UNIT 3 APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS
ONE DIMENSIONAL EQUATION OF HEAT CONDUCTION

One dimensional Heat Equation [parabolic]

One DHE [Rod]

The one dimensional heat equation is

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

where $a^2 = \frac{\text{Thermal conductivity}}{\text{Density} \times \text{Specific Heat}}$

(i) $a^2 = \frac{k}{\rho c}$ which is called diffusivity

Possible Solutions of ODHE

1) $u(x,t) = (A_1 e^{px} + A_2 e^{-px}) A_3 e^{a^2p^2t}$

2) $u(x,t) = (A_4 \cos px + A_5 \sin px) A_6 e^{-a^2p^2t}$

3) $u(x,t) = (A_7 x + A_8) A_9$

Suitable solution:

$u(x,t) = (A\cos px + B\sin px) e^{-a^2p^2t}$

Assumptions of deriving the ODHE:

1. Heat flows from higher to lower temperature
2. The amount of heat required to produce a given temperature change in a body is proportional to the mass of the body, and to the temperature change.
3. The rate at which heat flows through an area is proportional to the area and to the temperature gradient normal to the curve.

This constant of proportionality is known as the thermal conductivity (k) of the material. It is known as fourier's law of heat conduction.