UNI II - QUANTITATIVE ABILITY II

Problems on Ages,

Problem 1

If the sum of the ages of a father and his son is 50 years and the father's age is five times the son's age, what are their respective ages?

Solution:

Let the son's age be (x). Then the father's age is (5x).

According to the problem:

[x + 5x = 50]

[6x = 50]

[x = frac{50}{6} approx 8.33]

So, the son's age is approximately 8.33 years and the father's age is (5 times 8.33 = 41.67) years. However, ages are usually whole numbers, so let's assume there's a mistake or reframe the problem with whole numbers.

Problem 2:

A mother is 24 years older than her daughter. In 8 years, the mother will be twice as old as her daughter. How old are they now?

Solution:

Let the daughter's current age be (x). The mother's age is (x + 24).

In 8 years:

[(x + 24) + 8 = 2(x + 8)]

[x + 32 = 2x + 16]

[32 - 16 = 2x - x]

[16 = x]

So, the daughter is 16 years old, and the mother is (16 + 24 = 40) years old.

Problem 3:

Five years ago, a father was 7 times as old as his son. In 5 years, the father will be 3 times as old as his son. What are their current ages?

Solution:

Let the son's current age be (x). Then the father's age is (7x) (from 5 years ago).

Five years ago:

[7x - 5 = 7(x - 5)]

In 5 years:

[7x + 5 = 3(x + 5)]

[7x + 5 = 3x + 15]

[4x = 10]

[x = 2.5]

So, the son is 2.5 years old and the father is (7 times 2.5 = 17.5) years old.

Problem 4:

The age of a grandfather is twice that of his grandson. In 12 years, the grandfather will be 3 times as old as the grandson. How old are they now?

Solution:

Let the grandson's age be (x). Then the grandfather's age is (2x).

In 12 years:

[2x + 12 = 3(x + 12)]

[2x + 12 = 3x + 36]

[12 - 36 = 3x - 2x]

[-24 = x]

This indicates an issue with the problem framing, as ages cannot be negative.

Problem 5:

A brother is twice as old as his sister. In 10 years, he will be one and a half times as old as she will be. How old are they now?

Solution:

Let the sister's current age be (x). Then the brother's age is (2x).

In 10 years:

[2x + 10 = 1.5(x + 10)]

[2x + 10 = 1.5x + 15]

[2x - 1.5x = 15 - 10]

[0.5x = 5]

[x = 10]

So, the sister is 10 years old and the brother is (2 times 10 = 20) years old.

Problems on Averages

Problem 1:

Find the average of the following set of numbers: 12, 15, 20, 25, and 30.

Solution:

To find the average, sum up the numbers and then divide by the count of numbers.

Sum = 12 + 15 + 20 + 25 + 30 = 102

Count = 5

Average = Sum / Count = 102 / 5 = 20.4

Problem 2:

The average of three numbers is 20. If two of the numbers are 18 and 22, find the third number.

Solution:

Let the third number be (x).

The average of the three numbers is given by:

[frac{18 + 22 + x}{3} = 20]

Solving for (x):

[18 + 22 + x = 60]

[40 + x = 60]

[x = 20]

The third number is 20.

Problem 3:

The average age of a group of 10 people is 30 years. If a new person whose age is 40 years joins the group, what will be the new average age?

Solution:

Initial total age = Average age × Number of people = 30 × 10 = 300 years

Total age after adding the new person = 300 + 40 = 340 years

New number of people = 10 + 1 = 11

New average age = Total age / Number of people = 340 / 11 ≈ 30.91 years

Problem 4:

A student scored 75, 85, and 90 in three exams. If the student wants to have an average of 85 after a fourth exam, what score does the student need on the fourth exam?

Solution:

Let the score of the fourth exam be (x).

The average of four exams should be 85, so:

[frac{75 + 85 + 90 + x}{4} = 85]

Solving for (x):

[75 + 85 + 90 + x = 340]

[250 + x = 340]

[x = 90]

The student needs to score 90 on the fourth exam.

Problem 5:

If the average of five numbers is 12, what is the total sum of these numbers?

Solution:

Average = Total sum / Number of items

Total sum = Average × Number of items = 12 × 5 = 60

Problems on Clocks

Problem 1:

What is the angle between the hour and minute hands of a clock at 3:15?

Solution:

At 3:00, the hour hand is at 90 degrees (3 hours × 30 degrees/hour).

By 3:15, the hour hand has moved further. It moves at a rate of 0.5 degrees per minute (30 degrees/hour ÷ 60 minutes/hour).

In 15 minutes, the hour hand moves:

[15 times 0.5 = 7.5 text{ degrees}]

So at 3:15, the hour hand is at:

[90 + 7.5 = 97.5 text{ degrees}]

The minute hand at 15 minutes is at:

[15 times 6 = 90 text{ degrees}]

The angle between them is:

[|97.5 - 90| = 7.5 text{ degrees}]

Problem27:

How many degrees does the hour hand move in 1 hour?

Solution:

The hour hand moves 360 degrees in 12 hours.

So, in 1 hour:

[frac{360}{12} = 30 text{ degrees}]

Problem 3:

What is the angle between the hour and minute hands at 7:30?

Solution:

At 7:00, the hour hand is at:

[7 times 30 = 210 text{ degrees}]

By 7:30, the hour hand has moved further. It moves 0.5 degrees per minute.

In 30 minutes, the hour hand moves:

[30 times 0.5 = 15 text{ degrees}]

So at 7:30, the hour hand is at:

[210 + 15 = 225 text{ degrees}]

The minute hand at 30 minutes is at:

[30 times 6 = 180 text{ degrees}]

The angle between them is:

[|225 - 180| = 45 text{ degrees}]

Problem 4:

At what time between 2:00 and 3:00 is the angle between the hour and minute hands exactly 15 degrees?

Solution:

Let (t) be the number of minutes past 2:00.

The angle of the hour hand from 12:00 is:

[60 + 0.5t text{ degrees}]

The angle of the minute hand from 12:00 is:

[6t text{ degrees}]

The angle between them is:

$$|60 + 0.5t - 6t| = 15$$

Solving for t:

$$|60 - 5.5t| = 15$$

This results in two equations:

1. $60 - 5.5t = 15$

   $$5.5t = 45$$

   $$t = \frac{45}{5.5} \approx 8.18 \text{ minutes}$$

2. $60 - 5.5t = -15$

   $$5.5t = 75$$

   $$t = \frac{75}{5.5} \approx 13.64 \text{ minutes}$$

So, the times are approximately 2:08 and 2:14.

Problem 5:

How many times do the hour and minute hands overlap in a 24-hour period?

Solution:

In a 12-hour period, the hands overlap 11 times. This is because in each hour, they overlap once except for the 11th hour, where the hands overlap at 12:00, which is counted once at the start of the next period.

Therefore, in 24 hours:

$$11 \times 2 = 22 \text{ times}$$

UNI II - QUANTITATIVE ABILITY II

Calender

Problem 1: Day of the Week Calculation

Determine the day of the week for March 15, 2024.

Solution:

We can use Zeller's Congruence formula to find the day of the week. Here's the formula:

[h = left(q + leftlfloor frac{13(m + 1)}{5} rightrfloor + K + leftlfloor frac{K}{4} rightrfloor + leftlfloor frac{J}{4} rightrfloor - 2J right) mod 7]

where:

- (h) is the day of the week (0 = Saturday, 1 = Sunday, ..., 6 = Friday),

- (q) is the day of the month,

- (m) is the month (March = 3),

- (K) is the year of the century,

- (J) is the zero-based century.

For March 15, 2024:

- (q = 15),

- (m = 3),

- (K = 24),

- (J = 20).

Plugging into the formula:

[h = left(15 + leftlfloor frac{13 times (3 + 1)}{5} rightrfloor + 24 + leftlfloor frac{24}{4} rightrfloor + leftlfloor frac{20}{4} rightrfloor - 2 times 20 right) mod 7]

[h = left(15 + leftlfloor frac{52}{5} rightrfloor + 24 + 6 + 5 - 40 right) mod 7]

[h = left(15 + 10 + 24 + 6 + 5 - 40 right) mod 7]

[h = 20 mod 7]

[h = 6]

So, March 15, 2024, is a Friday.

Problem 2: Number of Days Between Dates

How many days are there between August 1, 2023, and November 1, 2023?

Solution:

Count the days in each month:

- August: 31 days

- September: 30 days

- October: 31 days

- November 1: 1 day

Total days:

[31 + 30 + 31 + 1 = 93 text{ days}]

Problem 3: Find the Number of Leap Years Between Two Years

Determine the number of leap years between 2000 and 2020, inclusive.

Solution:

A year is a leap year if:

1. It is divisible by 4.2. If it is divisible by 100, it must also be divisible by 400.

Count the leap years between 2000 and 2020:

- 2000 (divisible by 400)

- 2004 (divisible by 4 but not 100)

- 2008 (divisible by 4 but not 100)

- 2012 (divisible by 4 but not 100)

- 2016 (divisible by 4 but not 100)

So, there are 5 leap years between 2000 and 2020, inclusive.

Problem 4: Day of the Week for a Given Date

What day of the week was July 4, 1776?

Solution:

Use Zeller's Congruence:

For July 4, 1776:

- (q = 4)

- (m = 5) (July is treated as the 7th month of the previous year, so for the formula, we use (m = 5))

- (K = 76)

- (J = 17)

Plugging into the formula:

[h = left(4 + leftlfloor frac{13 times (5 + 1)}{5} rightrfloor + 76 + leftlfloor frac{76}{4} rightrfloor + leftlfloor frac{17}{4} rightrfloor - 2 times 17 right) mod 7]

[h = left(4 + leftlfloor frac{78}{5} rightrfloor + 76 + 19 + 4 - 34 right) mod 7]

[h = left(4 + 15 + 76 + 19 + 4 - 34 right) mod 7]

[h = 84 mod 7]

[h = 0]

So, July 4, 1776, was a Thursday.

Problem 5: Number of Days in a Month

How many days are there in the month of February in a leap year?

Solution:

In a leap year, February has 29 days. This is because leap years have an extra day in February compared to non-leap years.

Problem 6: Determine the Day of the Week for a Future Date

What day of the week will be on November 22, 2030?

Solution:

Using Zeller's Congruence:

For November 22, 2030:

- (q = 22)

- (m = 11)

- (K = 30)

- (J = 20)

Plugging into the formula:

[h = left(22 + leftlfloor frac{13 times (11 + 1)}{5} rightrfloor + 30 + leftlfloor frac{30}{4} rightrfloor + leftlfloor frac{20}{4} rightrfloor - 2 times 20 right) mod 7]

[h = left(22 + leftlfloor frac{156}{5} rightrfloor + 30 + 7 + 5 - 40 right) mod 7]

[h = left(22 + 31 + 30 + 7 + 5 - 40 right) mod 7]

[h = 55 mod 7]

[h = 6]

So, November 22, 2030, will be a Friday.