SNS College of Technology, Coimbatore – 641035
(An Autonomous Institution)

UNIT- I

PARTIAL DIFFERENTIAL EQUATIONS

PART-B

(TWO MARKS)

1. Find the PDE of all planes having equal intercepts on the x and y axis.

Ans: The equation of such planes is $\frac{x}{a}+\frac{y}{a}+\frac{z}{c}=1$ …… (1)

Partially differentiating (1) w.r.to x and y, we get

$\frac{1}{a}+\frac{p}{b}=0$ …… (2)

$\frac{1}{a}+\frac{q}{b}=0$ …… (3)

$$\left.\begin{aligned}(2)\Rightarrow \frac{p}{b}&=\frac{-1}{a}\\ (3)\Rightarrow \frac{q}{b}&=\frac{-1}{a}\end{aligned}\right\} \text{.....(4)}$$

From (4) , we get $\frac{p}{b}=\frac{q}{b}$

$p = q$

2. Form the PDE by eliminating a and b from $z=(x^2+a^2)(y^2+b^2)$

Ans: Given $z=(x^2+a^2)(y^2+b^2)$ …….. (1)

$p=\frac{\partial z}{\partial x}=2x(y^2+b^2)$ …..… (2)

$q=\frac{\partial z}{\partial y}=2y(x^2+a^2)$ …….. (3)

$(2)\Rightarrow y^2+b^2=\frac{p}{2x}$ …… (4)

$(3)\Rightarrow x^2+a^2=\frac{q}{2y}$ …… (5)

Sub (4) and (5) in (1), we get

$z=\frac{q}{2y}\frac{p}{2x}$

pq=4xyz

3. Eliminate the arbitrary function f from $z = f\left(\frac{xy}{z}\right)$ **and from PDE**

Ans: Given $z = f\left(\frac{xy}{z}\right)$

$$p = f'\left(\frac{xy}{z}\right)\frac{zy - xyp}{z^2} \quad \ldots\ldots\ldots (1)$$

$$q = f'\left(\frac{xy}{z}\right)\frac{zx - xyq}{z^2} \quad \ldots\ldots\ldots (2)$$

From (1) we get

$$f'\left(\frac{xy}{z}\right) = \frac{pz^2}{zy - xyp} \quad \ldots\ldots\ldots (3)$$

Sub (3) in (2) ,we get

$$q = \frac{pz^2}{zy - xyp}\frac{(zx - xyq)}{z^2}$$

4. Find the solution of $px^2 + qy^2 = z^2$

Ans: The S.E is $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2}$

Taking I two members ,we get

$$\frac{dx}{x^2} = \frac{dy}{y^2}$$

Integrating we get

$$\frac{-1}{x} = \frac{-1}{y} + c_1$$

$$u\left(\frac{1}{y} - \frac{1}{x}\right) = c_1$$

Taking last two members ,we get

$$\frac{dy}{y^2} = \frac{dz}{z^2}$$

Integrating we get

$$\frac{-1}{y} = \frac{-1}{z} + c_2$$

$$v\left(\frac{1}{z} - \frac{1}{y}\right) = c_2$$

The complete solution is

$$\phi\left(\frac{1}{y}-\frac{1}{x},\frac{1}{z}-\frac{1}{y}\right)=0$$

5. Find the singular integral of the partial differential equation $z=px+qy+p^2-q^2$

Ans: The complete solution is $z=ax+by+a^2-b^2$ …….. (1)

Now $\left.\begin{aligned}\frac{\partial z}{\partial a}=x+2a=0\Rightarrow a=\frac{-x}{2}\\ \frac{\partial z}{\partial b}=y-2b=0\Rightarrow b=\frac{y}{2}\end{aligned}\right\}$ .....(2)

Sub (2) in (1) , we get

$$z=\frac{-x^2}{2}+\frac{y^2}{2}+\frac{x^2}{4}-\frac{y^2}{4}$$

$$=\frac{-x^2}{4}+\frac{y^2}{4}$$

$y^2-x^2=4z$ which is S.I

6.Form the PDE by eliminating the arbitrary constants from $z=(x-a)^2+(y-b)^2+1$

Ans: $z=(x-a)^2+(y-b)^2+1$ ……. (1)

$$p=\frac{\partial z}{\partial x}=2(x-a) \quad \text{…… (2)}$$

$$q=\frac{\partial z}{\partial y}=2(y-b) \quad \text{…… (3)}$$

Sub (2) and (3) in (1) we get

$$z=\frac{p^2}{4}+\frac{q^2}{4}+1$$

7.Write the complete solution of p+q=x+y

Ans: Let p+q=x+y=k

p-x=k , y-q=k

p=k+x q=y-k

$$z=\int p\,dx+\int q\,dy$$

$$z=\int (k+x)\,dx+\int (y-k)\,dy$$

$$=\frac{(k+x)^2}{2}+\frac{(y-k)^2}{2}+c$$

8.Find the partial integral of $(D^2-2DD'+D'^2)z=e^{x-y}$

Ans: $P.I = \frac{1}{D^2 - 2DD' + D'^2} e^{x-y}$

$= \frac{1}{4} e^{x-y}$

9. **Find the complete integral of q=2px**

Ans: q=2px

$xp = \frac{q}{2} = k$

$p = \frac{k}{x}, \qquad q = 2k$

$z = \int p\, dx + \int q\, dy$

$z = k \log x + 2ky + c$

10.Solve $(D^3 - 3DD'^2 + 2D'^3)z = 0$

Ans: A.E is $(m^2 - 3m + 2) = 0$

(m-1)(m-1)(m+2)=0

m=1,1,-2

$z = f_1(y + x) + xf_2(y + x) + f_3(y - 2x)$

11.Find the complete solution of the PDE $\sqrt{p} + \sqrt{q} = 1$

Ans: Given $\sqrt{p} + \sqrt{q} = 1$ ……. (1)

Let z = ax+by+c …… (2) be the solution of (1)

$\left.\begin{aligned} \frac{\partial z}{\partial x} = p = a \\ \frac{\partial z}{\partial y} = q = b \end{aligned}\right\}$ .....(3)

Sub (3) in (1)

$\sqrt{a} + \sqrt{b} = 1$ ……. (4)

z = ax+by+c is a solution of (1)

From (4) $\sqrt{a} = 1 - \sqrt{b}$ ……(5)

Sub (5) in (2)

$z = (1 - \sqrt{b})^2 x + by + c$ which is complete integral

12. Find the Complete integral of the partial differential equation $z = px + qy + p^2 + q^2$

Ans: Sub p = a and q = b

$z = ax + by + a^2 + b^2$

13. Eliminate the arbitrary function f from $z = f\left(\frac{x}{y}\right)$ **and from PDE**

Ans: Given $z = f\left(\frac{x}{y}\right)$

$p = f'\left(\frac{x}{y}\right)\frac{1}{y}$ …….. (1)

$q = f'\left(\frac{x}{y}\right)\frac{-x}{y^2}$ …….. (2)

Sub (1) in (2) ,we get

$q = py\left(\frac{-x}{y^2}\right)$

$px + qy = 0$

14.Solve $(D^2 - 3DD' + 2D'^2)z = 0$

Ans: A.E is $m^2 - 3m + 2 = 0$

m=1,2

$z = f_1(y + x) + f_2(y + 2x)$

15.Find the PDE by eliminating the arbitrary constants in $z = a(x + y) + b$

Ans: $z = a(x + y) + b$ ….. (1)

Partially differentiating (1) w.r.to 'x' and 'y' we get

$\frac{\partial z}{\partial x} = p = a$ …… (2)

$\frac{\partial z}{\partial y} = q = a$ …… (3)

From (2) and (3) , p = q

16. Solve $(D^3 - 2D^2D')z = 0$

Ans: A.E is $m^3 - 2m^2 = 0$

m=0,0,2

$$z = f_1(y) + xf_2(y) + f_3(y+2x)$$

17.Find the complete solution of pq=xy

Ans: pq=xy

$$\frac{p}{x} = \frac{y}{q}$$

Let $\frac{p}{x} = \frac{y}{q} = k$

$$p = kx \,, q = \frac{y}{k}$$

$$z = \int p\,dx + \int q\,dy$$

$$= \int kx\,dx + \int \frac{y}{k}dy$$

$$= k\frac{x^2}{2} + \frac{1}{k}\frac{y^2}{2} + c$$

18. Solve $\frac{\partial^2 z}{\partial x^2} - \frac{4\partial^2 z}{\partial x\,\partial y} + \frac{4\partial^2 z}{\partial y^2} = 0$

Ans: A.E is $m^2 - 4m + 4 = 0$

$$(m-2)^2 = 0$$

m=2,2

$$z = f_1(y+2x) + 2f_2(y+2x)$$

20. Write Particular integral of $\frac{\partial^2 z}{\partial x^2} - \frac{5\partial^2 z}{\partial x\,\partial y} + \frac{6\partial^2 z}{\partial y^2} = e^{x+y}$

Ans: $P.I = \frac{1}{D^2 - 5DD' + 6D'^2} e^{x+y}$ Sub $D = 1$ and $D' = 1$

$$= \frac{1}{1-5+6} e^{x+y}$$

$$= \frac{1}{2} e^{x+y}$$

21. Write Particular integral of $(D^2 + 3DD' + 2D'^2)z = \sin(x+5y)$

Ans: $P.I = \frac{1}{D^2 + 3DD' + 2D'^2} \sin(x+5y)$ Sub $D^2 = -1$, $DD' = -5$ and $D'^2 = -25$

$$= \frac{1}{-1-15-50} \sin(x+5y)$$

$$= \frac{1}{-66} \sin(x+5y)$$

22.Find the singular Integral of $z = px+qy+p^2$

Ans: Complete solution is $z = ax+by+a^2$ …….. (1)

$\frac{\partial z}{\partial a} = x + 2a = 0$ …….. (2)

$\frac{\partial z}{\partial b} = y = 0$ …….. (3)

From (2) we get

$x = -2a$ (or) $a = -x/2$ ……. (4)

Sub (3) and (4) in (1) , we get

$$z = \frac{-x^2}{2} + \frac{x^2}{4} = \frac{-x^2}{4}$$

$4z = -x^2$

23. **Write Particular integral of** $(D^2 - DD')z = \sin(x+y)$

Ans: $P.I = \frac{1}{D^2 - DD'}\sin(x+y)$ Sub $D^2 = -1$, $DD' = -1$

$$= \frac{1}{-1+1}\sin(x+y)$$

$$= \frac{x}{2D}\sin(x+y)$$

$$= \frac{-x}{2}\cos(x+y)$$

24. **Write Particular integral of** $(D^2 + 2DD' + D'^2)z = e^{(x-y)}$

Ans: $P.I = \frac{1}{D^2 + 2DD' + D'^2}e^{(x-y)}$ Sub $D = 1$, $D' = -1$

$$= \frac{1}{1-2+1}e^{(x-y)}$$

$$= \frac{x}{2D + 2D'}e^{(x-y)}$$

$$= \frac{x^2}{2}e^{(x-y)}$$

25.Find the complete solution of p-q=0

Ans: p-q=0

This is of the type F(p,q)=0 …… (1)

Let z=ax+by+c ….. (2) be the solution of PDE

From (2) we get p=a , q=b ……. (3)

Sub (3) in (1), we get

a-b=0 $\Rightarrow$ a=b ……. (4)

Sub (4) in (2) , we get z= a(x+y)+c

UNIT-III

PARTIAL DIFFERENTIAL EQUATIONS

PART-C

1.		Solve $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$	
2.		Solve $(D^2+DD'-6D'^2)z=y\cos x$	
3.		Solve $z=px+qy+\sqrt{p^2+q^2+1}$	
4.		Solve $(D^3-7DD'^2-6D'^3)z=\sin(2x+y)$	
5.		Find the singular integral of $z=px+qy+p^2+pq+q^2$	
6.		Solve the PDE $(x-2z)p+(2z-y)q=y-x$	
7.		Solve $(D^2+3DD'-4D'^2)z=\cos(2x+y)+xy$	
8.		Solve $(D^2-DD'+2D)z=e^{2x+y}+4$	
9.		Solve (mz - ny) p + (nx – lz) q=ly - mx	
10.		Solve $(D^3+D^2D'-DD'^2-D'^3)z=e^x\cos 2y$	
11.		Solve p(1+q)=qz	
12.		Solve $(D^2-DD'-30D'^2)z=xy+e^{2x+y}$	
13.		Solve $(3z-4y)p+(4x-2z)q=2y-3x$	
14.		From the PDE by eliminating the arbitrary function f & g from z=f(x+ct)+g(x-ct)	
15.		Solve $z=1+p^2+q^2$	
16.		From the PDE by eliminating the arbitrary function ϕ from $\phi(x^2+y^2+z^2, ax+by+cz)=0$	
17.		Solve $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$	
18.		Solve $(D^3+D^2D'-4DD'^2-4D'^3)z=\cos(2x+y)$	

19.		Solve $(2D^2 - DD' - D'^2 + 6D + 3D')z = xe^y$	
20.		Solve $(x - y)p + (y - x - z)q = z$	
21		Solve $(1 + y)p + (1 + x)q = z$	
22.		Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$	
23.		Solve $(D^2 + 4DD' - 5D'^2)z = x + y^2 + \pi$	
24.		Solve $(D^2 - 6DD' + 5D'^2)z = e^x \sinh y + xy$	
25.		Solve $(D^3 - 4D^2D' + 4DD'^2)z = 6\sin(3x + 6y)$	
26.		Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} = \cos 2x \cos y$	
27.		Solve $(D^2 + 4DD' - 5D'^2)z = e^{2x-y} + \sin(x - 2y)$	
28.		Solve $(D^2 - 2DD' + D'^2)z = x^2y^2e^{x+y}$	