \right\rceil = 35$$

Now, the no. q integer btwn 1 to 250 that are divisible by 2 & 3 $= |A \cap B| = \left\lceil \frac{250}{2 \times 3} \right\rceil$
$$= 41$$

the no. q integer divisible by 2 & 5 $= |A \cap C| = \left\lceil \frac{250}{2 \times 5} \right\rceil$
$$= 25$$

Similarly, $|A \cap D| = \left[\dfrac{250}{2 \times 7}\right] = 17$

$|B \cap C| = \left[\dfrac{250}{3 \times 5}\right] = 16$

$|B \cap D| = \left[\dfrac{250}{3 \times 7}\right] = 11$

$|C \cap D| = \left[\dfrac{250}{5 \times 7}\right] = 7$

∴ No. q integers divisible by 2,3 & 5 $= |A \cap B \cap C|$

$\qquad = \left[\dfrac{250}{2 \times 3 \times 5}\right] = 8$

Similarly, $|A \cap B \cap D| = \left[\dfrac{250}{2 \times 3 \times 7}\right] = 5$

$|A \cap C \cap D| = \left[\dfrac{250}{2 \times 5 \times 7}\right] = 3$

$|B \cap C \cap D| = \left[\dfrac{250}{3 \times 5 \times 7}\right] = 2$

∴ $|A \cap B \cap C \cap D| = \left[\dfrac{250}{2 \times 3 \times 5 \times 7}\right] = 1$

∴ No. q integers b'ton. 1–250 that are divisible by integers 2, 3, 5 & 7 is $|A \cup B \cup C \cup D|$

∴ By principle Inclusion- Exclusion ,

$|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| - |A \cap B \cap C \cap D|$

$= 125 + 83 + 50 + 35 - 41 - 25 - 17 - 16 - 11 - 7 + 8 + 5 + 3 + 2 - 1$

$= 193$

∴ No. q integers b'ton. 1 to 250 that are not divisible by 2, 3, 5 & 7 is $250 - |A \cup B \cup C \cup D|$

$\qquad = 250 - 193 = 57$