



**SNS COLLEGE OF TECHNOLOGY**  
**An Autonomous Institution**  
**Coimbatore-35**



Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade  
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

**DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING**

**23ECB221 – DIGITAL ELECTRONICS**

II YEAR/ III SEMESTER

**UNIT 1 – MINIMIZATION TECHNIQUES AND LOGIC GATES**

TOPIC – KARNAUGH MAP



## KARNAUGH MAP



- The Karnaugh map technique provides a systematic method for simplifying and manipulation of Boolean expressions.
- A K-map is a diagram made up of squares, with each square representing one min term of the function that is to be minimized. For  $n$  variables on a Karnaugh map there are  $2^n$  numbers of squares.
- Each square or cell represents one of the min terms. It can be drawn directly from either min term (sum-of-products) or max term (product-of-sums) Boolean expressions.



# KARNAUGH MAP

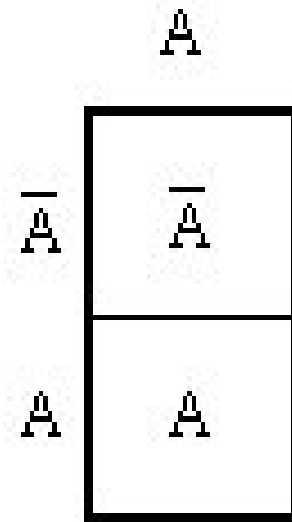


## Two- Variable, Three Variable and Four Variable Maps

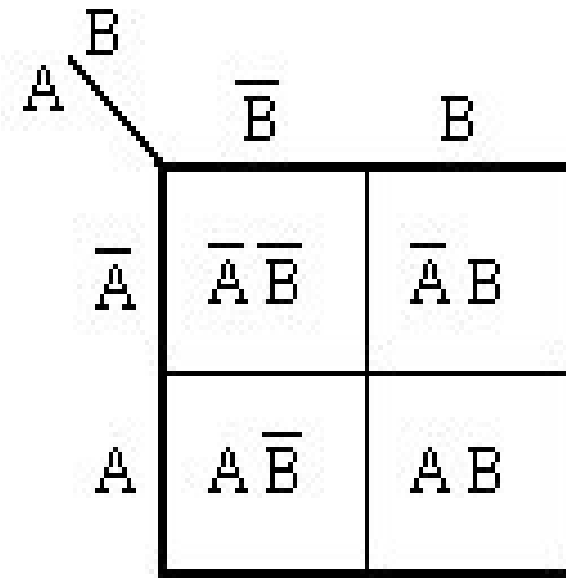
- Karnaugh maps can be used for expressions with two, three, four and five variables. The number of cells in a Karnaugh map is equal to the total number of possible input variable combinations as is the number of rows in a truth table.
- For three variables, the number of cells is  $2^3 = 8$ . For four variables, the number of cells is  $2^4 = 16$ .



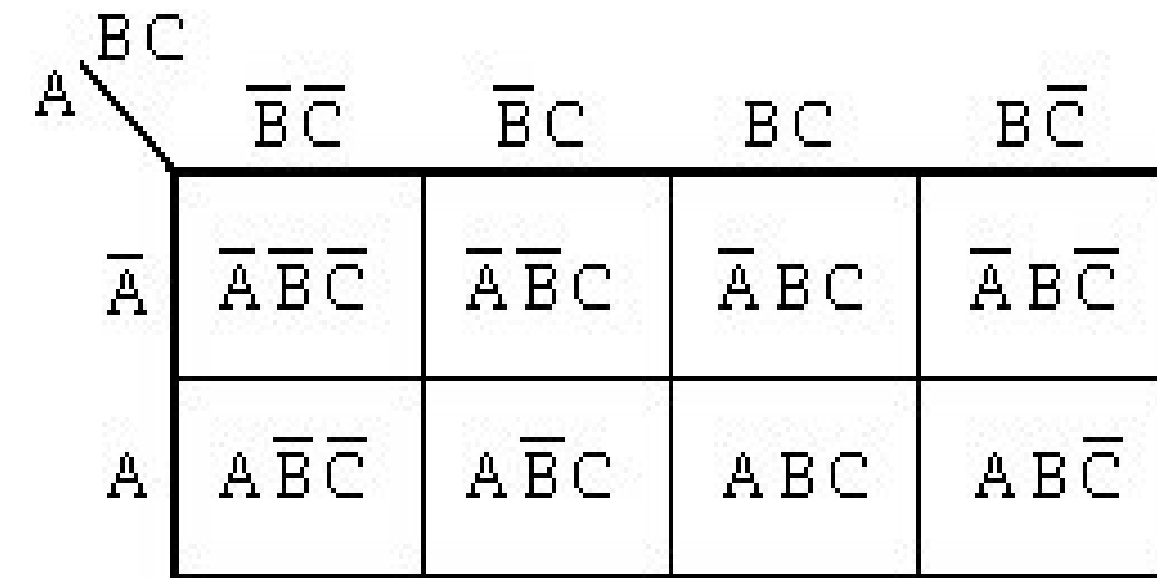
# KARNAUGH MAP VARIABLES



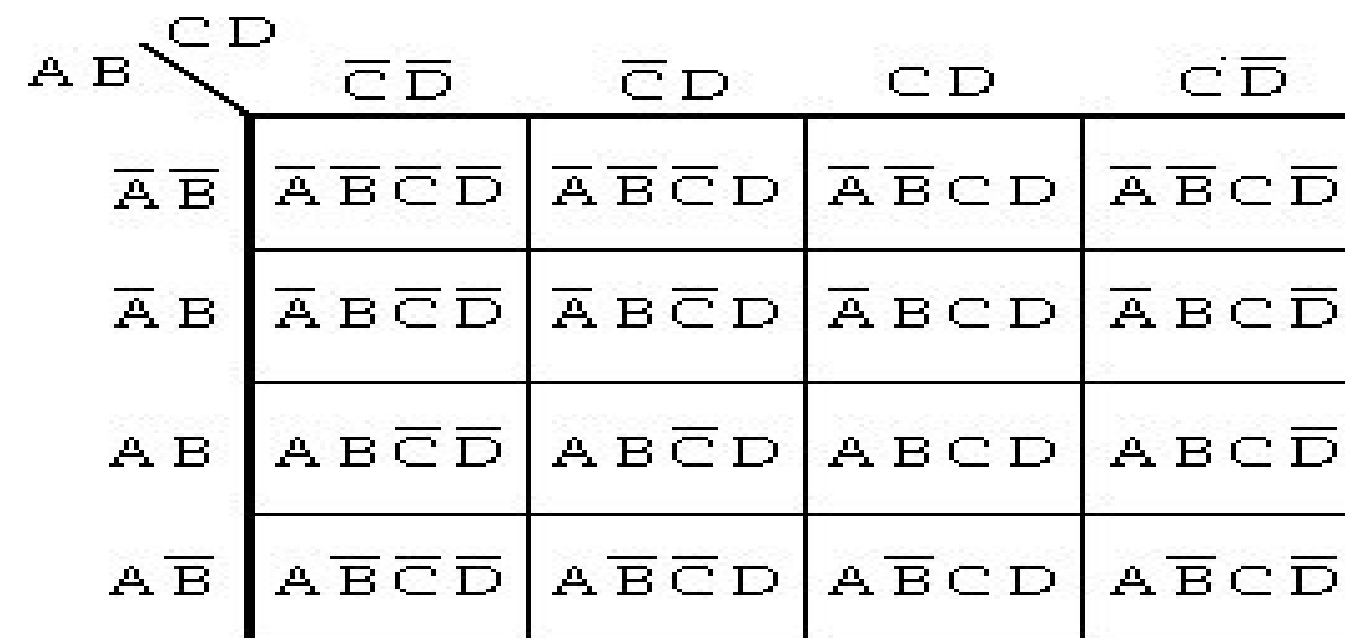
1-Variable map



2-Variable map



3-Variable map



4-Variable map



# KARNAUGH MAP VARIABLES

	0
0	m <sub>0</sub>
1	m <sub>1</sub>

1 -Variable map

		B	0	1
	A	0	m <sub>0</sub>	m <sub>1</sub>
	A	1	m <sub>2</sub>	m <sub>3</sub>

2 -Variable map

			Gray code Sequence			
			00	01	11	10
	A	BC	00	01	11	10
	0		m <sub>0</sub>	m <sub>1</sub>	m <sub>3</sub>	m <sub>2</sub>
	1		m <sub>4</sub>	m <sub>5</sub>	m <sub>7</sub>	m <sub>6</sub>

3 -Variable map

				Gray code Sequence			
				00	01	11	10
		A B	C D	00	01	11	10
	Gray code Sequence	00		m <sub>0</sub>	m <sub>1</sub>	m <sub>3</sub>	m <sub>2</sub>
		01		m <sub>4</sub>	m <sub>5</sub>	m <sub>7</sub>	m <sub>6</sub>
		11		m <sub>12</sub>	m <sub>13</sub>	m <sub>15</sub>	m <sub>14</sub>
		10		m <sub>8</sub>	m <sub>9</sub>	m <sub>11</sub>	m <sub>10</sub>

4 - Variable map



# KARNAUGH MAP GROUPING



## Grouping cells for Simplification:

- The grouping is nothing but combining terms in adjacent cells. The simplification is achieved by grouping adjacent 1's or 0's in groups of  $2^i$ , where  $i = 1, 2, \dots, n$  and  $n$  is the number of variables.
- When adjacent 1's are grouped then we get result in the sum of product form; otherwise we get result in the product of sum form.

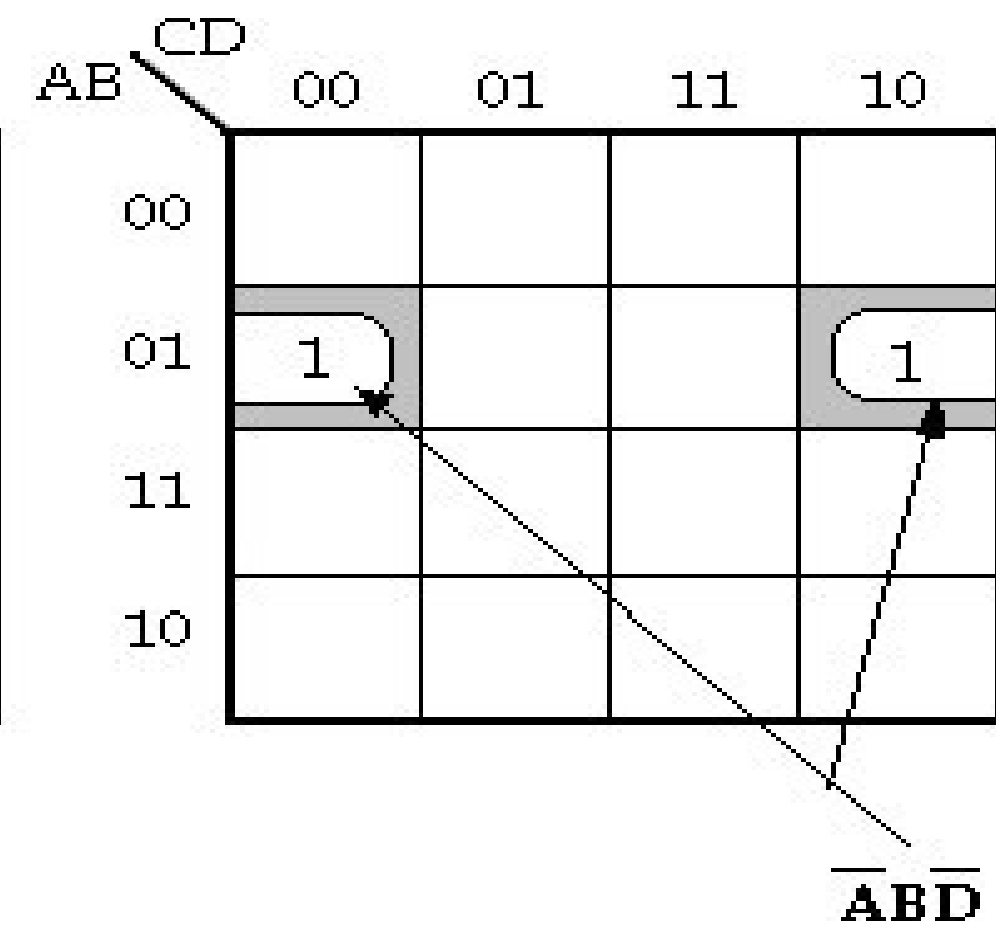
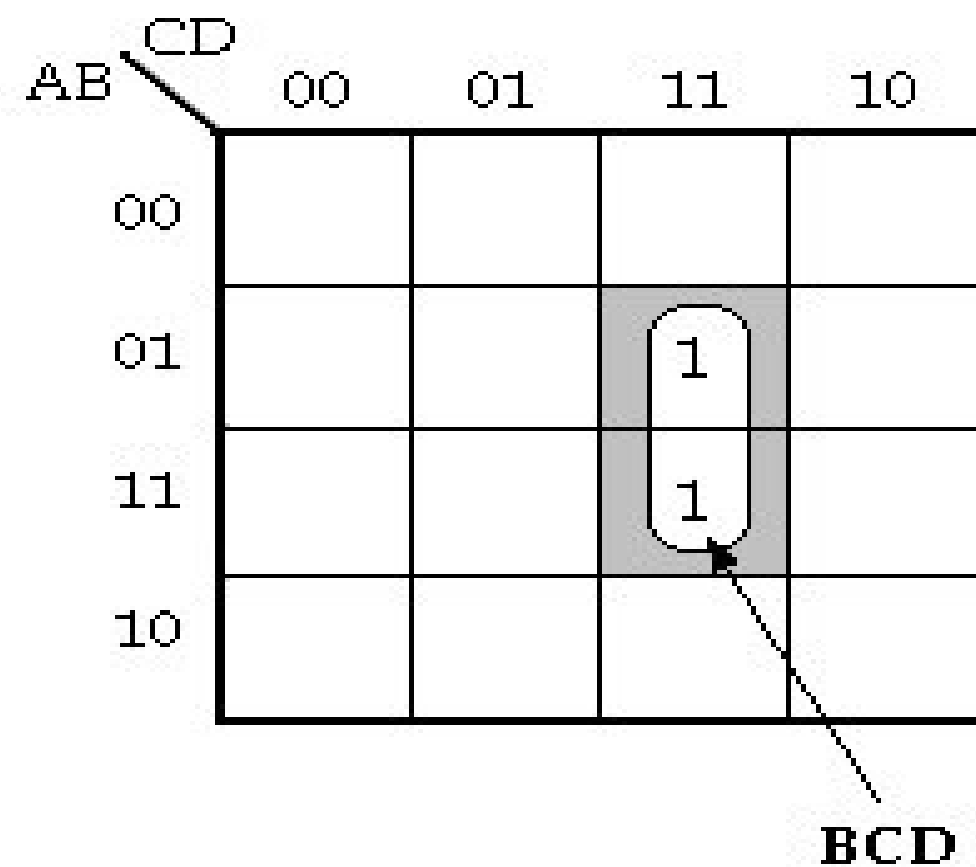
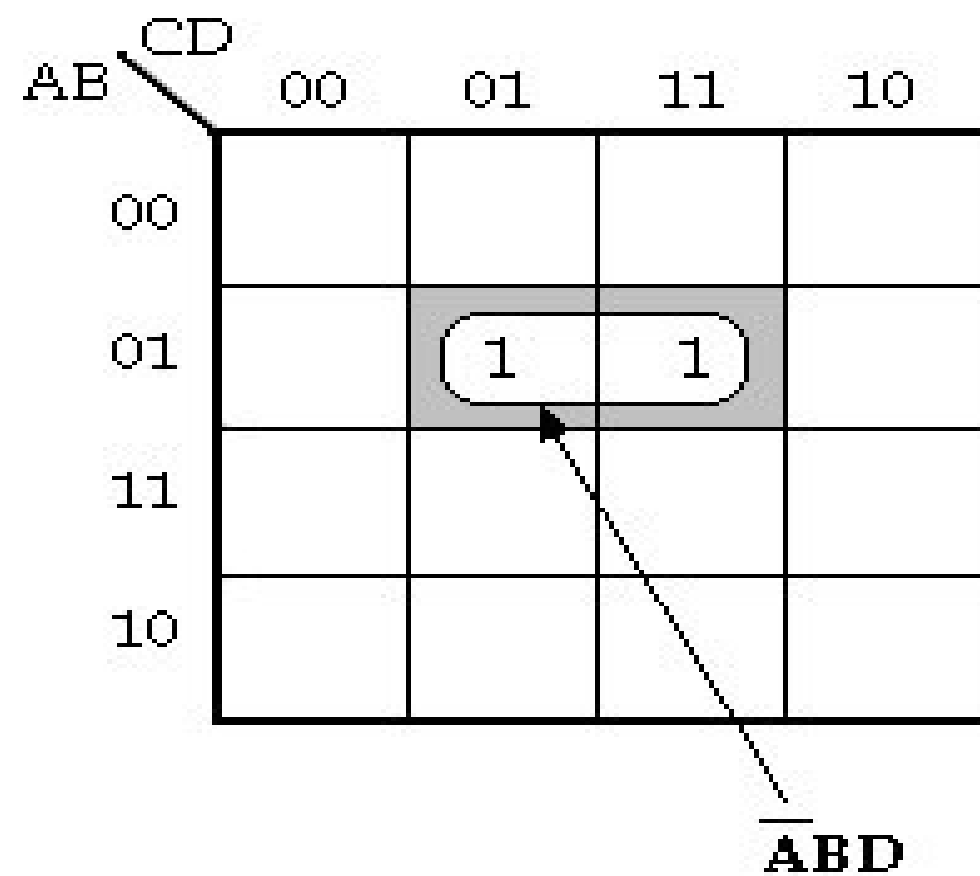


# KARNAUGH MAP GROUPING



## Grouping Two Adjacent 1's: (Pair)

In a Karnaugh map we can group two adjacent 1's. The resultant group is called Pair.



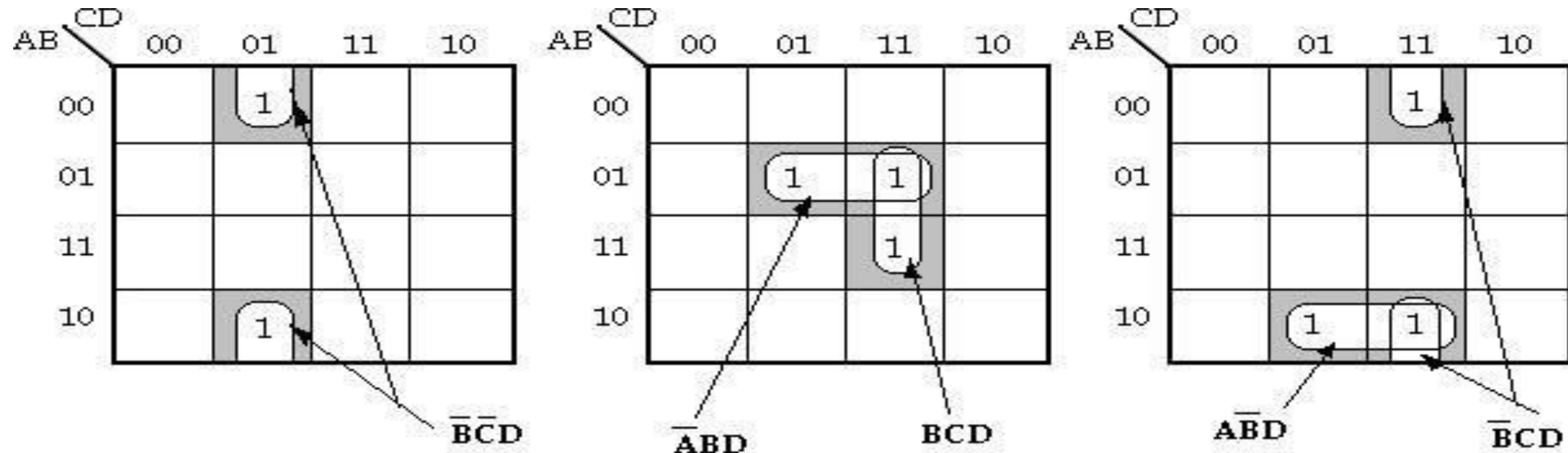


# KARNAUGH MAP GROUPING



## Grouping Two Adjacent 1's: (Pair)

In a Karnaugh map we can group two adjacent 1's. The resultant group is called Pair.





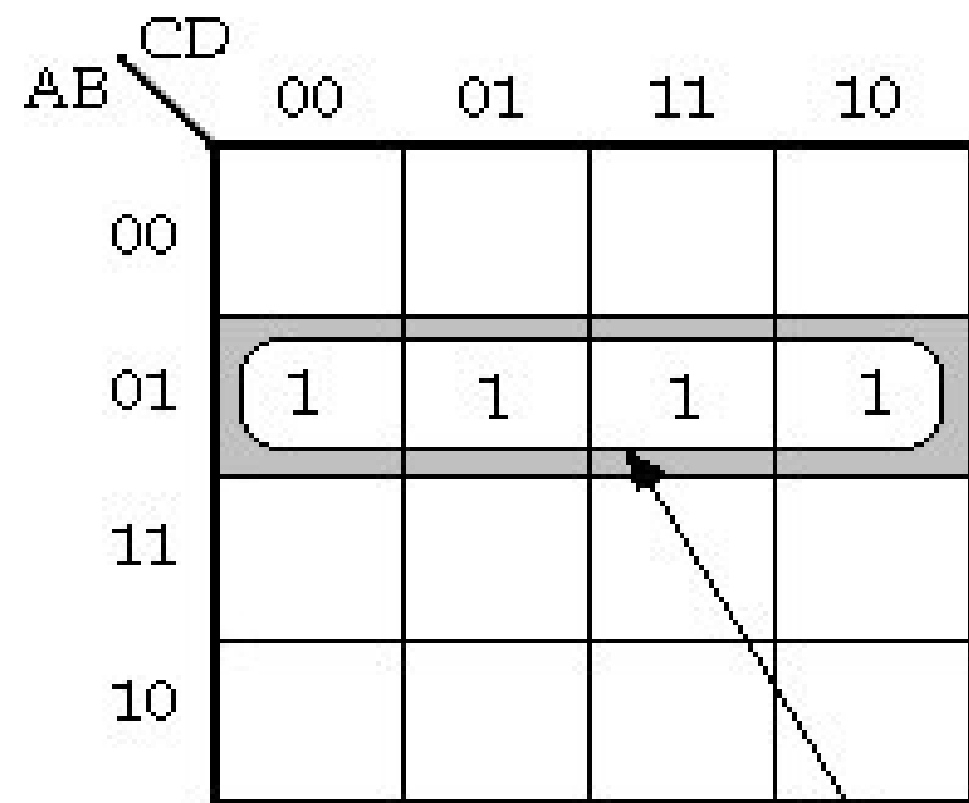


# KARNAUGH MAP GROUPING

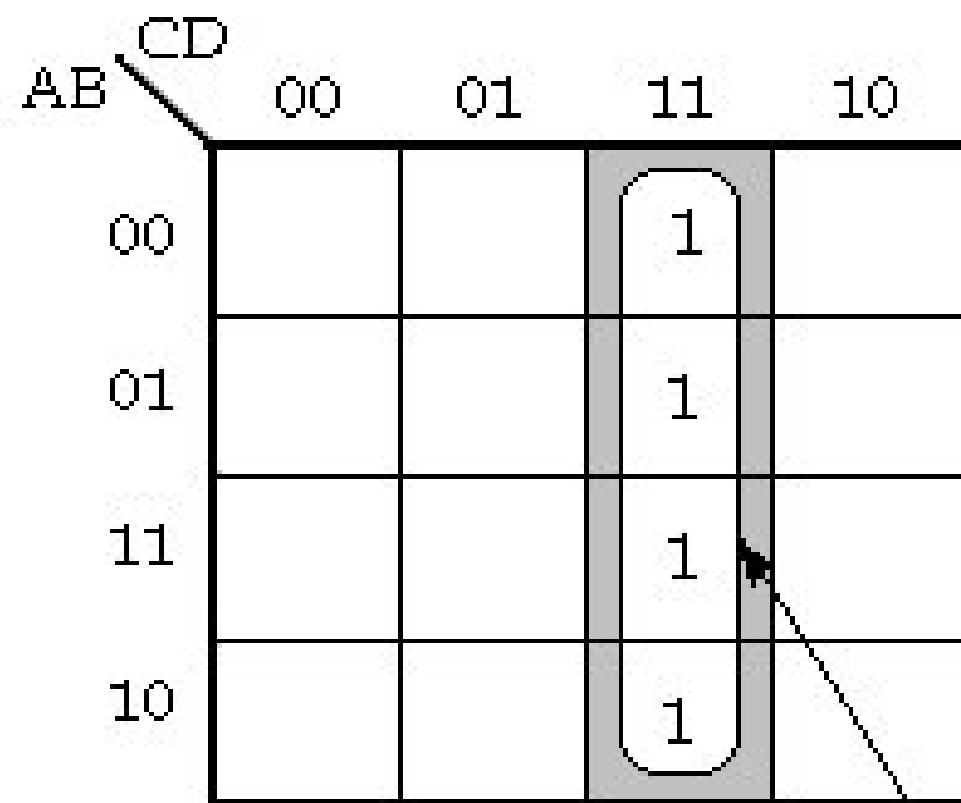


## Grouping Four Adjacent 1's: (Quad)

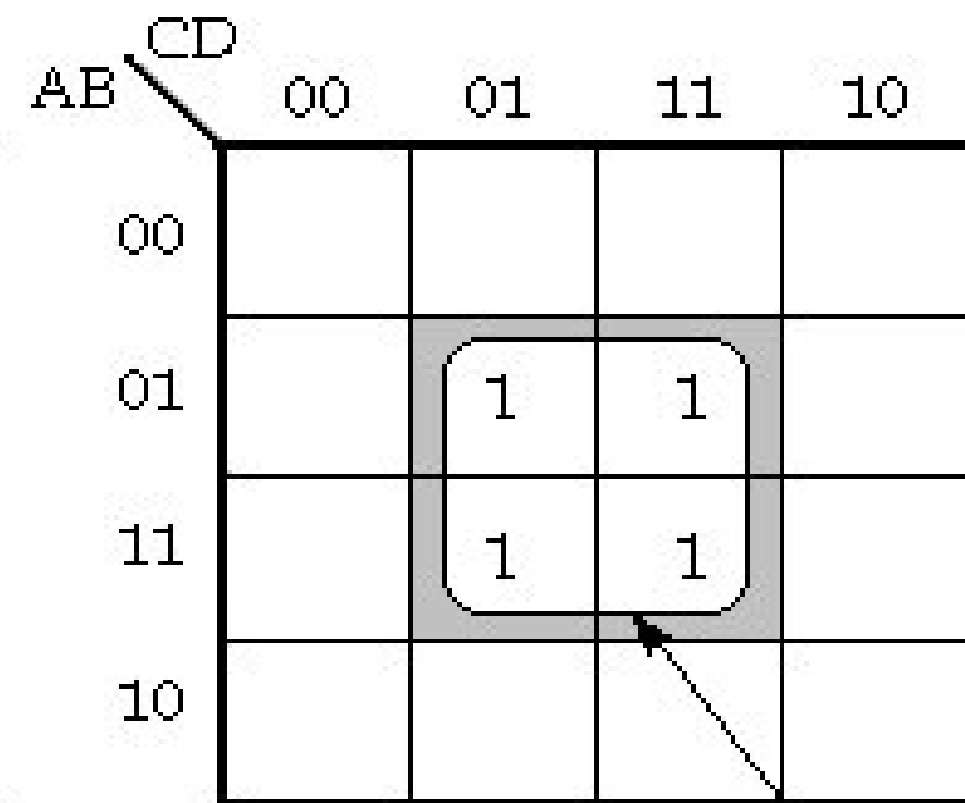
In a Karnaugh map we can group four adjacent 1's. The resultant group is called Quad.



(a)  $\bar{A}B$



(b)  $CD$



(c)  $BD$

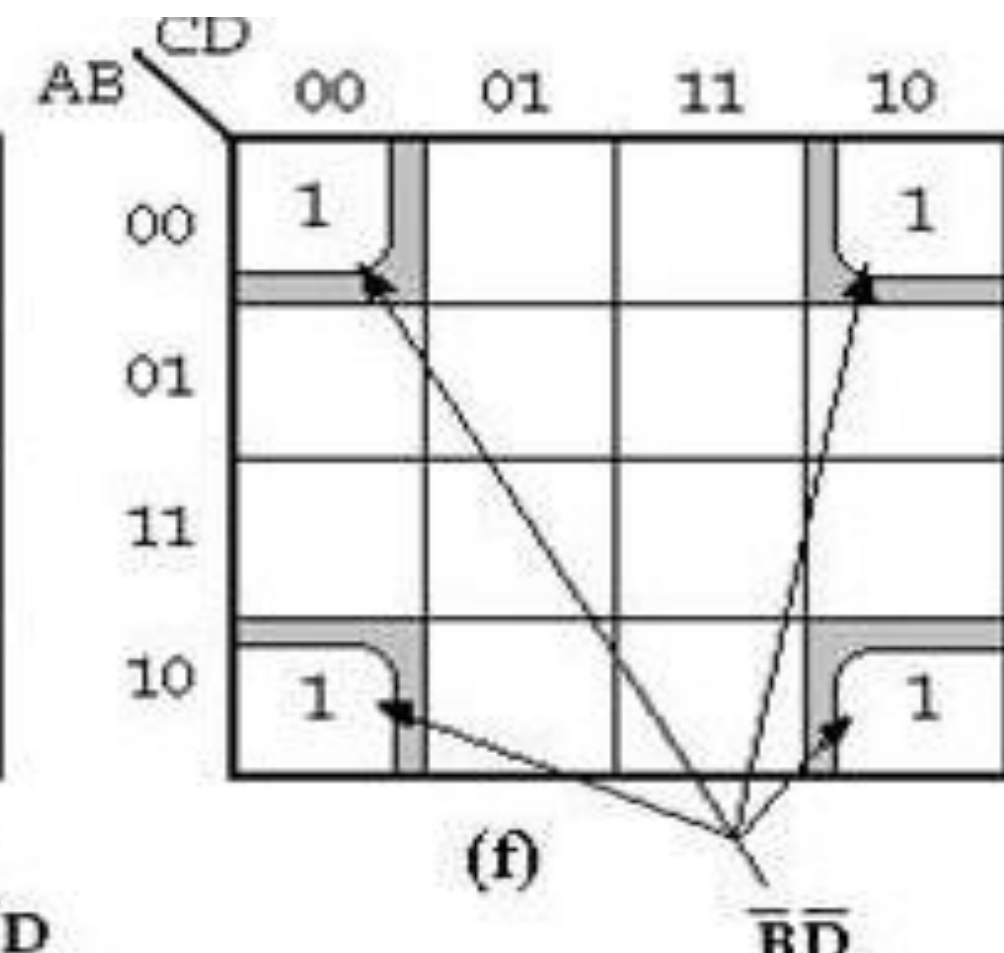
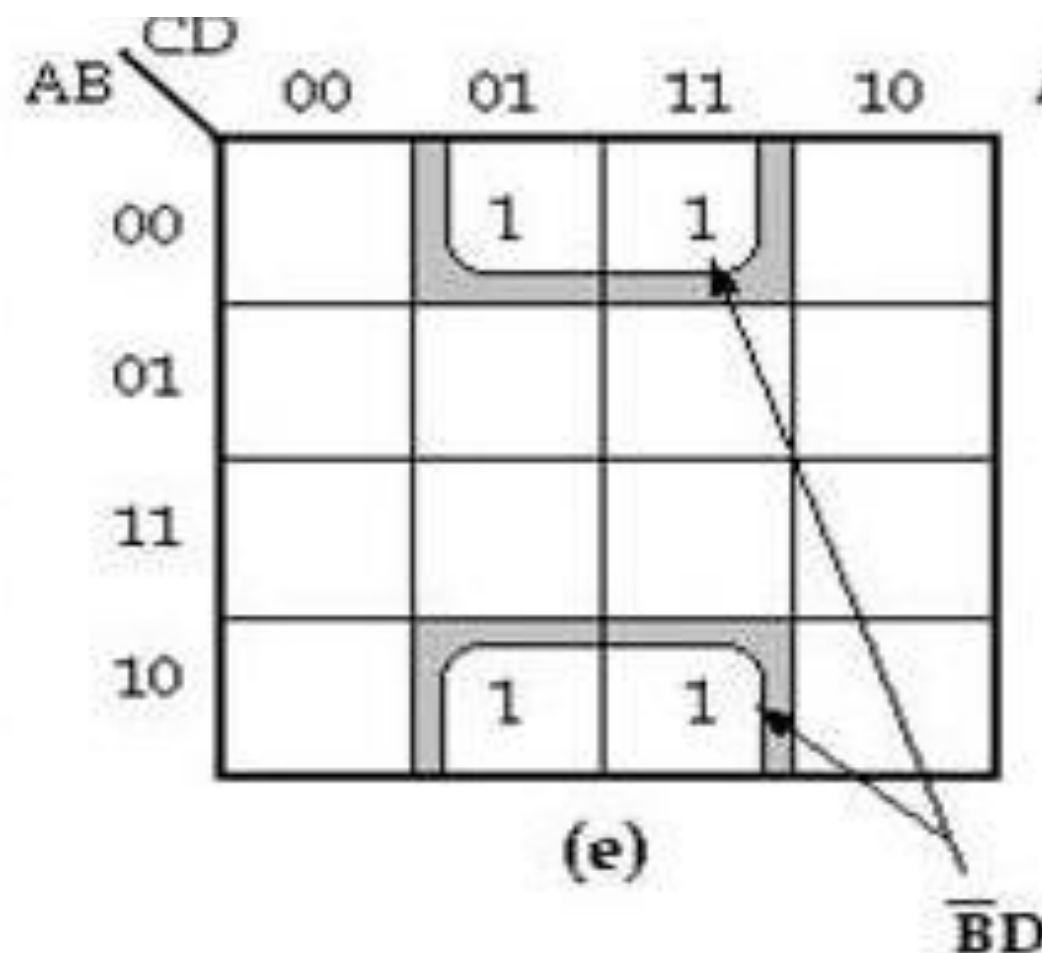
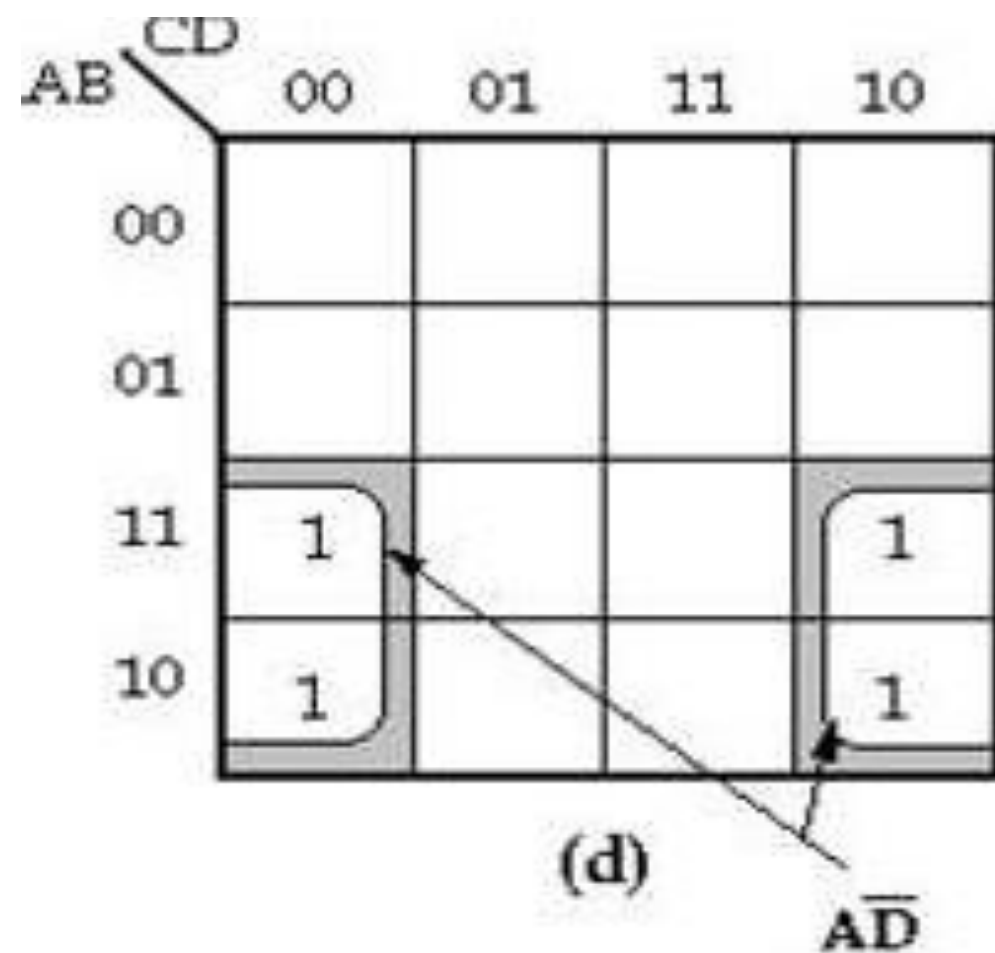


# KARNAUGH MAP GROUPING



## Grouping Four Adjacent 1's: (Quad)

In a Karnaugh map we can group four adjacent 1's. The resultant group is called Quad.



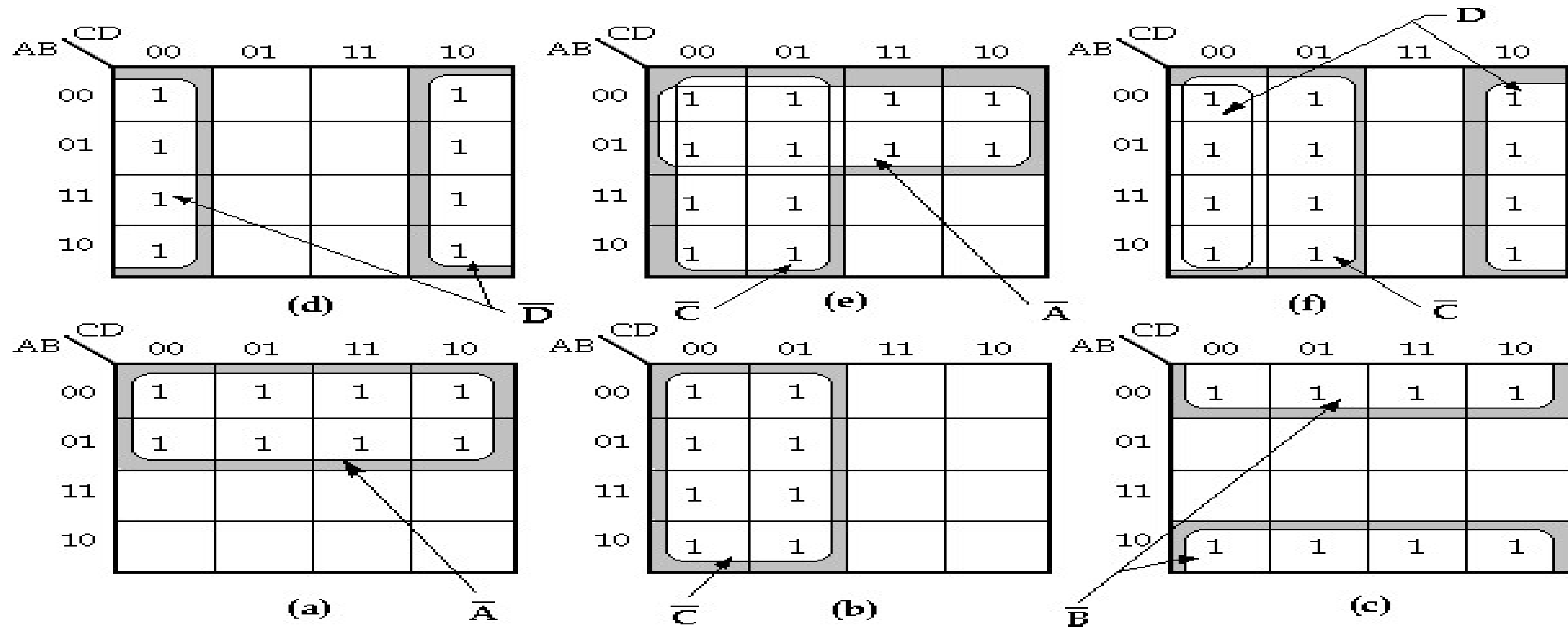


# KARNAUGH MAP GROUPING



## Grouping Eight Adjacent 1's: (Octet)

In a Karnaugh map we can group eight adjacent 1's. The resultant group is called Octet.





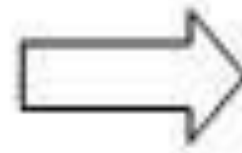
## KARNAUGH MAP PROBLEMS



Simplify the Boolean expression,

$$F(x, y, z) = \sum m (3, 4, 6, 7).$$

x \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	$yz$	$y\bar{z}$
	00	01	11	10
$\bar{x}$ 0	0 0	0 1	1 3	0 2
x 1	1 4	0 5	1 7	1 6



x \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	$yz$	$y\bar{z}$
	00	01	11	10
$\bar{x}$ 0	0	0	1	0
x 1	1	0	1	1

Groupings in the simplified map:  
- A group of two cells (1,3) and (1,7) is circled and labeled  $yz$ .  
- A group of two cells (1,4) and (1,6) is circled and labeled  $x\bar{z}$ .

$$F = yz + xz'$$



## KARNAUGH MAP PROBLEMS

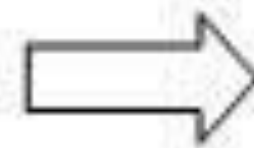


$$AB'C + A'B'C + A'BC + AB'C' + A'B'C'$$

$$= m_5 + m_1 + m_3 + m_4 + m_0$$

$$= \sum m(0, 1, 3, 4, 5)$$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	$BC$	$B\bar{C}$
	00	01	11	10
$\bar{A}$ 0	1 <sub>0</sub>	1 <sub>1</sub>	1 <sub>3</sub>	0 <sub>2</sub>
A 1	1 <sub>4</sub>	1 <sub>5</sub>	0 <sub>7</sub>	0 <sub>6</sub>



A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	$BC$	$B\bar{C}$
	00	01	11	10
$\bar{A}$ 0	1	1	1	0
A 1	1	1	0	0

Groupings in the simplified map:  
- A group of four 1s in the top row is labeled  $\bar{A}C$ .  
- A group of two 1s in the bottom-left cells is labeled  $\bar{B}$ .

$$F = A'C + B'$$



## ASSESSMENT



1. Define Karnaugh Map.
2. What is meant by Quad in K-Map Variable.
3. List the Variables of Karnaugh Map.
4. Group of Four adjacent 1's is called -----
5. In a Karnaugh map we can group eight adjacent 1's. The resultant group is called -----
6. For three variables, the number of cells is  $2^3 = \text{-----}$  . For four variables, the number of cells is  $2^4 = \text{-----}$
7. Simplify the Boolean expression:

$$Y = A'BC'D' + A'BC'D + ABC'D' + ABC'D + AB'C'D + A'B'CD'$$



# THANK YOU