Subject Code & Name: **19AST302 FLIGHT DYNAMICS** Date: 10.8.2024

DAY: **21** TOPIC: **THE V-n DIAGRAM**

6.5 THE V–n DIAGRAM

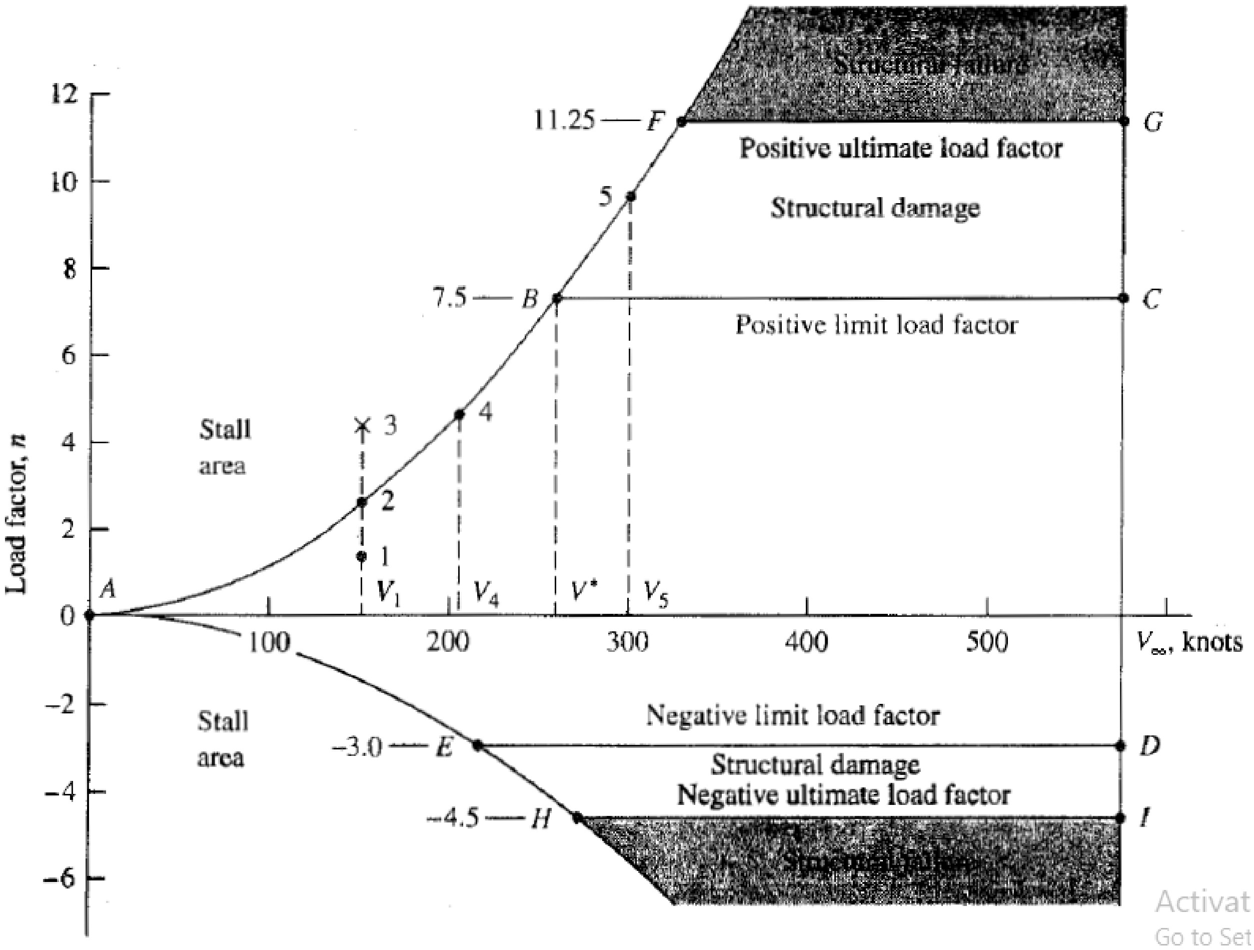
There are structural limitations on the maximum load factor allowed for a given airplane. These structural limitations were not considered in the previous sections; let us examine them now.

There are two categories of structural limitations in airplane design:

1. *Limit load factor*. This is the boundary associated with *permanent* structural deformation of one or more parts of the airplane. If n is less than the limit load factor, the structure may deflect during a maneuver, but it will return to its original state when $n = 1$. If n is greater than the limit load factor, then the airplane structure will experience a permanent deformation, that is, it will incur *structural damage*.
2. *Ultimate load factor*. This is the boundary associated with outright *structural failure*. If n is greater than the ultimate load factor, parts of the airplane will break.

Both the aerodynamic and structural limitations for a given airplane are illustrated in the *V–n diagram*, a plot of load factor versus flight velocity, as given in Fig. 6.7. A V–n diagram is a type of "flight envelope" for a given airplane; it establishes the maneuver boundaries. Let us examine Fig. 6.7 in greater detail.

The curve between points A and B in Fig. 6.7 represents the aerodynamic limit on load factor imposed by $(C_L)_{max}$. This curve is literally a plot of Eq. (6.23). The region above curve AB in the V–n diagram is the stall region. To understand the significance of curve AB better, consider an airplane flying at velocity V_1, where V_1 is shown in Fig. 6.7. Assume the airplane is at an angle of attack such that $C_L < (C_L)_{max}$. This flight condition is represented by point 1 in Fig. 6.7. Now assume the angle of attack is increased to that for $(C_L)_{max}$, keeping the velocity constant at V_1. The lift increases to its maximum value for the given V_1, and hence the local factor $n = L/W$ reaches its maximum value for the given V_1. This value of n_{max} is given by Eq. (6.23), and the corresponding flight condition is given by point 2 in Fig. 6.7. If the angle of attack is increased further, the wing stalls and the load factor decreases. Therefore, point 3 in Fig. 6.7 is unobtainable in flight. Point 3 is in the stall region of the V–n diagram. Consequently, point 2 represents the highest possible load factor that can be obtained



at the given velocity V_1. As V_∞ is increased, say, to a value of V_4, then the maximum possible load factor n_{max} also increases, as given by point 4 in Fig. 6.7. However, n_{max} cannot be allowed to increase indefinitely. It is constrained by the structural limit load factor, given by point B in Fig. 6.7.

The horizontal line BC denotes the *positive limit load factor* in the V–n diagram. The flight velocity corresponding to B is designated as V^*. At velocities higher than V^*, say, V_5, the airplane must fly at values of C_L less than $(C_L)_{max}$ so that the positive limit load factor is not exceeded. If flight at $(C_L)_{max}$ is obtained at velocity V_5, corresponding to point 5 in Fig. 6.7, then structural damage or possibly structural failure will occur. The right-hand side of the V–n diagram, line CD, is a high-speed limit. At flight velocities higher than this limit (to the right of line CD), the dynamic pressure is higher than the design range for the airplane. This will exacerbate the consequences of other undesirable phenomena that may occur in high-speed flight, such as encountering a critical gust and experiencing destructive flutter, aileron reversal, wing or surface divergence, and severe compressibility buffeting. Any one of these phenomena in combination with the high dynamic pressure could cause structural damage or failure. The high-speed limit velocity is a *red-line speed* for the airplane; it should never be exceeded. By design, it is higher than the level flight maximum cruise velocity V_{max}, determined in Chapter 5, by at least a factor of 1.2. It may be as high as the terminal dive velocity of the aircraft. The bottom part of the V–n diagram, given by curve AE and the horizontal line ED in Fig. 6.7, corresponds to negative absolute angles of attack, that is, negative lift, and hence the load factors are negative quantities. Curve *AE* defines the stall limit. (If the wing is pitched downward to a large enough negative angle of attack, the flow will separate from the bottom surface of the wing and the negative lift will decrease in magnitude; that is, the wing "stalls.") Line *ED* gives the negative limit load factor, beyond which structural damage will occur. Line *HI* gives the negative ultimate load factor beyond which structural failure will occur.

For instantaneous maneuver performance, point *B* on the V–n diagram in Fig. 6.7 is very important. This point is called the *maneuver point*. At this point, both C_L and n are simultaneously at their highest possible values that can be obtained anywhere throughout the allowable flight envelope of the airplane. In turn, from Eqs. (6.52) and (6.53), this point simultaneously corresponds to the smallest possible instantaneous turn radius and the largest possible instantaneous turn rate for the airplane. The velocity corresponding to point *B* is called the *corner velocity* and is designated by V^* in Fig. 6.7. The corner velocity can be obtained by solving Eq. (6.23) for velocity, yielding

$$V^* = \sqrt{\frac{2n_{\max}}{\rho_\infty (C_L)_{\max}} \frac{W}{S}} \qquad \textbf{[6.54]}$$

In Eq. (6.54), the value of $n_{\max}$ corresponds to that at point *B* in Fig. 6.7. The corner velocity is an interesting dividing line. At flight velocities less than V^*, it is not possible to structurally damage the airplane due to the generation of too much lift. In contrast, at velocities greater than V^*, lift can be obtained that can structurally

damage the aircraft (e.g., point 5 in Fig. 6.7), and the pilot must make certain to avoid such a case.

If the values of δ_e and α_s are substituted into equation (6–42), the final stick force equation results.

$$F_s = -GS_e c_e \frac{1}{2}\rho V^2 \eta_t \left[C_{h0} + C_{h_\alpha}\left(\alpha_0 + \frac{C_L}{a_w}\left(1 - \frac{d\epsilon}{d\alpha}\right) - i_w + i_t\right) + C_{h\delta}\left(\delta_{e0} - \left(\frac{dC_m}{dC_L}\right)_{\text{Fix}} \frac{C_L}{C_{m\delta}}\right) + C_{h\delta_t}\delta_t\right] \quad (6\text{–}45)$$

letting $\quad K = -GS_e c_e \eta_t$

and $\quad A = C_{h0} + C_{h\alpha}(\alpha_0 - i_w + i_t) + C_{h\delta}\delta_{e0}$

Equation (6–45) becomes

$$F_s = K\frac{1}{2}\rho V^2\left[A + \frac{C_{h\alpha}C_L}{a_w}\left(1 - \frac{d\epsilon}{d\alpha}\right) - \frac{C_{h\delta}}{C_{m\delta}}C_L\left(\frac{dC_m}{dC_L}\right)_{\text{Fix}} + C_{h\delta_t}\delta_t\right] \quad (6\text{–}46)$$

Rearranging

$$F_s = K\frac{1}{2}\rho V^2\left[A + C_{h\delta_t}\delta_t - C_L\left(\frac{dC_m}{dC_L}\right)_{\text{Free}}\frac{C_{h\delta}}{C_{m\delta}}\right] \quad (6\text{–}47)$$

for unaccelerated flight, $C_L = \dfrac{2W/S}{\rho V^2}$, substitution into (6–47) gives

$$F_s = K\frac{1}{2}\rho V^2(A + C_{h\delta_t}\delta_t) - K\frac{W}{S}\frac{C_{h\delta}}{C_{m\delta}}\left(\frac{dC_m}{dC_L}\right)_{\text{Free}} \quad (6\text{–}48)$$

Equation (6–48) brings out the interesting fact that the stick force variation with speed is dependent on the first term only and independent in general of the stability level. The slope of the stick force versus speed curve is simply

$$\frac{dF_s}{dV} = K\rho V(A + C_{h\delta_t}\delta_t) \quad (6\text{–}49)$$

A plot of elevator stick force, F_s, versus velocity is shown in Figure 6–18 and is made up of a constant force springing from the second or stability term of equation (6–48) plus a variable force proportional to the velocity squared, introduced through the constant A and the tab term $C_{h\delta_t}\delta_t$.

For a given center of gravity, then, a stable or negative value of the stability criterion (stick-free) will introduce a constant pull force, while an unstable value will introduce a push force. It can be seen from

Figure 6–18 that an airplane possessing stick-free stability will require a nose-up tab setting to trim out the stick force ($F_s = 0$) for a given trim speed, and the resultant variation of stick force with air speed will be stable. If $(dC_m/dC_L)_{\text{Free}}$ is unstable, then in order to trim the airplane out at the given trim speed a nose-down tab is required, giving an unstable variation of stick force with air speed. In other words the tab creates the required slope, but the static stability criterion stick-free is essential to allow the tab to move in a stable direction for trim. It is important to notice again that a stable slope is of interest only if equilibrium or trim is established.

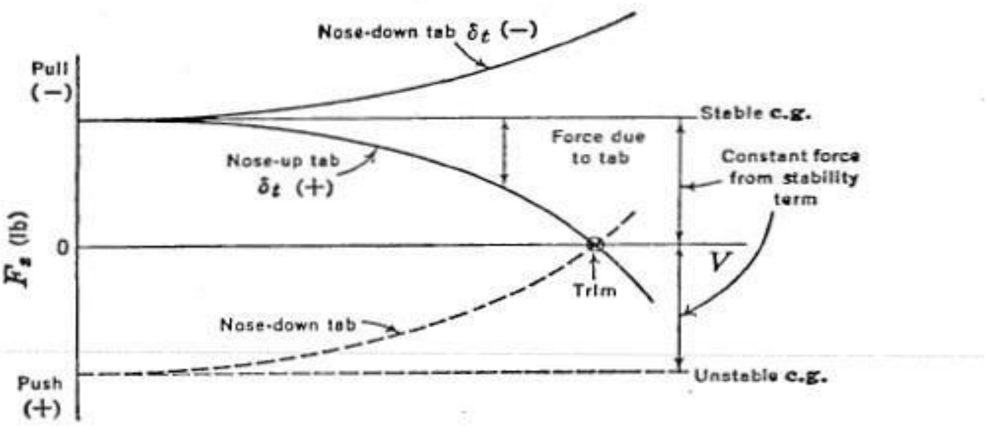


FIGURE 6–18. Stick force build-up.

From this discussion it can be seen that the stability criterion $(dC_m/dC_L)_{\text{Free}}$ plays an important but rather complex role in establishment of the flight condition of a stable stick force variation with speed.

It is interesting to note how explicitly $(dC_m/dC_L)_{\text{Free}}$ can be brought into the picture, if it is required that the trim tab always be deflected to trim the airplane out ($F_s = 0$) to a given speed (V_{Trim}).

The value of $C_{h\delta_t}\delta_t$ for this trim condition can be obtained from equation (6–48) by substituting V_{Trim} for V and equating F_s to zero.

$$C_{h\delta_t}\delta_t = \frac{2W/S}{\rho {V_{\text{Trim}}}^2} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{dC_m}{dC_L}\right)_{\text{Free}} - A \qquad (6\text{–}50)$$

Substituting (6–50) into (6–48) gives

$$F_s = K \frac{W}{S} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{dC_m}{dC_L}\right)_{\text{Free}} \left(\frac{V^2}{{V_{\text{Trim}}}^2} - 1\right) \qquad (6\text{–}51)$$

and

$$\frac{dF_s}{dV} = 2K \frac{W}{S} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{dC_m}{dC_L}\right)_{\text{Free}} \frac{V}{{V_{\text{Trim}}}^2} \qquad (6\text{–}52)$$

The slope when $V = V_{\text{Trim}}$ will be

$$\frac{dF_s}{dV} = 2K \frac{W}{S} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{dC_m}{dC_L}\right) \frac{1}{V_{\text{Trim}}} \tag{6-53}$$

Equation (6–53) indicates that the slope F_s versus V varies with c.g position if the tab is rolled to maintain the trim speed (V_{Trim}), the slope becoming more stable as the c.g. is moved forward, and more

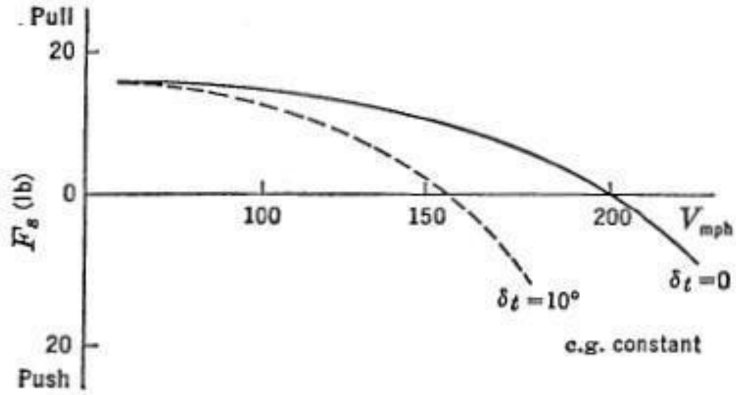


FIGURE 6–19. Stick force versus velocity for different tab angles.

unstable as the c.g. is moved aft. Equation (6–53) also shows that the slope dF_s/dV varies inversely with the trim speed, being higher at the lower speeds. See Figure 6–19.