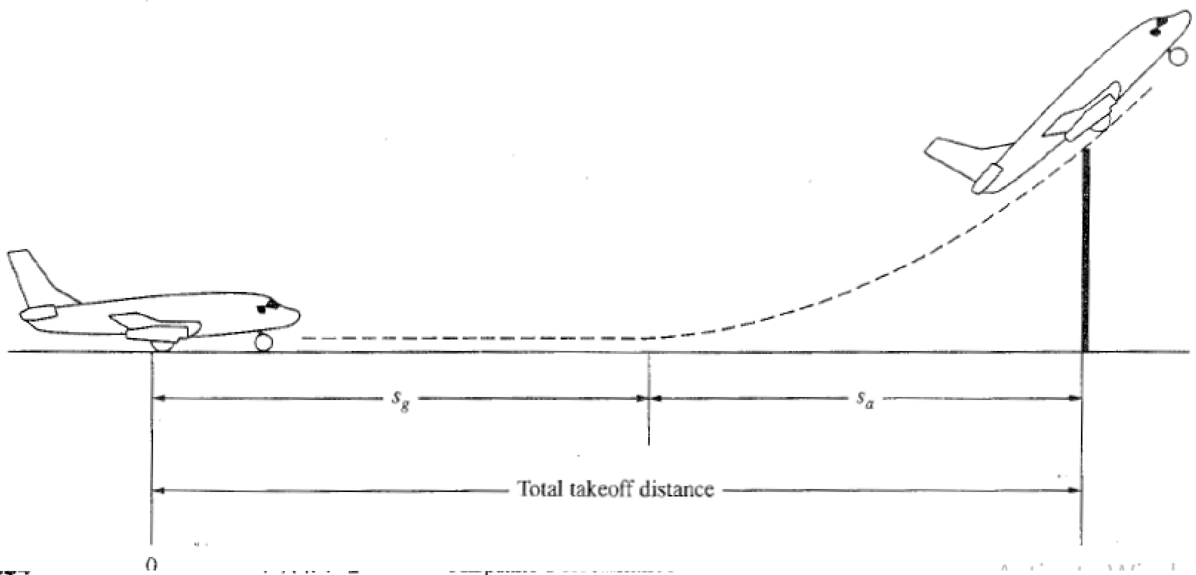
6.7 TAKEOFF PERFORMANCE

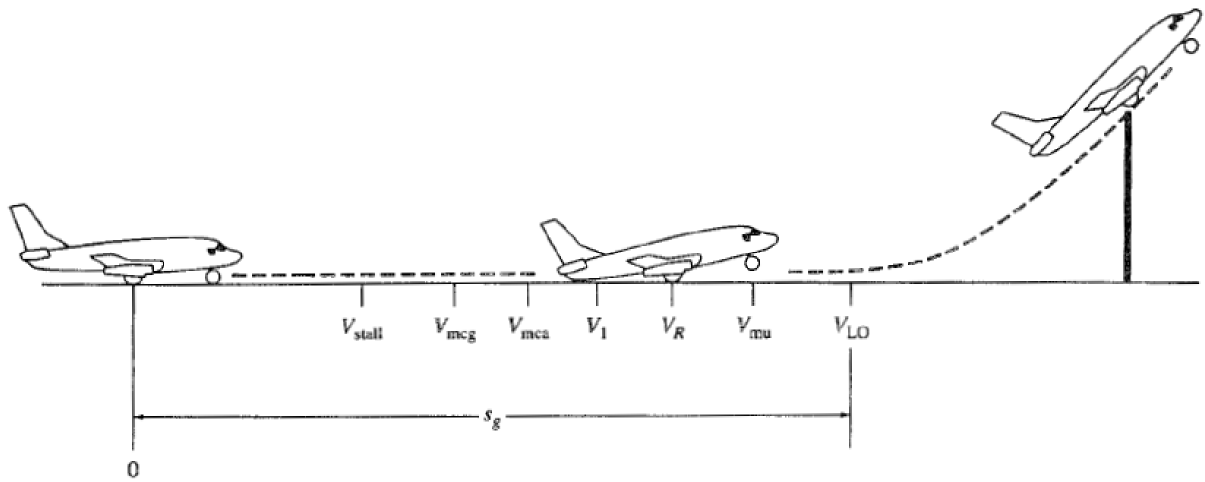
For the performance characteristics discussed so far in this book, we have considered the airplane in full flight in the air. However, for the next two sections, we come back to earth, and we explore the characteristics of takeoff and landing, many of which are concerned with the airplane rolling along the ground. These are accelerated performance problems of a special nature.

Consider an airplane standing motionless at the end of a runway. This is denoted by location 0 in Fig. 6.12. The pilot releases the brakes and pushes the throttle to maximum takeoff power, and the airplane accelerates down the runway. At some distance from its starting point, the airplane lifts into the air. How much distance does the airplane cover along the runway before it lifts into the air? This is the central question in the analysis of takeoff performance. Called the *ground roll* (or sometimes the *ground run*) and denoted by s_g in Fig. 6.12, it is a major focus of this section. However, this is not the whole consideration. The total takeoff distance also includes the extra distance covered over the ground after the airplane is airborne but before it clears an obstacle of a specified height. This is denoted by s_a in Fig. 6.12. The height of the obstacle is generally specified to be 50 ft for military aircraft and 35 ft for commercial aircraft. The sum of s_g and s_a is the total takeoff distance for the airplane.

The ground roll s_g is further divided into intermediate segments, as shown in Fig. 6.13. These segments are defined by various velocities, as follows:

1. As the airplane accelerates from zero velocity, at some point it will reach the stalling velocity V_{stall}, as noted in Fig. 6.13.


s_g
s_a
Total takeoff distance
0


V_{stall}
V_{mcg}
V_{mca}
V_1
V_R
V_{mu}
V_{LO}
s_g
0

2. The airplane continues to accelerate until it reaches the *minimum control speed on the ground*, denoted by V_{mcg} in Fig. 6.13. This is the minimum velocity at which enough aerodynamic force can be generated on the vertical fin with rudder deflection while the airplane is still rolling along the ground to produce a yawing moment sufficient to counteract that produced when there is an engine failure for a multiengine aircraft.

3. If the airplane were in the air (without the landing gear in contact with the ground), the minimum speed required for yaw control in case of engine failure is slightly greater than V_{mcg}. This velocity is called the *minimum control speed in the air*, denoted by V_{mca} in Fig. 6.13. For the ground roll shown in Fig. 6.13, V_{mca} is essentially a reference speed—the airplane is still on the ground when this speed is reached.

4. The airplane continues to accelerate until it reaches the *decision speed*, denoted by V_1 in Fig. 6.13. This is the speed at which the pilot can successfully continue the takeoff even though an engine failure (in a multiengine aircraft) would occur at that point. This speed must be equal to or larger than V_{mcg} in order to maintain control of the airplane. A more descriptive name for V_1 is the *critical engine failure speed*. If an engine fails before V_1 is achieved, the takeoff must be stopped. If an engine fails after V_1 is reached, the takeoff can still be achieved.

5. The airplane continues to accelerate until the takeoff rotational speed, denoted by V_R in Fig. 6.13, is achieved. At this velocity, the pilot initiates by elevator deflection a rotation of the airplane in order to increase the angle of attack, hence to increase C_L. Clearly, the maximum angle of attack achieved during rotation should not exceed the stalling angle of attack. Actually, all that is needed is an angle of attack high enough to produce a lift at the given velocity larger than the weight, so that the airplane will lift off the ground. However, even this angle of attack may not be achievable because the tail may drag the ground. (Ground clearance for the tail after rotation is an important design feature for the airplane, imposed by takeoff considerations.)

6. If the rotation of the airplane is limited by ground clearance for the tail, the airplane must continue to accelerate while rolling along the ground after rotation is achieved, until a higher speed is reached where indeed the lift becomes larger than the weight. This speed is called the *minimum unstick speed*, denoted by V_{mu} in Fig. 6.13. For the definition of V_{mu}, it is assumed that the angle of attack achieved during rotation is the maximum allowable by the tail clearance.

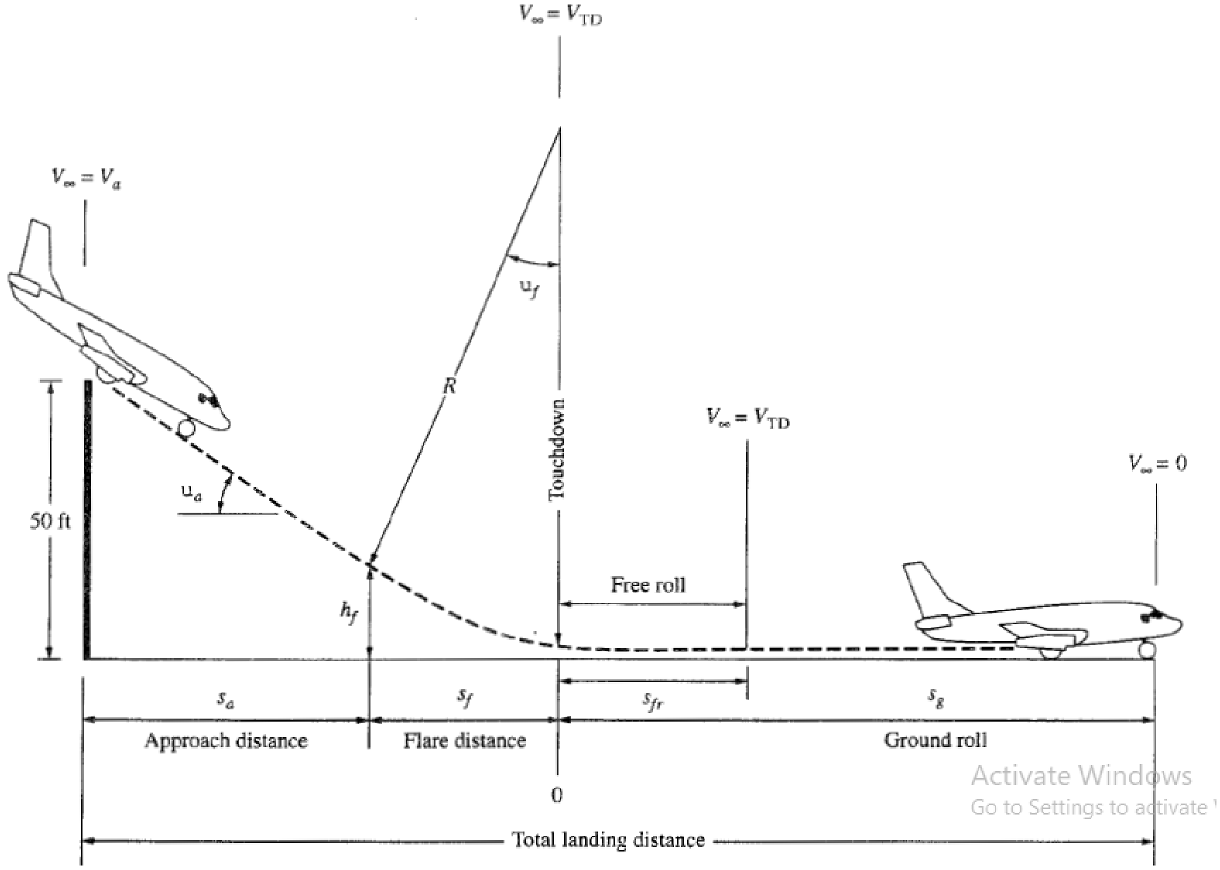
7. However, for increased safety, the angle of attack after rotation is slightly less than the maximum allowable by tail clearance, and the airplane continues to accelerate to a slightly higher velocity, called the *liftoff speed*, denoted by V_{LO} in Fig. 6.13. This is the point at which the airplane actually lifts off the ground. The total distance covered along the ground to this point is the ground roll s_g.

The relative values of the various velocities discussed above, and noted on Fig. 6.13, are all sandwiched between the value of V_{stall} and that for V_{LO}, where usually $V_{LO} \approx 1.1V_{stall}$. A nice discussion of the relative values of the velocities noted in Fig. 6.13 is contained in Ref. 41, which should be consulted for more details.

Related to the above discussion is the concept of *balanced field length*, defined as follows. The decision speed V_1 was defined earlier as the minimum velocity at which the pilot can successfully continue the takeoff even though an engine failure would occur at that point. What does it mean that the pilot "can successfully continue the takeoff" in such an event? The answer is that when the airplane reaches V_1, if an engine fails at that point, then the additional distance required to clear the obstacle at the end of takeoff is exactly the same distance as required to bring the airplane to a stop on the ground. If we let A be the distance traveled by the airplane along the ground from the original starting point (point 0 in Fig. 6.13) to the point where V_1 is reached, and we let B be the additional distance traveled with an engine failure (the same distance to clear an obstacle or to brake to a stop), then the balanced field length is by definition the *total* distance $A + B$.

6.8 LANDING PERFORMANCE

The analysis of the landing performance of an airplane is somewhat analogous to that for takeoff, only in reverse. Consider an airplane on a landing approach. The landing distance, as sketched in Fig. 6.17, begins when the airplane clears an obstacle, which is taken to be 50 ft in height. At that instant the airplane is following a straight approach path with angle θ_a, as noted in Fig. 6.17. The velocity of the airplane at the instant it clears the obstacle, denoted by V_a, is required to be equal to $1.3V_{stall}$ for commercial airplanes and $1.2V_{stall}$ for military airplanes. At a distance h_f above the ground, the airplane begins the flare, which is the transition from the straight approach path to the horizontal ground roll. The flight path for the flare can be considered a circular arc with radius R, as shown in Fig. 6.17. The distance measured along the ground from the obstacle to the point of initiation of the flare is the approach distance s_a. Touchdown occurs when the wheels touch the ground. The distance over the ground covered during the flare is the flare distance s_f. The velocity at the touchdown V_{TD} is $1.15V_{stall}$ for commercial airplanes and $1.1V_{stall}$ for military airplanes. After touchdown, the airplane is in free roll for a few seconds before the pilot applies the brakes and/or thrust reverser. The free-roll distance is short enough that the velocity over this length is assumed constant, equal to V_{TD}. The distance that the airplane rolls on the ground from touchdown to the point where the velocity goes to zero is called the *ground roll* s_g.



6.8.1 Calculation of Approach Distance

Examining Fig. 6.17, we see that the approach distance s_a depends on the approach angle θ_a and the flare height h_f. In turn, θ_a depends on T/W and L/D. This can be seen from Fig. 6.18, which shows the force diagram for an aircraft on the approach flight path. Assuming equilibrium flight conditions, from Fig. 6.18,

$$L = W \cos\theta_a \qquad [6.101]$$

$$D = T + W \sin\theta_a \qquad [6.102]$$

From Eq. (6.102),

$$\sin\theta_a = \frac{D - T}{W} = \frac{D}{W} - \frac{T}{W} \qquad [6.103]$$

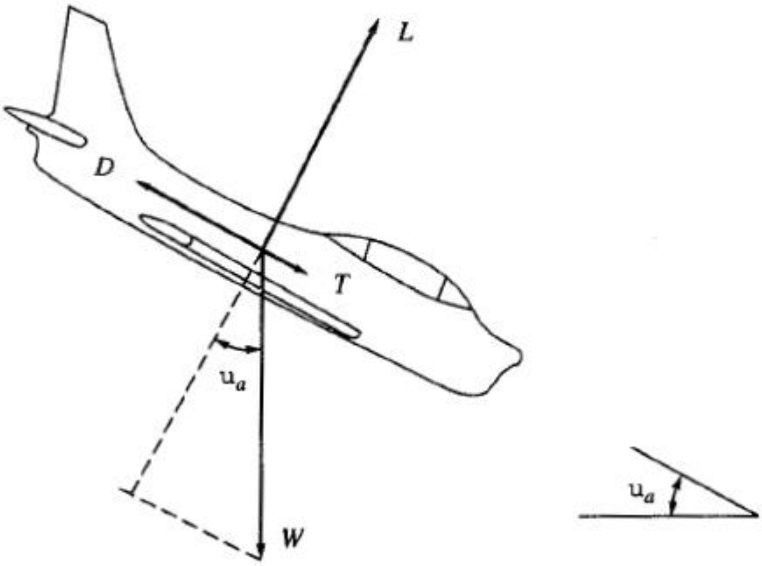


Figure 6.18 Force diagram for an airplane on the landing approach flight path.

The approach angle is usually small for most cases. For example, Raymer (Ref. 25) states that for transport aircraft $\theta_a \leq 3°$. Hence, $\cos\theta_a \approx 1$ and from Eq. (6.101), $L \approx W$. In this case, Eq. (6.103) can be written as

$$\sin\theta_a = \frac{1}{L/D} - \frac{T}{W} \qquad \textbf{[6.104]}$$

The flare height h_f, shown in Fig. 6.17, can be calculated from the construction shown in Fig. 6.19 as follows.

If the values of δ_e and α_s are substituted into equation (6–42), the final stick force equation results.

$$F_s = -GS_e c_e \frac{1}{2}\rho V^2 \eta_t \left[C_{h_0} + C_{h_\alpha}\left(\alpha_0 + \frac{C_L}{a_w}\left(1 - \frac{d\epsilon}{d\alpha}\right) - i_w + i_t\right) + C_{h_\delta}\left(\delta_{e0} - \left(\frac{dC_m}{dC_L}\right)_{\text{Fix}} \frac{C_L}{C_{m_\delta}}\right) + C_{h_{\delta_t}}\delta_t \right] \quad (6\text{–}45)$$

letting $\quad K = -GS_e c_e \eta_t$

and $\quad A = C_{h_0} + C_{h_\alpha}(\alpha_0 - i_w + i_t) + C_{h_\delta}\delta_{e0}$

Equation (6–45) becomes

$$F_s = K\frac{1}{2}\rho V^2 \left[A + \frac{C_{h_\alpha} C_L}{a_w}\left(1 - \frac{d\epsilon}{d\alpha}\right) - \frac{C_{h_\delta}}{C_{m_\delta}} C_L \left(\frac{dC_m}{dC_L}\right)_{\text{Fix}} + C_{h_{\delta_t}}\delta_t \right] \quad (6\text{–}46)$$

Rearranging

$$F_s = K\frac{1}{2}\rho V^2 \left[A + C_{h_{\delta_t}}\delta_t - C_L \left(\frac{dC_m}{dC_L}\right)_{\text{Free}} \frac{C_{h_\delta}}{C_{m_\delta}} \right] \quad (6\text{–}47)$$

for unaccelerated flight, $C_L = \dfrac{2W/S}{\rho V^2}$, substitution into (6–47) gives

$$F_s = K\frac{1}{2}\rho V^2 (A + C_{h_{\delta_t}}\delta_t) - K\frac{W}{S}\frac{C_{h_\delta}}{C_{m_\delta}}\left(\frac{dC_m}{dC_L}\right)_{\text{Free}} \quad (6\text{–}48)$$

Equation (6–48) brings out the interesting fact that the stick force variation with speed is dependent on the first term only and independent in general of the stability level. The slope of the stick force versus speed curve is simply

$$\frac{dF_s}{dV} = K\rho V (A + C_{h_{\delta_t}}\delta_t) \quad (6\text{–}49)$$

A plot of elevator stick force, F_s, versus velocity is shown in Figure 6–18 and is made up of a constant force springing from the second or stability term of equation (6–48) plus a variable force proportional to the velocity squared, introduced through the constant A and the tab term $C_{h_{\delta_t}}\delta_t$.

For a given center of gravity, then, a stable or negative value of the stability criterion (stick-free) will introduce a constant pull force, while an unstable value will introduce a push force. It can be seen from

Figure 6–18 that an airplane possessing stick-free stability will require a nose-up tab setting to trim out the stick force ($F_s = 0$) for a given trim speed, and the resultant variation of stick force with air speed will be stable. If $(dC_m/dC_L)_{\text{Free}}$ is unstable, then in order to trim the airplane out at the given trim speed a nose-down tab is required, giving an unstable variation of stick force with air speed. In other words the tab creates the required slope, but the static stability criterion stick-free is essential to allow the tab to move in a stable direction for trim. It is important to notice again that a stable slope is of interest only if equilibrium or trim is established.

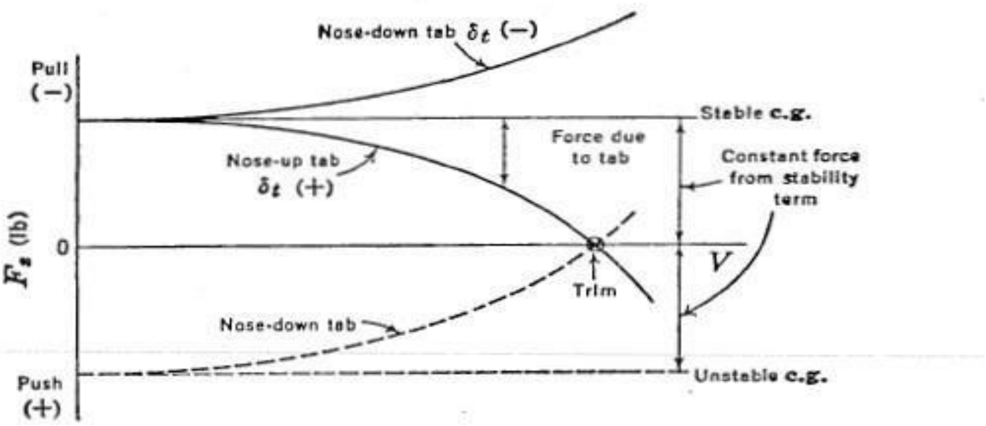


FIGURE 6–18. Stick force build-up.

From this discussion it can be seen that the stability criterion $(dC_m/dC_L)_{\text{Free}}$ plays an important but rather complex role in establishment of the flight condition of a stable stick force variation with speed.

It is interesting to note how explicitly $(dC_m/dC_L)_{\text{Free}}$ can be brought into the picture, if it is required that the trim tab always be deflected to trim the airplane out ($F_s = 0$) to a given speed (V_{Trim}).

The value of $C_{h\delta_t}\delta_t$ for this trim condition can be obtained from equation (6–48) by substituting V_{Trim} for V and equating F_s to zero.

$$C_{h\delta_t}\delta_t = \frac{2W/S}{\rho V_{\text{Trim}}^2} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{dC_m}{dC_L}\right)_{\text{Free}} - A \qquad (6\text{–}50)$$

Substituting (6–50) into (6–48) gives

$$F_s = K \frac{W}{S} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{dC_m}{dC_L}\right)_{\text{Free}} \left(\frac{V^2}{V_{\text{Trim}}^2} - 1\right) \qquad (6\text{–}51)$$

and

$$\frac{dF_s}{dV} = 2K \frac{W}{S} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{dC_m}{dC_L}\right)_{\text{Free}} \frac{V}{V_{\text{Trim}}^2} \qquad (6\text{–}52)$$

The slope when $V = V_{\text{Trim}}$ will be

$$\frac{dF_s}{dV} = 2K \frac{W}{S} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{dC_m}{dC_L}\right) \frac{1}{V_{\text{Trim}}} \tag{6-53}$$

Equation (6–53) indicates that the slope F_s versus V varies with c.g position if the tab is rolled to maintain the trim speed (V_{Trim}), the slope becoming more stable as the c.g. is moved forward, and more

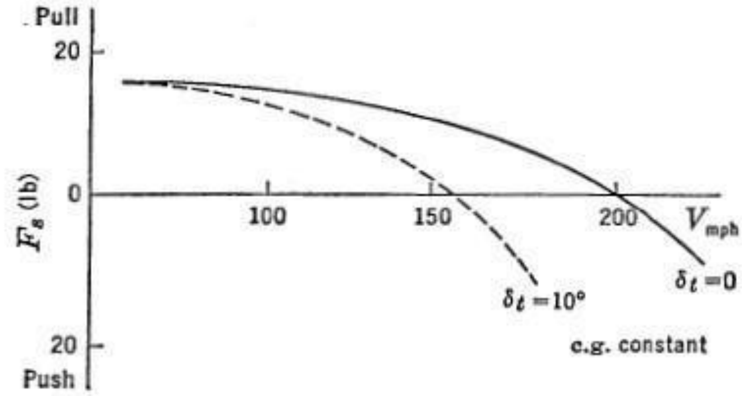


FIGURE 6–19. Stick force versus velocity for different tab angles.

unstable as the c.g. is moved aft. Equation (6–53) also shows that the slope dF_s/dV varies inversely with the trim speed, being higher at the lower speeds. See Figure 6–19.