Puzzle 1: The Seven Bridges of Königsberg

The city of Königsberg (now Kaliningrad, Russia) was set on both sides of the Pregel River, and included two large islands connected to each other and to the mainland by seven bridges. The residents of the city wondered whether it was possible to walk through the city, crossing each of the seven bridges exactly once and returning to the starting point.

The layout of the bridges can be represented as a graph, where the land areas are vertices, and the bridges are edges connecting those vertices.

Puzzle Question:

1. **Is it possible to start at any point in the city, cross each of the seven bridges exactly once, and return to the starting point?**
2. **If not, explain why this is impossible using graph theory concepts.**

Puzzle 2: The Handshake Problem

You are at a party where some people shake hands with each other. Represent this scenario as a graph, where each person at the party is a vertex, and an edge exists between two vertices if those two people shook hands.

Puzzle Question:

1. **Prove that in any group of people, the number of people who shook hands with an odd number of others is always even.**
2. **Explain this result using the concept of graph theory, particularly focusing on vertex degrees.**