



UNIT 1 PARTIAL DIFFERENTIAL EQUATIONS

Solutions of standard types of first order partial differential equations

Type - iii $f(z, p, q) = 0$.

1. Solve: $p(1+q) = qz$

Soln: $p(1+q) = qz \Rightarrow \textcircled{1}$

Let $u = x + ay$

Then $p = \frac{dz}{du}$ and $q = a \frac{dz}{du}$

$\textcircled{1} \Rightarrow \frac{dz}{du} (1 + a \frac{dz}{du}) = a \frac{dz}{du} z$

$$1 + a \frac{dz}{du} = az$$

$$a \frac{dz}{du} = az - 1$$

$$\frac{dz}{du} = \frac{az-1}{a} \Rightarrow \frac{du}{dz} = \frac{a}{az-1}$$

$$du = \frac{a}{az-1} dz$$

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Integrating, $\int du = \int \frac{a}{az-1} dz$
 $u = \log(az-1) + \log c$
 $x+ay = \log(c(az-1))$

2) Solve: $z^2 = 1+p^2+q^2$

Sol: $z^2 = 1+p^2+q^2 \rightarrow \textcircled{1}$

Let $u = x+ay$ $p = \frac{dz}{du}$, $q = a \frac{dz}{du}$

$\textcircled{1} \Rightarrow z^2 = 1 + \left(\frac{dz}{du}\right)^2 + \left(a \frac{dz}{du}\right)^2$

$z^2 = 1 + \left(\frac{dz}{du}\right)^2 (1+a^2)$

$z^2 - 1 = \left(\frac{dz}{du}\right)^2 (1+a^2)$

$\left(\frac{dz}{du}\right)^2 = \frac{z^2-1}{1+a^2} \Rightarrow \frac{dz}{du} = \sqrt{\frac{z^2-1}{1+a^2}} = \frac{\sqrt{z^2-1}}{\sqrt{1+a^2}}$

$\frac{dz}{\sqrt{1-z^2}} = \frac{du}{\sqrt{1+a^2}}$

Integrating on both sides

$\cosh^{-1} z = \frac{1}{\sqrt{1+a^2}} u + c = \frac{1}{\sqrt{1+a^2}} (x+ay) + c$

Type-IV $f_1(x, p) = f_2(y, q)$

for this type, there is no singular integral.

1. Solve $q^2 - p = y - x$

Given: $q^2 - p = y - x = k$ (constant)

Now, $q^2 - y = k$

$q^2 = k + y$

$q = \sqrt{k+y}$

$p - x = k$

$p = k + x$